HYDROMAGNETIC OSCILLATORY FLOW PAST A NON-CONDUCTING INFINITE POROUS FLAT PLATE WITH VARIABLE SUCTION

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Unsteady laminar boundary layer flow of a viscous incompressible and electrically conducting fluid past a non-conducting infinite porous flat plate with variable suction in the presence of a uniform transverse magnetic field is considered. General formulae for velocity and skin-friction are obtained under the assumption that the magnetic Reynolds number is small. The dependence of velocity and skin-friction on Hartmann number (M) is discussed graphically.

Stuart[1] has studied the flow of a viscous incompressible fluid past an infinite porous flat plate subjected to a constant uniform suction when the free stream velocity oscillates in magnitude but not in direction. The corresponding problem when the fluid is electrically conducting and the applied magnetic field is normal to the plate has been discussed by Suryaprakasarao[2] for small magnetic Reynolds number and with no applied external electric field.

The purpose of this paper is to study the effect of magnetic field on skin-friction and velocity profile in case of oscillatory flow of viscous conducting fluid past a non-conducting infinite porous flat plate with variable suction. The variable suction velocity is taken to be of the form $v'_0 \ [1+\epsilon A e^{i\omega' t'}]$, where v_0' is a non-zero negative constant mean suction velocity, ϵ is taken to be small and A is a positive constant such that $\epsilon A \leqslant 1$, as has been considered recently by Messiha[3] who limited his problem only to non-conducting fluid in the absence of magnetic field.

General formulae for velocity and skin-friction are obtained for small magnetic Reynolds number. Graphs showing variations of velocity profile and fluctuating parts of velocity profile against the distance from the plate, phase and amplitude of skin-friction against frequency parameter are plotted for $A = 0, 1$ and Hartmann number (M) equal to 0 and 3.

It may be pointed out that the previous results[1—3] can be derived from our results as particular cases. In this paper we have not discussed the temperature field variation which will be taken up in a subsequent paper.

BASIC EQUATIONS AND THEIR SOLUTION

Two-dimensional motion has been considered in which we take x'-axis along the plate and y'-axis perpendicular to it, u' and v' are the corresponding velocity components. Since the plate, which is assumed to be non-magnetic and non-conducting, is infinite, all the physical variables, except pressure, depend on y' only. The suction velocity normal to the plate varies periodically with time about a non-zero constant mean value v'_0. The momentum and continuity equations relevant to the problem are

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = - \frac{1}{\rho} \frac{\partial p}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u', \qquad (1)$$

$$\frac{\partial v'}{\partial y'} = 0, \qquad (2)$$

where t', ρ, σ, p, ν and B_0 are the time, the density, the electrical conductivity, the pressure, the kinematic coefficient of viscosity and the magnetic induction respectively.

In deriving (1), as in reference[2], it is assumed that the magnetic Reynolds number is small so that the induced magnetic field is negligible in comparison with the imposed magnetic field. Further, since no external electric field is applied, the effect of polarization of ionized fluid is negligible, hence it can be assumed that the electric field is zero.

From (2) it is clear that v' is a function of time only and following Messiha we may take

$$v' = v_0' [1 + \epsilon A e^{i\omega' t'}], \qquad (3)$$

where A is a real positive constant and ϵ is small such that $\epsilon A \leqslant 1$.

From (1) one gets, for the main stream velocity

$$\frac{dU'}{dt'} = - \frac{1}{\rho} \frac{\partial p}{\partial x'} - \frac{\sigma B^2_0}{\rho} U', \qquad (4)$$

where U' (t) is the free stream velocity.

Using (3) and (4), (1) becomes

$$\frac{\partial u'}{\partial t'} + v_0' \left[1 + \epsilon A e^{i\omega' t'} \right] \frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + \nu \frac{\partial^2 u'}{\partial y'^2} + \frac{\sigma B_0^2}{\rho} \left(U' - u' \right), \qquad (5)$$

with the boundary conditions

$$u' = 0 \text{ at } y' = 0 \text{ and } u' \rightarrow U' \text{ as } y' \rightarrow \infty .$$

We now define the following non-dimensional parameters

$$y = \frac{y' \mid v_0' \mid}{\nu}, \; t = \frac{v_0'^2 t'}{4\nu}, \; \omega = \frac{4\nu\omega'}{v_0'^2}, \; u = \frac{u'}{U_0'}, \; U = \frac{U'}{U_0'},$$

where U'_0 is a reference velocity and ω' is the frequency.

