INERTIA EFFECTS OF SQUEEZE FILMS BETWEEN TWO PARALLEL PLATES

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In this paper, a theoretical study of inertia effects in squeeze films between rectangular and elliptical plates in the presence of transverse magnetic field has been made. It is shown that lead capacity and time of approach, for sufficiently large M_0, increase as the inertia increases sufficiently.

NOMENCLATURE

a—major axis of the ellipse

b—minor axis of the ellipse

B^*—length of the plate

h—fluid film thickness

h_0—initial film thickness

H_0—applied magnetic field

H_x—induced magnetic field in x-direction

H_z—induced magnetic field in z-direction

L—width of the plate

M_0—initial Hartmann number $(\sigma\mu_e\ h_0^2 H_0^2/\mu)^{\frac{1}{2}}$

p—pressure inside the bearing

p_0—pressure inside the bearing when inertia effects are neglected

p_c—correction term in pressure due to inertia

$\bar{t}_1$ or $\bar{t}_2$—time of approach for infinite rectangular or finite elliptical plates

u, v, w—velocity components in x, y and z directions respectively

$\overline{W}_1$ or $\overline{W}_2$—load capacity for infinite rectangular or finite elliptical plates.

It has been found that liquid metals can be used in high temperature bearings[1] because they have several advantages over the conventional lubricants. The most obvious one is, of course, due to their high thermal conductivity which helps in maintaining a more uniform temperature and thereby a constant viscosity. But one disadvantage of using liquid metals as lubricants is the low load capacity which can be achieved in comparison to the conventional lubricants by classical methods. This difficulty can be removed by the application of magnetic field as the liquid metals are good conductors of electricity.

Recently Kuzma[2] & Shukla[3] have studied the hydromagnetic squeeze films between two plane or curved surfaces in the presence of a transverse magnetic field and have shown that load capacity and time of approach can be increased by increasing the strength of the magnetic field. They considered the plates to be infinite in length and neglected the induced magnetic field.

Later on Kuzma, Maki & Donnelly[4] studied experimentally the problem of squeeze films between two circular plates in the presence of axial magnetic field and compared their results with theoretical one. Then Shukla & Prakash[5] studied again the problem of squeeze films between rectangular and elliptical plates in the presence of magnetic field including the effects of induced field.

In this paper, the problems of squeeze films between infinite rectangular and finite elliptical plates have been studied theoretically considering the effects of inertia in the presence of axial magnetic field. It is seen that in this case the load capacity and time approach is sufficiently increased in comparison to the values computed without inertia.

BASIC EQUATIONS

The configuration of the bearing is shown in Fig. 1. The coordinate system is fixed in the bearing and applied magnetic field is in y direction.

The basic equations governing the hydromagnetic flow of a lubricant are :

$$\left.\begin{aligned} \rho \frac{D\bar{v}}{Dt} + \rho(\bar{v}.\nabla)\bar{v} &= -\nabla p + \mu\nabla^2\bar{v} + \mu_e \bar{J}\times\bar{H} \\ \text{curl}(\bar{v}\times\bar{H}) + \eta\nabla^2 H &= 0 \\ \nabla.\bar{H} &= 0 \\ \nabla.\bar{v} &= 0 \end{aligned}\right\} \quad (1)$$

In order to approximate effective inertia, consider the usual assumptions of ordinary hydrodynamic lubrication[6], vertical velocities are ignored[7]. The following assumptions regarding magnetic field are made :

(*i*) The Lorentz force in the only dominating force due to presence of magnetic field.

(*ii*) The permeability and conductivity are constant scalar quantities.

(*iii*) The induced magnetic field is small in comparison with the applied magnetic field H_0 such that their squares and higher powers can be neglected.

Under these assumptions equations (1) are :

$$\rho\left(u\frac{\partial u}{\partial x} + \omega\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} + \mu_e\left(H_0\frac{\partial H_x}{\partial y}\right) \quad (2)$$

$$\frac{\partial p}{\partial y} = 0 \text{ implies } p = p(x, z) \quad (3)$$

$$\rho\left(u\frac{\partial \omega}{\partial x} + \omega\frac{\partial \omega}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\frac{\partial^2 \omega}{\partial y^2} + \mu_e\left(H_0\frac{\partial H_z}{\partial y}\right) \quad (4)$$

$$H_0\frac{\partial u}{\partial y} + \eta\frac{\partial^2 H_x}{\partial y^2} = 0 \quad (5)$$

$$H_0\frac{\partial \omega}{\partial y} + \eta\frac{\partial^2 H_z}{\partial y^2} = 0 \quad (6)$$

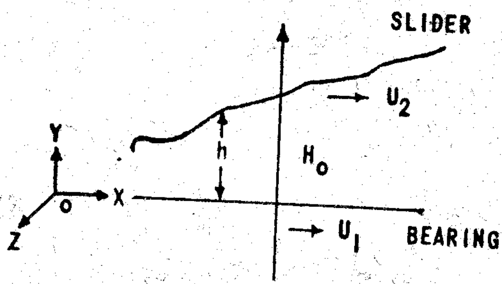

Fig. 1—Configuration of the magnetohydrodynamic slider bearing

We now solve this system of equations under the following boundary conditions:

$$u = U_1 \;, \quad \omega = 0 \;, \quad \text{at} \;\; y = 0 \;,$$
$$u = U_2 \;, \quad \omega = 0 \quad \text{at} \;\; y = h \tag{7}$$

and

$$H_x = 0 = H_z \quad \text{at} \;\; y = 0 \,,\, h \tag{8}$$

using the usual perturbation technique for which we assume

$$\left.\begin{aligned} u &= u_c + u_0 \;, \\ \omega &= \omega_c + \omega_0 \;, \\ p &= p_c + p_0 \;, \end{aligned}\right\} \tag{9}$$

where

u_0, ω_0, p_0 — are velocities in x and z directions and pressure when inertia effects are neglected. These have been found by Shukla & Prakash[5],

u_c, ω_c, p_c— are correction terms on account of inertia effects,

and u, ω, p — are solutions in which both viscous and inertia forces are present.

