BEARING CAPACITY OF SNOW

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A method for determining the modulus of deformation due to cohesive ingredients of snow 'K_c' the modulus of deformation due to frictional component of snow 'K_ϕ' and an exponent of deformation 'n' has been described. An apparatus for determining these values has been devised. The results obtained indicate that the sinkage does not increase appreciably when the pressure exceeds 0·35 kg/cm². It has been found that the value of 'n' varies from 0·5 to 1·4, K_ϕ from 0·011 to 0·021 and K_ϕ from 0·04 to 0·71 if breadth 'b' of the loading area is measured in cms.

If a plate which represents the ground contact area of a pneumatic tyre or track is forced into ground with a uniformly distributed load, then the relationship between static sinkage 'Z' and pressure σ as proposed by earlier investigators[1] takes the following form

$$\sigma = KZ^n \tag{1}$$

where K is a modulus of soil deformation and 'n' is an exponent of deformation.

In civil engineering soil mechanics, it is sometimes assumed that a small deformation 'Z' of the ground under unit load σ of a structure having width 'b' (the smaller dimension) may be expressed by the following[2]

$$\sigma = \left(\frac{B}{b} + C \right) Z \tag{2}$$

where B is a modulus of deformation due to cohesive ingredients of soil and 'C' is a similar modulus due to the frictional component of the ground. Both the moduli 'B' and 'C' have been found independent of the size of the loading area for small 'Z'.

Equation (2) which represents a straight line, is applicable only to initial portion of sinkage curves where 'Z' is small compared to 'b', but will be invalid for large sinkage of a vehicle. To make the relation applicable to vehicle mobility, Bekker[3] proposed the relation in the following form :

$$\sigma = \left(\frac{k_c}{b} + k\phi \right) Z^n \tag{3}$$

where $B = k_c$ and $E = k\varphi$

Thus 'k' definess the relationship between sinkage and load. The values of 'k_c', '$k\varphi$' and 'n' have been measured for soil in detail but no data are available in respect of snow. Most of our troops are supposed to operate in snowbound areas during the winter, therefore, it was felt essential to carry out this basic research under the field conditions. Moreover

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these data along with 'c', the cohesion and 'φ', the angle of friction which have been determined by the authors[1] under the similar conditions, are required for calculating the motion resistance of tyres on fresh snow.

METHOD

To determine the values of 'k_c', 'k_φ' and 'n' of snow, an apparatus called "Snow Bearing Apparatus" similar to the apparatus "North Dakota Cone" which is used for measuring soil strength, was constructed in the Workshop of Defence Laboratory, Jodhpur. It consisted of a circular plate to which a cylindrical shaft was attached and supported on a triangular frame as shown in Fig. 1. The shaft could move up and down freely without any resistance. Plates of mild steel thickness 1/8″ and of different areas, circular, rectangular and square were constructed. The whole apparatus was placed over the undisturbed snow surface outside. A plate was placed gently without disturbing the snow surface under the shaft and weights were placed over the circular plate. The sinkage of the plate for various loads was measured with the help of a scale attached to the apparatus. The apparatus had a locking device so that the position of the shaft could be adjusted for initial position. Observations were taken for plates of various sizes on different conditions of snow and are shown in Figs 2—12.