Using the non-dimensional parameters, (5) becomes

$$\frac{\partial^2 u}{\partial y^2} + [1 + \epsilon A e^{i\omega t}] \frac{\partial u}{\partial y} - \frac{1}{4} \frac{\partial u}{\partial t} + \frac{1}{4} \frac{dU}{dt} + \frac{M}{4} \left(U - u \right) = 0 , \qquad (6)$$

where $M = \dfrac{4\nu\sigma B_0^2}{\rho v_0'^2}$ is the Hartmann number. The boundary conditions reduce to

$$u = 0 \text{ at } y = 0 \text{ and } u \to U \quad \text{as } y \to \infty .$$

Let us now consider a periodic free stream velocity of the form

$$U(t) = 1 + \epsilon e^{i\omega t}, \tag{7}$$

and let the velocity in the neighbourhood of the plate be

$$u(y,t) = \{1 - f_1(y)\} + \epsilon\{1 - f_2(y)\} e^{i\omega t}. \tag{8}$$

Substituting these values of U and u in (6) and comparing harmonic terms, neglecting coefficient of ϵ^2, we get

$$\frac{d^2 f_1}{dy^2} + \frac{df_1}{dy} - \frac{M}{4} f_1 = 0, \tag{9}$$

$$\frac{d^2 f_2}{dy^2} + \frac{df_2}{dy} - \frac{1}{4}(M + i\omega) f_2 = -A \frac{df_1}{dy}, \tag{10}$$

with the boundary conditions

$$\left.\begin{array}{ll} f_1 = 1,\ f_2 = 1 & \text{at } y = 0 \\ f_1 \to 0,\ f_2 \to 0 & \text{as } y \to \infty \end{array}\right\} \tag{11}$$

The solutions of (9) and (10), satisfying the prescribed boundary conditions (11), are

$$f_1 = e^{-ny}, \tag{12}$$

$$f_2 = (1 - S) e^{-ny} + S e^{-hy}, \tag{13}$$

where $\quad S = 1 - \dfrac{4inA}{\omega},\ n = \dfrac{1}{2}\left[1 + (1 + M)^{\frac{1}{2}}\right]$

and $\quad h = h_r + ih_i = \dfrac{1}{2}\left[1 + (1 + M + i\omega)^{\frac{1}{2}}\right],$

$$= \frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}\left\{\left((1+M)^2 + \omega^2\right)^{\frac{1}{2}} + (1 + M)\right\}\right]^{\frac{1}{2}}$$

$$+ \frac{i}{2}\left[\frac{1}{2}\left\{\left((1+M)^2 + \omega^2\right)^{\frac{1}{2}} - (1 + M)\right\}\right]^{\frac{1}{2}} \tag{14}$$

Hence the expression for velocity profile is given by

$$u\,(y, t) = 1 - e^{-ny} + \epsilon \left\{ 1 - (1 - S)\, e^{-ny} - Se^{-hy} \right\} e^{i\omega t}. \tag{15}$$

If we take

$$\left.\begin{aligned} M_r &= 1 - e^{-h_r y} \left\{ \cos(h_i\, y) - \frac{4nA}{\omega} \sin(h_i\, y) \right\}, \\ M_i &= -\frac{4nA}{\omega}\, e^{-ny} + e^{-h_r y} \left\{ \frac{4nA}{\omega} \cos(h_i\, y) + \sin\,(h_i y) \right\} \end{aligned}\right\} \tag{16}$$

where h_r and h_i are given by (14), the equation (15) reduces to

$$u\,(y,t) = 1 - e^{-ny} + \epsilon\,\{M_r \cos \alpha t - M_i \sin \omega t\} \tag{17}$$

The expression for non-dimensional skin-friction τ_0 is given by

$$\begin{aligned} \tau_0 &= n + \epsilon\,(n - ns + hs)\, e^{i\omega t}, \\ &= n + \epsilon\, |\, B \,|\, \cos\,(\omega t + \alpha), \end{aligned} \tag{18}$$

where $\tan \alpha = B_i/B_r$ and

$$\begin{aligned} B &= B_r + iB_i = n - ns + hs, \\ &= \left\{ h_r + \frac{4nA}{\omega} h_i \right\} + i \left\{ h_i + \frac{4nA}{\omega} (n - h_r) \right\} \end{aligned} \tag{19}$$

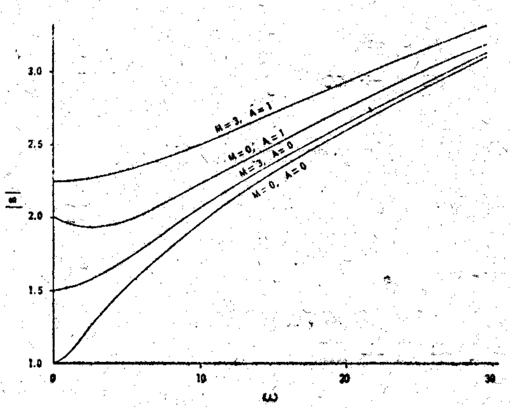

Fig. 1—Variation of skin-friction amplitude against frequency parameter for $A=0, 1$ and $M=0, 3$.

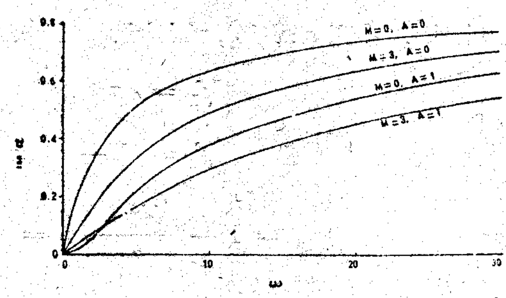

Fig. 2—Variation of skin-friction phase against frequency parameter for $A=0, 1$ and $M=0, 3$.