The solutions are found to be given by

$$u = u_0 + u_c$$

$$= \frac{h_0}{\mu M_0}\left\{\frac{h}{2}\frac{\partial p_0}{\partial x} - \frac{h_0}{M_0}\left(\frac{1-\cosh\frac{M_0 h}{h_0}}{\sinh\frac{M_0 h}{h_0}}\right)\frac{\partial p_c}{\partial x}\right\} A$$

$$+\frac{\rho h\; h_0^4}{24\mu^3 M_0^4}\cdot\left(\frac{\partial p_0}{\partial x}\right)\cdot\left(h\frac{\partial^2 p_0}{\partial x^2} + h'\frac{\partial p_0}{\partial x}\right)\left[\frac{\sinh^2\frac{M_0 h}{h_0}}{\left\{1-\cosh\frac{M_0 h}{h_0}\right\}^2}\cdot\left\{B+8C-\frac{6M_0}{h_0}D-9E\right\}\right.$$

$$\left.-2\cdot\frac{\sinh\frac{M_0 h}{h_0}}{\left(1-\cosh\frac{M_0 h}{h_0}\right)}\cdot\left\{\frac{3M_0}{h_0}F-G\right\}+\left(B-4C+3D\right)\right]$$

$$-\frac{\rho\, h'\, h^2 h_0^3}{24\mu^3 M_0^3}\left(\frac{\partial p_0}{\partial x}\right)^2\left[\frac{\sinh\frac{M_0 h}{h_0}}{\left(1-\cosh\frac{M_0 h}{h_0}\right)}\cdot\left\{B+8C-\frac{6M_0}{h_0}D-9E\right\}-\left\{\frac{\frac{3M_0}{h_0}F-G}{1-\cosh\frac{M_0 h}{h_0}}\right\}\right]$$

$$+\frac{\rho\,(U_1-U_2)\,hh'\,h_0^2}{24\mu^2 M_0^2}\left(\frac{1+\cosh\frac{M_0 h}{h_0}}{\sinh\frac{M_0 h}{h_0}}\right)\left[\frac{\sinh\frac{M_0 h}{h_0}}{\left(1-\cosh\frac{M_0 h}{h_0}\right)}\cdot\left\{\frac{3M_0}{h_0}F-G\right\}-(B-3E-8C)\right]$$

$$+\frac{\rho(U_1-U_2)\,h_0^3}{24\mu^2 M_0^3}\left(h\frac{\partial^2 p_0}{\partial x^2}+\frac{h'2p_0}{\partial x}\right)\left[\frac{\sinh\frac{M_0h}{h_0}}{\left(1-\cosh\frac{M_0h}{h_0}\right)}\left\{\frac{3M_0D}{h_0}-2C-B-3E\right\}-\frac{3M_0}{h_0}\times\right.$$

$$\left.\left\{\frac{1+\cosh\frac{M_0h}{h_0}}{1-\cosh\frac{M_0h}{h_0}}\right\}F-2\cdot\frac{\cosh\frac{M_0h}{h_0}}{\left(1-\cosh\frac{M_0h}{h_0}\right)}\cdot G+\frac{1+\cosh\frac{M_0h}{h_0}}{\sinh\frac{M_0h}{h_0}}\cdot\left\{B+3E-4C\right\}\right]$$

$$-\frac{\rho hh'h_0^2(U_1-U_2)}{24\mu^2 M_0^2}\cdot\frac{\partial p_0}{\partial x}\left[\left\{\frac{\frac{3M_0}{h_0}D-B+3E-2C}{1-\cosh\frac{M_0h}{h_0}}\right\}-\frac{1}{\sinh\frac{M_0h}{h_0}}\left(\frac{1+\cosh\frac{M_0h}{h_0}}{1-\cosh\frac{M_0h}{h_0}}\right)\cdot\right.$$

$$\left.\left\{\frac{6M_0}{h_0}F-G\right\}\right]$$

$$+\frac{\rho(U_1-U_2)^2 h_0h'}{24\mu M_0}\left(\frac{1+\cosh\frac{M_0h}{h_0}}{\sinh\frac{M_0h}{h_0}}\right)\left[G-\frac{1+\cosh\frac{M_0h}{h_0}}{\sinh\frac{M_0h}{h_0}}\cdot\left\{B+3E-4C\right\}\right]$$

$$-\frac{\rho(U_1+U_2)h_0^2}{8\mu M_0^2}\left[\frac{h_0}{2\mu M_0}\left(h\frac{\partial^2 p_0}{\partial x^2}+\frac{h'\partial p_0}{\partial x}\right)\left\{\frac{\sinh\frac{M_0h}{h_0}}{\left(1-\cosh\frac{M_0h}{h_0}\right)}\cdot\left(\frac{2M_0}{h_0}D+4E-4C\right)+\frac{2M_0}{h_0}F\right\}\right.$$