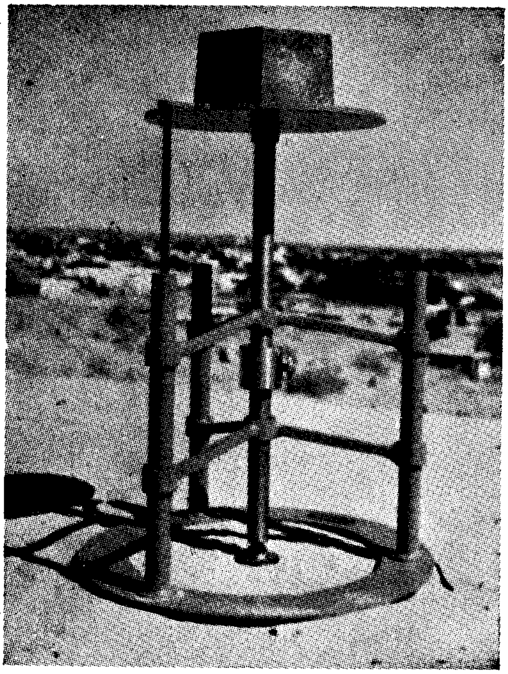

Fig 1—Snow bearing apparatus

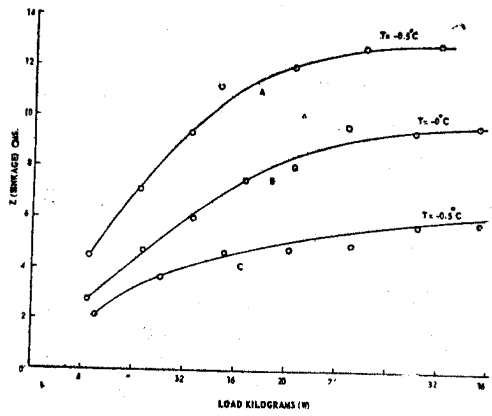

Fig 2—Variation of sinkage Z (cm) against load kilograms (W) for square plates, Size of plates (a) 2″ × 2″, (b) 3″ × 3″, (c) 5″ × 5″

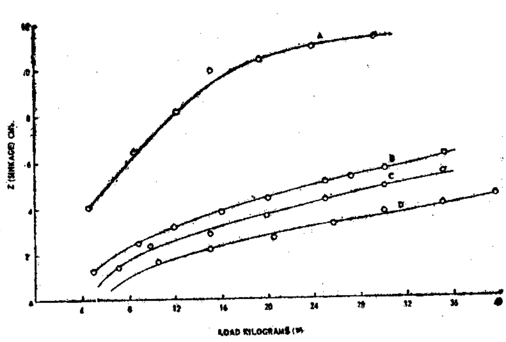

Fig 3—Variation of sinkage Z (cms) against load kilograms (W) for circular plates, Sizes of diameter (a) 2″ (b) 5″ (c) 6″ (d) 8″

RESULTS AND DISCUSSIONS

Fig 2 shows the variation of sinkage with load for different square plates (size 2″× 2″, 3″×3″ and 5″×5″) at different temperatures. It is seen that the curves are linear at low loads while at higher loads these are concave towards the load axis and at still higher pressures they become parallel to the load axis. It is also observed that as the size of the plates increases, the initial linear portion of the curves decreases. The flat portion of the curves indicates that at this sinkage the snow is well compacted and it can bear more loads without any further appreciable sinkage. Therefore, it is concluded that the motion resistances of the vehicles which are supposed to ply over snow, should be calculated at that sinkage. The reproducible and reliable results can be obtained only at these sinkages. In the linear portion, the sinkage would be dependent on the load and would be variable. Moreover the bearing capacity (pressure/area) is very low and such pressures are not important for the vehicle point of view. Fig. 3 shows the variation of sinkage with load for different circular plates under different conditions of snow. It is observed that the nature of the curves is similar to those of square plates.

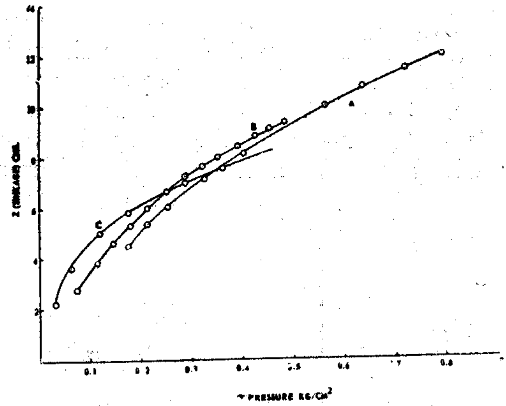

Fig 4—Variation of sinkage Z (cm) against pressure σ kg/cm² for square plates Sizes of plates (a) 2″ × 2″ (b) 3″ × 3″ (c) 5″ × 5″.