CONCLUSION

Fig. 1 and 2 show respectively the variation of skin-friction amplitude and phase with ω for $A = 0, 1$ and $M = 0, 3$. The variation of fluctuating part of velocity profile with y for $\omega = 1$, $A = 0, 1$ and $M = 0, 3$ is shown in Fig. 3 and 4. Fig. 5 and 6 give the variation of transient velocity profile with y for $\omega = 1$, $\omega t = \frac{\pi}{2}$, $\epsilon = 0{\cdot}284$, $A = 0$, and $M = 0, 3$.

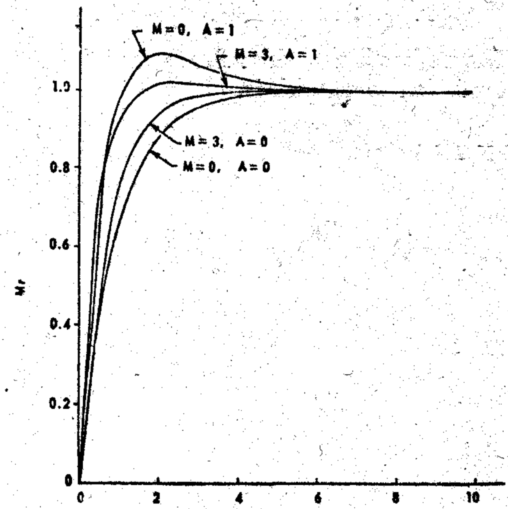

Fig. 3—Variation of fluctuating part of velocity profile (M_r) for $\omega=1$, $A=0$, 1 and $M=0$, 3.

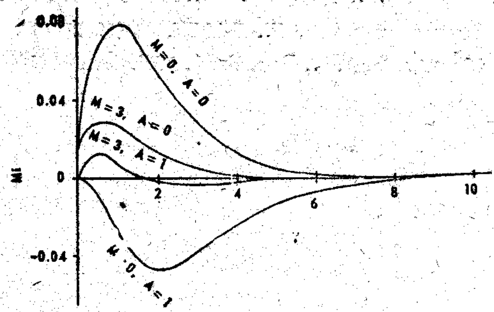

Fig. 4—Variation of fluctuating part of velocity profile (M_i) for $\omega=1$, $A=0$, 1 and $M=0$, 3.

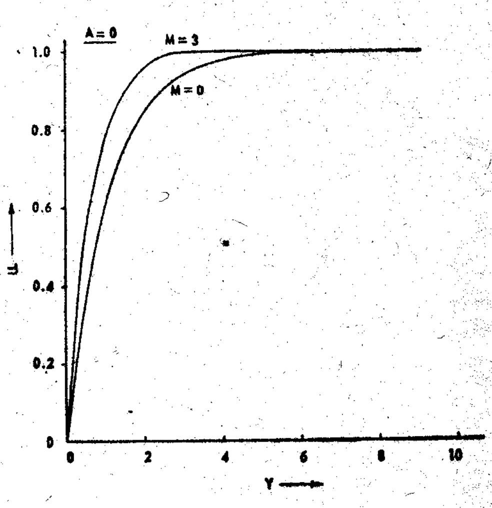

Fig. 5—Variation of velocity profile $u=1-e^{-ny}-\varepsilon M_i$ for $\omega=1$, $\omega t=\pi/2$, $\varepsilon=0.284$, $A=0$ and $M=0$, 3.

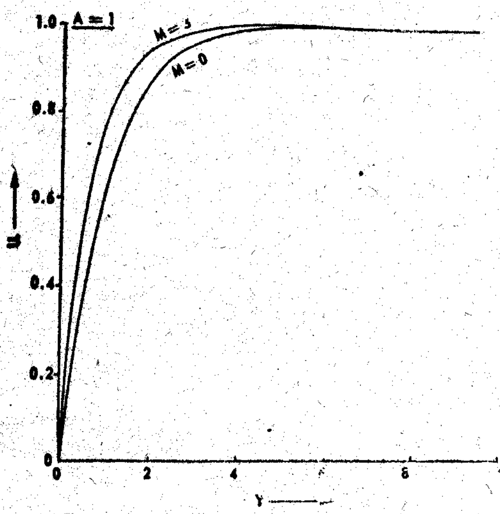

Fig. 6—Variation of velocity profile $u=1-e^{-ny}-\varepsilon M_i$ for $\omega=1$, $\omega t=\pi/2$, $\varepsilon=0.284$, $A=1$ and $M=0$, 3.

In case of constant suction velocity ($A=0$), it is clear from the graphs that the amplitude of skin-friction fluctuations, the transient velocity profile and M_r increase with M while the phase of skin-friction fluctuations and M_i decrease. On the other hand when the suction velocity is variable (*e.g.*, $A=1$) it is seen that the amplitude of the skin-friction fluctuations, the transient velocity profile and M_i increase with M. The phase of skin-friction fluctuations increases with M for small values of frequency while near the plate, M_r increases with M.

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