$$\left.-\frac{\rho hh'}{2\mu^2}\cdot\frac{\partial p_0}{\partial x}\cdot\frac{\left\{\frac{2M_0}{h_0}D+4E-4C\right\}}{\left(1-\cosh\frac{M_0h}{h_0}\right)}-(U_1-U_2)\frac{hM_0^2}{h_0^2}\left(\frac{1+\cosh\frac{M_0h}{h_0}}{\sinh\frac{M_0h}{h_0}}\right)F\right]$$

$$+\frac{U_1-U_2}{2}H^*+\frac{U_1+U_2}{2} \tag{10}$$

$$\omega=\omega_0+\omega_c$$

$$=\frac{h_0}{\mu M_0}=\left\{\frac{h}{2}\frac{\partial p_0}{\partial z}-\frac{h_0}{M_0}-\left(\frac{1-\cosh\frac{M_0h}{h_0}}{\sinh\frac{M_0h}{h_0}}\right)\frac{\partial p_c}{\partial z}\right\}A$$

$$+\frac{\rho h h_0^4}{24\mu^3M_0^4}\frac{\partial p_0}{\partial z}\left(h\frac{\partial^2 p_0}{\partial z^2}+h'\frac{\partial p_0}{\partial z}\right)\left[\frac{\sinh^2\frac{M_0h}{h_0}}{\left(1-\cosh\frac{M_0h}{h_0}\right)^2}\cdot\left\{B+8C-\frac{6M_0}{h_0}D-9E\right\}\right.$$

$$- \frac{\sinh \frac{M_0 h}{h_0}}{1 - \cosh \frac{M_0 h}{h_0}} \left\{ \frac{6M_0}{h_0} F - 2G \right\} + (B - 2C + 3D) \Bigg]$$

$$- \frac{\rho h h^2 \; h_0^3}{24\mu^3 M_0^3} \left(\frac{\partial p_0}{\partial z} \right)^2 \left[\frac{\sinh \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)} B + 8C - \frac{6M_0}{h_0} D - 9E \right\} - \left\{ \frac{\frac{2M_0}{h_0} F - G}{1 - \cosh \frac{M_0 h}{h_0}} \right\} \Bigg] \quad (11)$$

Now substituting the values of u and w from (13) and (14) into (5) and (6) and then solving under the boundary conditions (8), we get

$$H_x = \frac{\mu_0}{H_0} \cdot \frac{M_0}{h_0} \left\{ \frac{h}{2} \frac{\partial p_0}{\partial x} - \frac{h_0}{M_0} \cdot \left(\frac{1 - \cosh \frac{M_0 h}{h_0}}{\sinh \frac{M_0 h}{h_0}} \right) \frac{\partial p_c}{\partial x} \right\} \bar{\bar{A}}$$

$$+ \frac{\rho h \; h_0^2 \mu_e}{24\mu^2 M_0^2 H_0} \frac{\partial p_0}{\partial x} \left(h \frac{\partial^2 p_0}{\partial x^2} + h' \frac{\partial p_0}{\partial x} \right) \left[\frac{\sinh^2 \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)} \cdot \left\{ \bar{\bar{B}} - 8\bar{\bar{C}} - \frac{6M_0}{h_0} \bar{\bar{D}} - 9\bar{\bar{E}} \right\} \right.$$

$$\left. - \frac{\sinh \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)} \cdot \left\{ 6 \frac{M_0}{h_0} \bar{\bar{F}} - \bar{\bar{G}} \right\} + (\bar{\bar{B}} - 4\bar{\bar{C}} + 3\bar{\bar{D}}) \right]$$

$$- \frac{\rho h' \; h^2 \; h_0 \mu_e}{24\mu^2 \; H_0 M_0} \cdot \left(\frac{p_0}{x} \right)^2 \left[\frac{\sinh \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)^2} \left\{ \frac{3M_0}{h_0} \bar{\bar{F}} - \bar{\bar{G}} \right\} - (\bar{\bar{B}} + 3\bar{\bar{E}} - 8\bar{\bar{C}}) \right]$$

$$+ \frac{\rho (U_1 - U_2) h_0 \mu_e}{24\mu^2 \; H_0 M_0} \left(h \frac{\partial^2 p_0}{\partial x^2} + h' \frac{\partial p_0}{\partial x} \right) \left[\frac{\sinh \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)} \left(\frac{3M_0}{h_0} \bar{\bar{D}} - 2\bar{\bar{C}} - \bar{\bar{B}} - 3\bar{\bar{E}} \right) \right.$$

$$\left. - \frac{3M_0}{h_0} \left\{ \frac{1 + \cosh \frac{M_0 h}{h_0}}{1 - \cosh \frac{M_0 h}{h_0}} \right\} \bar{\bar{F}} + 2 \cdot \frac{\cosh \frac{M_0 h}{h_0}}{\left(1 + \cosh \frac{M_0 h}{h_0}\right)} \cdot \bar{\bar{G}} + \frac{\left(1 + \cosh \frac{M_0 h}{h_0}\right)}{\sinh \frac{M_0 h}{h_0}} \cdot \left\{ \bar{\bar{B}} + 2\bar{\bar{F}} - 4\bar{\bar{C}} \right\} \right]$$

$$- \frac{\rho h h' (U_1 - U_2) \mu_e}{24\mu H_0} \cdot \frac{\partial p_0}{\partial x} \left[\frac{\left\{ \frac{3M_0}{h_0} \bar{\bar{D}} - \bar{\bar{B}} + 3\bar{\bar{E}} - 2\bar{\bar{C}} \right\}}{\left(1 - \cosh \frac{M_0 h}{h_0} \right)} - \frac{1}{\sinh \frac{M_0 h}{h_0}} \cdot \left\{ \frac{1 + \cosh \frac{M_0 h}{h_0}}{1 - \cosh \frac{M_0 h}{h_0}} \right\} \right.$$