Fig. 4 shows the variation of sinkage (cms) with pressure σ (kg/cm²) for different square plates (2″×2″, 3″×3″ and 5″×5″). It is seen that sinkage increases linearly at low values of σ while at higher values, the curves Z $Vs\,\sigma$ become concave towards σ axis indicating that the bear'ng capacity

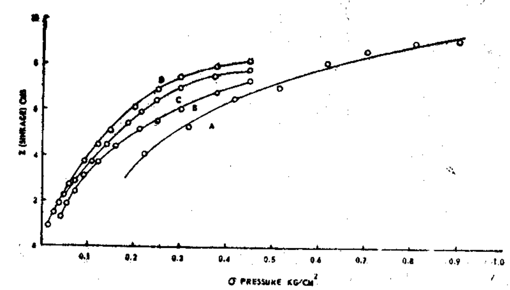

Fig 5—Variation of sinkage Z (cms) against pressure σ kg/cm² for circular plates, Sizes of diameter (a) 2″ (b) 5″ (c) 6″ (d) 8″

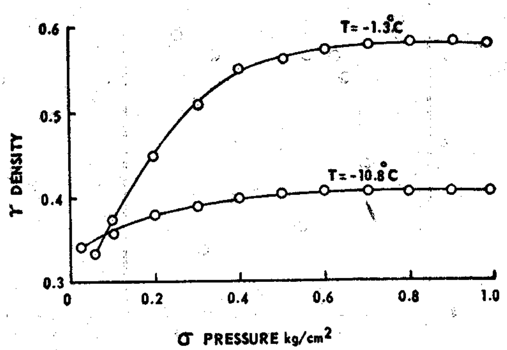

Fig 6—Variation of density γ *vs* σ pressure kg/cm² (Taken from theory of Land Locomotion by M.G. Bekker)

of the surface increases at larger values of sinkage. Fig 5 shows the variation of sinkage with σ for circular plates (size 2″, 5″, 6″ and 8″ dia). It is seen that the nature of the curves is similar to those of square plates. The mechanical characteristics of snow are more affected by changes in structure due to compression than those of soils. According to the experiments performed by the workers[5], the change in snow density with reference to the pressure applied varies in the way as shown in Fig 6. Fig 6 indicates the existence of practical limits of snow compaction at two ranges of temperatures *i.e.* it shows that for these temperatures, pressure higher than approximately $0\cdot4$ kg/cm^2 (or $5\cdot6$ lb/sq in.) do not change the snow structure as far as γ is considered. The present results also show that the sinkage does not increase appreciably when the pressure is higher than $0\cdot33$ kg/cm^2. It is also noted that this pressure is attained by the snow surface at a depth approximately 10 cms.

Sinkage (cms) has been plotted against σ (kg/cm^2) for different months as shown in Figs 7 and 9. It is seen that during the months of Dec, Jan and Feb, the bearing capacity which is defined as the load kg/cm^2 is not more than $0\cdot3$ kg/cm^2 while in the month of March its value has been recorded upto $1\cdot0$ kg/cm^2. It is also to be noted that the sinkage increases more than 12 cms in the early months of snowfall while in the month of Feb and March,

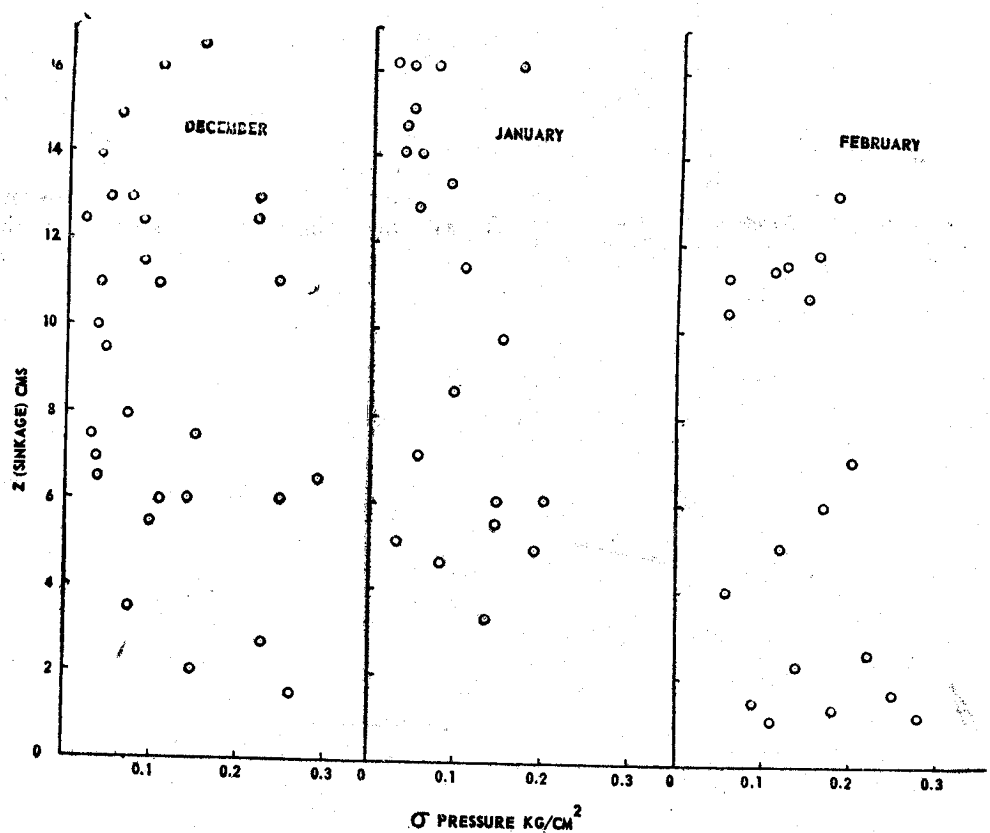

Fig 7—Variation of sinkage Z_s(cm) against σ pressure kg/cm^2 for square as well as circular plates for the months of December, January and February

this value has not exceeded more than 12 cms under the similar conditions. When the sinkage is more than 10 cms, it is found that the vehicles are bogged and the rolling resistance is very high. For climatic point of view the months of Feb and March are not very good due to the daily snowfall but temperature rises and snow starts to melt which helps in the compaction of snow, thereby increasing the density as well as the bearing capacity.

An attempt has been made to determine the values of K_c, K_φ and n. The curves plotted for each plate give a separate curve. For two plates having breadth b_1 and b_2 equation (3) becomes

$$\left.\begin{aligned} \sigma_1 &= \left(\frac{K_c}{b_1} + K_\varphi\right) Z_1^n \\ \sigma_2 &= \left(\frac{K_c}{b_2} + K_\varphi\right) Z_2^n \end{aligned}\right\} \tag{4}$$

where Z_1 and Z_2 are the sinkages in two cases. Plotting the above curves on log scale will produce practically straight lines as shown in fig. 8

$$\begin{aligned} \log \sigma_1 &= n \log Z_1 + \log \left(\frac{K_c}{b_1} + K_\varphi \right) \\ \log \sigma_2 &= n \log Z_2 + \log \left(\frac{K_c}{b_2} + K_\varphi \right) \end{aligned} \tag{5}$$

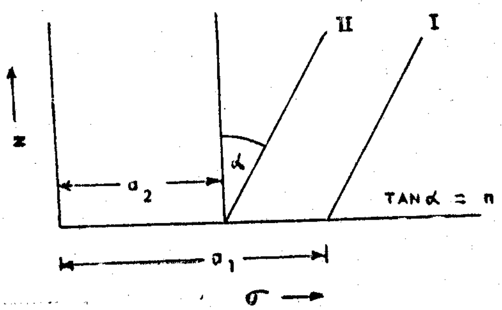

Fig. 8

The slope of the line with Z axis gives the value of $n = \tan \alpha$. Denoting

$$a_1 = \left(\frac{K_c}{b_1} + K_\varphi\right)$$

and $a_2 = \left(\frac{K_c}{b_2} + K_\varphi\right)$, the values of k_c and k_φ may be determined as

$$\begin{aligned} K_\varphi &= (a_2 b_2 - a_1 b_1) / (b_2 - b_1) \\ K_c &= (a_1 - a_2)\, b_1 b_2 / (b_2 - b_1) \end{aligned} \tag{6}$$