$$\times \left(\frac{3M_0}{h_0}\,\overline{\overline{F}} - \overline{\overline{G}}\right)\Bigg] + \frac{\rho h'(U_1 - U_2) M_0 \mu_e}{2H_0 h_0} \left(\frac{1+\cosh \frac{M_0 h}{h_0}}{\sinh \frac{M_0 h}{h_0}}\right)\left[\overline{\overline{G}} - \frac{\left(1+\cosh \frac{M_0 h}{h_0}\right)}{\sinh \frac{M_0 h}{h_0}}\right.$$

$$\left. .\,(\overline{\overline{B}} + 3\overline{\overline{E}} - 4\overline{\overline{C}})\right] - \frac{\rho\,(U_1 + U_2)\,\mu_e}{8H_0}\left[\frac{h_0}{2\mu M_0}\left(h\,\frac{\partial^2 p_0}{\partial x^2} + h'\,\frac{\partial p_0}{\partial x}\right)\right.$$

$$\left\{\frac{\sinh \frac{M_0 h}{h_0}}{\left(1 + \cosh \frac{M_0 h}{h_0}\right)}\left(\frac{2M_0}{h_0}\,\overline{\overline{D}} + 4\overline{\overline{E}} - 4\overline{\overline{C}}\right) + \frac{2M_0}{h_0}\,\overline{\overline{F}}\right\}$$

$$\left. - \frac{\rho h h'}{2\mu^2} \cdot \frac{\partial p_0}{\partial x} \cdot \frac{\left(\frac{2M_0}{h_0}\,\overline{\overline{D}} + 4\overline{\overline{E}} - 4\overline{\overline{C}}\right)}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)} - (U_1 - U_2)\left(\frac{1 + \cosh \frac{M_0 h}{h_0}}{\sinh \frac{M_0 h}{h_0}}\right)\overline{\overline{F}}\right].$$

$$+ \frac{1}{2}\,(U_1 - U_2)\,\overline{\overline{H}}^* \tag{12}$$

and

$$H_z = \frac{M_0 \mu_e}{h_0 H_0}\left\{\frac{h}{2}\,\frac{\partial p_0}{\partial z} - \frac{h_0}{M_0}\left(\frac{1 - \cosh \frac{M_0 h}{h_0}}{\sinh \frac{M_0 h}{h_0}}\right)\frac{\partial pc}{\partial z}\right\}\overline{\overline{A}} + \frac{\rho h h_0^2 \mu_e}{24\mu^2 H_0 M_0^2} \cdot \frac{p_0}{\partial z}$$

$$\left(h\,\frac{\partial^2 p_0}{\partial z^2} + h'\,\frac{\partial p_0}{\partial z}\right)\left[\frac{\sinh^2 \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)^2} \cdot \left\{\overline{\overline{B}} + 8\overline{\overline{C}} - \frac{6M_0}{h_0}\,\overline{\overline{D}} - 9\overline{\overline{E}}\right\}\right.$$

$$\left. - \frac{\sinh \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)}\left\{\frac{6M_0}{h_0}\,\overline{\overline{F}} - 2\overline{\overline{G}}\right\} + (\overline{\overline{B}} - 4\overline{\overline{C}} + 3\overline{\overline{D}})\right].$$

$$- \frac{\rho h' h^2 h_0 \mu_e}{24\mu^2 H_0 M_0} \cdot \left(\frac{\partial p_0}{\partial z}\right)^2\left[\frac{\sinh \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)}\left(\overline{\overline{B}} + 8\overline{\overline{C}} - \frac{6M_0}{h_0}\,\overline{\overline{D}} - 9\overline{\overline{E}}\right)\right.$$

$$\left. - \frac{\left\{\frac{3M_0}{h_0}\,\overline{\overline{F}} - \overline{\overline{G}}\right\}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)}\right] \tag{13}$$

where

$$A = \frac{\sinh \frac{M_0 h}{h_0}}{\left(1 - \cosh \frac{M_0 h}{h_0}\right)} \left(1 - \cosh \frac{M_0}{h_0} y\right) - \sinh \frac{M_0 y}{h_0}$$

$$B = \cosh \frac{2M_0 y}{h_0} - \cosh \frac{2M_0 h}{h_0}, \frac{\sinh \frac{M_0}{h_0} y}{\sinh \frac{M_0 h}{h_0}}$$

$$C = \cosh \frac{M_0 y}{h_0} - \coth \frac{M_0 h}{h_0} . \sinh \frac{M_0 y}{h_0}$$

$$D = (y - h) \sinh \frac{M_0}{h_0} y$$

$$E = \left[1 - \frac{\sinh \frac{M_0}{h_0} y}{\sinh \frac{M_0 h}{h_0}} \right]$$

$$F = y \cosh \frac{M_0 y}{h_0} - h \coth \frac{M_0 h}{h_0} . \sinh \frac{M_0 y}{h_0}$$

$$G = \sinh \frac{2M_0}{h_0} y - \sinh \frac{2M_0 h}{h_0} . \frac{\sinh \frac{M_0}{h_0} y}{\sinh \frac{M_0}{h_0} h}$$

$$H^* = \cosh \frac{M_0 y}{h_0} - \frac{\left(1 + \cosh \frac{M_0 h}{h_0}\right)}{\sinh \frac{M_0 h}{h_0}} \cdot \sinh \frac{M_0 y}{h_0}$$

and the (—) of the above functions represent results of integration as follows :

$$\bar{A}(y) = \int A\, dy, \quad \bar{B}(y) = \int B\, dy, \text{ and so on,}$$

$$\bar{A}(0,h) = \int_0^h A\, dy, \quad \bar{B}(0,h) = \int_0^h B\, dy, \text{ and so on.}$$

Also

$$\bar{\bar{A}} = \bar{A}(0,h) - A(y)$$

Similarly

$\bar{\bar{B}}$, $\bar{\bar{C}}$, $\bar{\bar{D}}$, $\bar{\bar{E}}$, $\bar{\bar{F}}$, $\bar{\bar{G}}$ and $\bar{\bar{H}}^*$ are defined.

Moreover primes represent differentiation with respect to x.

The equation of continuity under boundary conditions :

$$v = 0 \quad \text{at } y = 0$$

$$v = -V \quad \text{at } y = h$$

gives

$$V = \int_0^h \frac{\partial u}{\partial x}\, dy + \int_0^h \frac{\partial \omega}{\partial z}\, dy \tag{14}$$

Using equations (10), (11) and (14) we get the hydromagnetic form of Reynolds' equation as

$$\frac{\partial}{\partial x}\left[\frac{h}{\mu}\cdot\frac{\frac{M_0 h}{2h_0} - \tanh\frac{M_0 h}{2h_0}}{\left(\frac{M_0}{h_0}\right)^2 \tanh\frac{M_0 h}{2h_0}}\cdot\frac{\partial p_0}{\partial x} + \frac{2h_0^3}{\mu M_0^3}\left\{\frac{M_0 h}{2h_0} - \tanh\frac{M_0 h}{2h_0}\right\}\frac{\partial pc}{\partial x}\right.$$

$$+ \frac{\rho h}{24\mu^3}\cdot\frac{\partial p_0}{\partial x}\left(h\frac{\partial^2 p_0}{\partial x^2} + h'\frac{\partial p_0}{\partial x}\right)$$

$$\frac{\left\{8\tanh^3\frac{M_0 h}{2h_0} - 9\left(\frac{M_0 h}{h_0}\right)\tanh^2\frac{M_0 h}{2h_0} - 30\tanh\frac{M_0 h}{2h_0} + 15\frac{M_0 h}{h_0}\right\}}{\left(\frac{M_0}{h_0}\right)^5 \tanh^2\frac{M_0 h}{2h_0}} - \frac{\rho h h'}{24\mu^3}\left(\frac{\partial p_0}{\partial x}\right)^2$$

$$\frac{\left\{3\left(\frac{M_0 h}{h_0}\right)\tanh^4\frac{M_0 h}{2h_0} + 26\tanh^3\frac{M_0 h}{2h_0} - 18\left(\frac{M_0 h}{h_0}\right)\tanh^2\frac{M_0 h}{2h_0} - 30\tanh\frac{M_0 h}{2h_0} + 15\frac{M_0 h}{h_0}\right\}}{\left(\frac{M_0}{h_0}\right)^4 \tanh^3\frac{M_0 h}{2h_0}}$$

$$-\frac{\rho(U_1 - U_2)\, h h'}{48\mu^2}\left\{\frac{4\left(\frac{M_0 h}{h_0}\right)\tanh^4\frac{M_0 h}{2h_0} - 8\tanh^2\frac{M_0 h}{2h_0} + 12\tanh\frac{M_0 h}{2h_0} - 3\frac{M_0 h}{h_0}}{\left(\frac{M_0}{h_0}\right)^3 \tanh^2\frac{M_0 h}{2h_0}}\right\}$$

$$+\frac{\rho(U_1 - U_2)^2 h'}{48\mu}\left\{\frac{4\tanh^3\frac{M_0 h}{2h_0} - 3\left(\frac{M_0 h}{h_0}\right)\tanh\frac{M_0 h}{2h_0} - 6\tanh\frac{M_0 h}{2h_0} - \frac{3}{2}\frac{M_0 h}{h_0}}{\left(\frac{M_0}{h_0}\right)^2 \tanh^3\left(\frac{M_0 h}{2h_0}\right)}\right\}$$

$$+\frac{\rho(U_1 + U_2)\, h_0^2}{8\mu\, M_0^2}\left\{\frac{h_0^2}{\mu M_0^2}\left(h\frac{\partial^2 p_0}{\partial x^2} + h'\frac{\partial p_0}{\partial x}\right)\right.$$

$$\frac{\left\{\left(\frac{M_0 h}{h_0}\right)\tanh^2\frac{M_0 h}{2h_0} - 6\tanh\frac{M_0 h}{2h_0} - 3\frac{M_0 h}{h_0}\right\}}{\tanh^2\frac{M_0 h}{h_0}}$$

$$+\frac{\rho h h'}{2\mu^2}\cdot\frac{\partial p_0}{\partial x}\left\{\frac{4\tanh^3\frac{M_0 h}{2h_0} - 3\frac{M_0 h}{h_0}\tanh\frac{M_0 h}{2h_0} - 6\tanh\frac{M_0 h}{2h_0} + 3\frac{M_0 h}{h_0}}{\frac{M_0}{h_0}\tanh^2\frac{Moh}{2ho}}\right\}$$

$$+\frac{(U_1-U_2)h'}{4}\,\frac{\left\{\left(\frac{M_0h}{h_0}\right)\cdot\left(\tanh\frac{M_0h}{2h_0}-1\right)+2\tanh\frac{M_0h}{2h_0}\right\}}{\tanh\frac{M_0h}{2h_0}}\Bigg]\Bigg]$$

$$+\frac{\partial}{\partial z}\left[\frac{h}{\mu}\,\frac{\frac{M_0h}{2h_0}-\tanh\frac{M_0h}{2h_0}}{\left(\frac{M_0}{h_0}\right)^2\tanh\frac{M_0h}{2h_0}}\cdot\frac{\partial p_0}{\partial z}+\frac{2}{\mu}\,\frac{h_0^2}{M_0^2}\left\{\frac{M_0h}{2h_0}-\tanh\frac{M_0h}{2h_0}\right\}\frac{\partial p_c}{\partial z}\right.$$

$$+\frac{\rho h}{24\mu^3}\cdot\frac{\partial p_0}{\partial z}\left(h\,\frac{\partial^2 p_0}{\partial z^2}+h'\,\frac{\partial p_0}{\partial z}\right)$$

$$\frac{\left\{8\tanh^3\frac{M_0h}{2h_0}-9\left(\frac{M_0h}{h_0}\right)\tanh^2\frac{M_0h}{2h_0}-30\tanh\frac{M_0h}{2h_0}+15\,\frac{M_0h}{h_0}\right\}}{(M_0/h_0)^5\tanh^2\left(\frac{M_0h}{2h_0}\right)}\quad-\frac{\rho h^2h'}{48\mu^3}\cdot\left(\frac{\partial p_0}{\partial z}\right)^2$$

$$\left.\frac{\left\{3\left(\frac{M_0h}{h_0}\right)\tanh^4\frac{M_0h}{2h_0}+26\tanh^3\frac{M_0h}{2h_0}-18\tanh^2\frac{M_0h}{2h_0}-30\tanh\frac{M_0h}{2h_0}-15\,\frac{M_0h}{h_0}\right\}}{\left(\frac{M_0}{h_0}\right)^3\tanh^2\frac{M_0h}{2h_0}}\right]$$

$$+\frac{1}{2}\,\frac{\partial}{\partial x}\left[(U_1-U_2)\,h\right]=-V \qquad (15)$$

This is the most general equation for determining the pressure in two dimensional bearings considering the effects of inertia in the presence of transverse magnetic field. The values of p_0 have already found by Shukla & Prakash[5].

SQUEEZE FILMS

In the case of squeeze films between parallel plates, h is constant i.e. $h'=0$. There fore (15) reduces to :

$$\left(\frac{\partial^2 p_0}{\partial x^2}+\frac{\partial^2 p_0}{\partial z^2}\right)\left[\frac{h}{\mu}\cdot\frac{\left(\frac{M_0h}{2h_0}\right)-\tanh\frac{M_0h}{2h_0}}{\left(\frac{M_0}{h_0}\right)^2\tanh\frac{M_0h}{2h_0}}\right]$$

$$+\frac{2h_0^3}{\mu M_0^3}\left(\frac{\partial^2 p_c}{\partial x^2}+\frac{\partial^2 p_c}{\partial z^2}\right)\left[\left(\frac{M_0h}{2h_0}\right)-\tanh\frac{M_0h}{2h_0}\right]$$

$$+\frac{\rho h^2}{24\mu^3}\left\{\left(\frac{\partial^2 p_0}{\partial x^2}\right)^2+\left(\frac{\partial^2 p_0}{\partial z^2}\right)^2+\frac{\partial^2 p_0}{\partial x^2}\cdot\frac{\partial p_0}{\partial x}+\frac{\partial^2 p_0}{\partial z^2}\cdot\frac{\partial p_0}{\partial z}\right\}\times$$

$$\left[\frac{8\tanh^3\frac{M_0h}{2h_0}-9\left(\frac{M_0h}{h_0}\right)\tanh^2\frac{M_0h}{2h_0}-30\tanh\frac{M_0h}{2h_0}+15\,\frac{M_0h}{h_0}}{\left(\frac{M_0}{h_0}\right)^5\tanh^2\frac{M_0h}{2h_0}}\right]$$

$$=-V \qquad (16)$$

Infinite plates

Consider two non-conducting rectangular plates infinite in the z direction approaching with a squeeze velocity V, in the presence of uniform magnetic field perpendicular to the plates.

Substituting the values[5] of p_0, in (16), the equation for correction pressure p_c is given by

$$\frac{\partial^2 p_c}{\partial x^2} = -\frac{\rho V^2 M_0^2}{48 h_0^2}\left[\frac{\left\{8\tanh^3\frac{M_0 h}{2h_0} - 9\left(\frac{M_0 h}{h_0}\right)\tanh^2\frac{M_0 h}{2h_0} - 30\tanh\frac{M_0 h}{2h_0} + 15\frac{M_0 h}{2h_0}\right\}}{\left\{\left(\frac{M_0 h}{2h_0}\right) - \tanh\frac{M_0 h}{2h_0}\right\}^3}\right] \tag{17}$$

Solving the above equation under the conditions $p_c\,(\pm B^*/2) = 0$ and taking into considerationing the pressure p_0 without inertia, the total pressure is given by

$$p = \frac{1}{2}\cdot\frac{V\mu M_0^2}{h_0^2}\cdot\frac{\tanh\frac{M_0 h}{h_0}}{h\left\{\frac{M_0 h}{2h_0} - \tanh\frac{M_0 h}{2h_0}\right\}}\times$$

$$\left[1 + \frac{\rho V h}{48\mu}\left\{\frac{8\tanh^3\frac{M_0 h}{2h_0} - 9\left(\frac{M_0 h}{h_0}\right)\tanh^2\frac{M_0 h}{2h_0} - 30\tanh\frac{M_0 h}{2h_0} + 15\frac{M_0 h}{h_0}}{\tanh\left(\frac{M_0 h}{2h_0}\right)\cdot\left\{\frac{M_0 h}{2h_0} - \tanh\frac{M_0 h}{2h_0}\right\}}\right\}\right]\left(\frac{B^2}{4} - x^2\right) \tag{18}$$

Hence the load per unit length has been calculated by integrating (18) over the width of the plate i.e.,

$$\overline{W}_1 = \frac{W h_0^3}{\mu B^{*3} L V}$$

$$= \frac{M_0^2}{12}\left\{\frac{\tanh M_0\xi/2}{\xi\left(\frac{M_0\xi}{2} - \tanh\frac{M_0\xi}{2}\right)}\right\}\left\{1 + \frac{\eta_1\xi}{48}F^*(\alpha, M_0)\right\} \tag{19}$$

where

$$F^*(\alpha, M_0) = \frac{8\tanh^3\frac{M_0\xi}{2} - 9 M_0\xi\tanh^2\frac{M_0\xi}{2} - 30\tanh\frac{M_0\xi}{2} + 15 M_0\xi}{\left(\frac{M_0\xi}{2} - \tanh\frac{M_0\xi}{2}\right)^2\tanh\frac{M_0\xi}{2}}$$

$$\eta_1 = \frac{\rho V h_0}{\mu}$$

and ξ being the dimensionless film thickness.

From (19) on substituting

$$V = -\frac{dh}{dt} = h_0 \frac{d\xi}{dt}$$

and integrating under the assumption that M_0 is sufficiently large, we get

$$\bar{t_1} = \frac{\overline{W}_1 h_0^2 t_1}{\mu L B^{*3}} = \left(\frac{1}{\xi} - 1\right) M_0 + \frac{\eta_3}{2\sqrt{3}} \left[\tan^{-1}\left(\frac{\sqrt{3}\eta_3}{M_0}\right) - \tan^{-1}\left(\frac{\sqrt{3}\eta_3 \xi}{M_0}\right)\right] \tag{20}$$

where η_3 is a dimensionless quantity given by

$$\eta_3 = \frac{h_0^2}{\mu}\left\{\frac{\rho W_1}{L B^{*3}}\right\}^{\frac{1}{2}}$$

Elliptical plates

The case of two elliptical plates whose major and minor axes are a and b respectively, which are approaching each other with squeeze velocity V, is being considered. The equation for determining the pressure in this case is also given by (19). Integrating (19) with boundary conditions:

$$p_c = 0 \text{ at } \frac{x^2}{a^2} + \frac{z^2}{b^2} = 1,$$

and taking into consideration the value[5] of p_0, the total pressure distribution is given by

$$p(x, z) = \frac{\mu}{2} \cdot V \frac{M_0^2}{h_0^2} \cdot \frac{a^2 b^2}{(a^2+b^2)} \cdot \frac{\tanh \frac{M_0 \xi}{2}}{h_0 \xi \left(\frac{M_0 \xi}{2} - \tanh \frac{M_0 \xi}{2}\right)} \times \left[1 + \frac{\eta_2 \xi}{48} F^*(\alpha, M_0)\right]\left(1 - \frac{x^2}{a^2} - \frac{z^2}{b^2}\right) \tag{21}$$

where

$$\eta_2 = \frac{\rho V h_0}{\mu} \cdot \frac{(a^4+b^4)}{(a^2+b^2)^2}$$

The load capacity in this case is given by

$$W_2 = 4 \int_0^b \int_0^{a\sqrt{1-z^2/b^2}} p \, dx \, dz$$

Therefore

$$\overline{W}_2 = \frac{W_2 h_0^3}{3\pi\mu} \cdot \frac{a^2+b^2}{a^3 b^3}$$

$$= \frac{M_0^2}{12} \cdot \frac{\tanh \frac{M_0 \xi}{2}}{\left(\frac{M_0 \xi}{2} - \tanh \frac{M_0 \xi}{2}\right)} \left[1 + \frac{\eta_2 \xi}{48} F^*(\alpha, M_0)\right] \tag{22}$$

From (22) the time height relation, obtained by considering that M_0 is sufficiently large, is given as by

$$\bar{t}_2 = \frac{2W_2 h_0^2}{\pi\mu} \cdot \frac{a^2+b^2}{a^3\ b^3}\, t_2$$
$$= \left(\frac{1}{\xi} - 1\right) M_0 + \frac{1}{\sqrt{\pi\eta_4}}\left[\tan^{-1}\left(\frac{1}{M_0}\sqrt{\frac{\eta_4}{\pi}}\right) - \tan^{-1}\left(\frac{\xi}{M_0}\sqrt{\frac{\eta_4}{\pi}}\right)\right]$$

where η_4 is a dimensionless quantity which is given by

$$\eta_4 = \frac{\rho W_2\, h_0^4}{\mu^2} \cdot \frac{a^4\, b^4}{a^3\, b^3\, (a^2+b^2)}$$

If $a = b$, this reduces to circular plates approaching each other.

CONCLUSION

A graph between dimensionless load for both infinite rectangular or finite elliptical plates versus initial Hartmann number for various values of N^*, (η_1 or η_2) at a dimensionless thicness $\xi = 0.5$ is shown in Fig. 2. It is concluded that load capacity increases with the increase in inertia parameter η_1 or η_2 for infinite rectangular and finite elliptical plates, and for lower values of Hartmann number the increase in load capacity is lesser as inertia parameter increases, but for higher values of Hartmann number load capacity increases sufficiently as both inertia parameter and Hartmann number increase.

Figures 3, 4 and 5 are the graphs between the dimensionless time of approach of infinite rectangular or finite elliptical plates versus initial Hartmann number for various values of inertia and various values of dimensionless height between the plates. By comparing them, it is found that time of approach increases as both inertia increases and

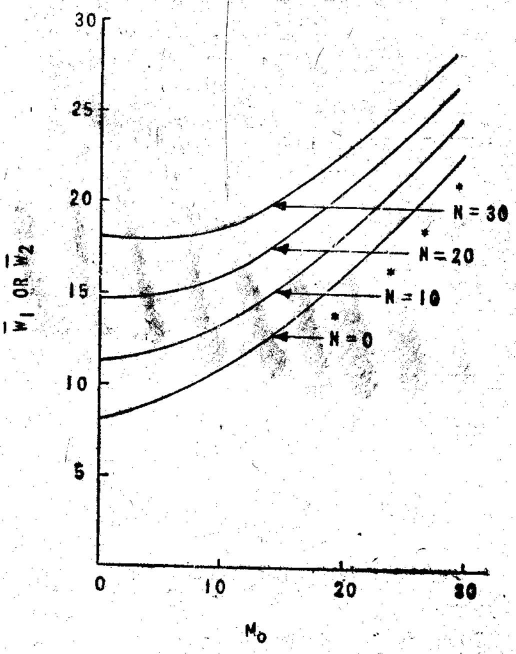

Fig. 2—Dimensionless load versus Hartmann Number for various values of N^* (η_1 or η_2), $\xi = 0{\cdot}5$

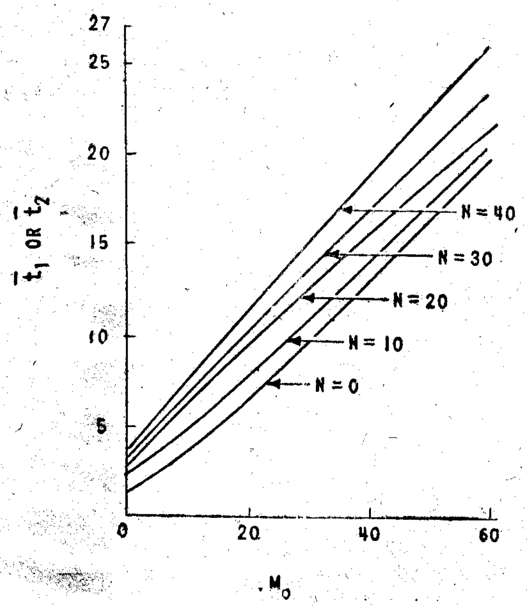

Fig. 3—Dimensionless time versus Hartmann Number for various Values of N (η_3 or η_4), $\xi = 0{\cdot}75$

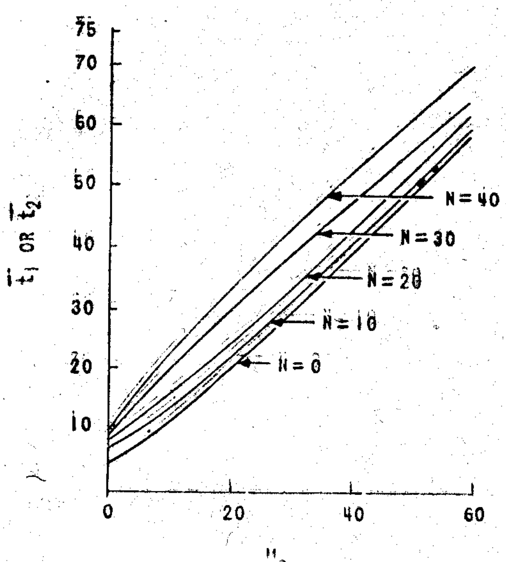

Fig. 4—Dimensionless time versus Hartmann Number for various values of N (η_3 or η_4), $\xi = 0 \cdot 5$

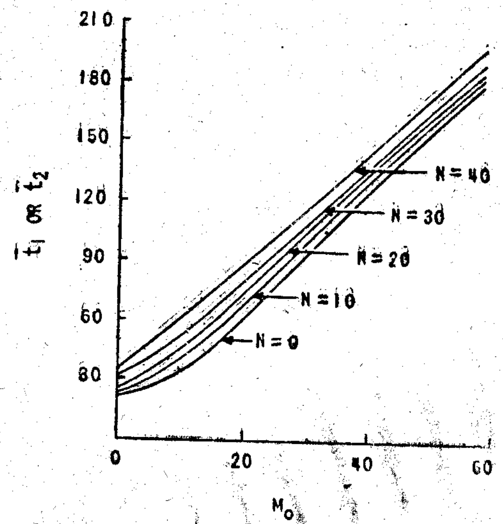

Fig. 5—Dimensionless time versus Hartmann Number for various values of N (η_3 or η_4), $\xi = 0 \cdot 25$

dimensionless height ξ in between the plates decreases. It is also concluded that there is a sufficient increase in the time of approach with the decrease in dimensionless height ξ between the plates (Infinite rectangular or finite elliptical). This implies that as the plates, whether infinite rectangular or elliptical, approach each other, the time of approach increases sufficiently and as ξ tends to zero, the time of approach becomes infinite. On the other hand it is also seen from Figs. 3, 4 and 5 that for various values of inertia, the time of approach comes closer and closer as the plates (Infinite rectangular or finite elliptical) approach each other together with the increase of Hartmann number i.e. with the increase of magnetic field.

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