TABLE 1

CALCULATED VALUES OF n, K_{ϕ}, K_c

Date & Month in 1964	n	K_{ϕ}	K_c	Units of b
Jan 14 ..	0·6, 0·5	0·021	0·71	cms
			0·28	inches
Feb 12	1·1, 0·8	—	—	—
20	0·8	0·042	0·81	cms
			0·32	inches
March 4	0·9	—	—	—
10	1·3, 0·6	—	—	—
19	1·4	0·017	0·04	cms
			0·02	inches
20	1·3	0·011	0·13	cms
			0·05	inches

The values have been calculated from the Figs 10, 11 & 12 and are tabulated in Table 1. It is seen that the values of 'n' vary from 0·5 to 1·4; K_{ϕ} from 0·011 to 0·021 and K_c from 0·02 to 0·28 if 'b' is taken in inches and from 0·04 to 0·71 if b is taken in cms. It

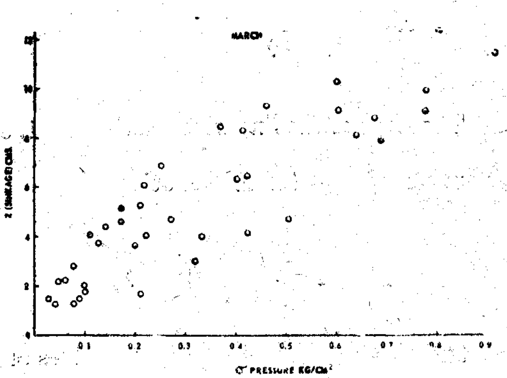

Fig 9—Variation of sinkage Z (cm) against σ pressure kg/cm² for square as well as circular plates for the month of March

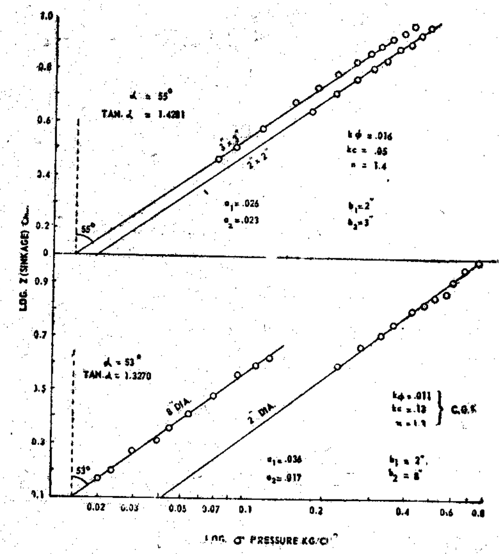

Fig 10—Variation of Log Z (sinkage) *vs* Log σ pressure kg/cm² for different plates shown on the curves

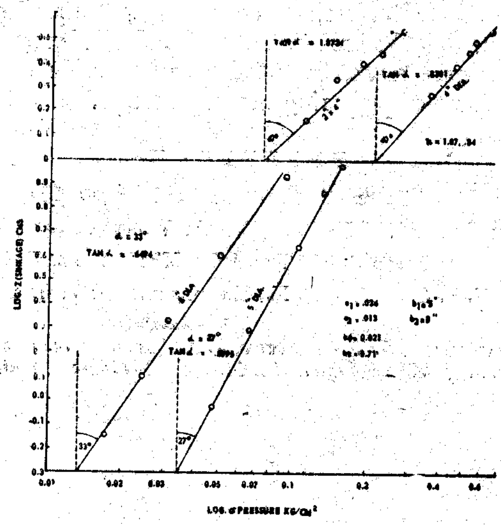

Fig 11—Variation of Log Z (Sinkage) cms *vs* Log σ pressure kg/cm² for different plates shown on the curves

may be concluded that, for practical reasons, only very approximate assumptions may be made in order to evaluate the nonelastic deformation of snow. In view of the above discussions, further work is needed to be done in order to obtain a more satisfactory solution of such an important problem of vehicle mobility over snow.

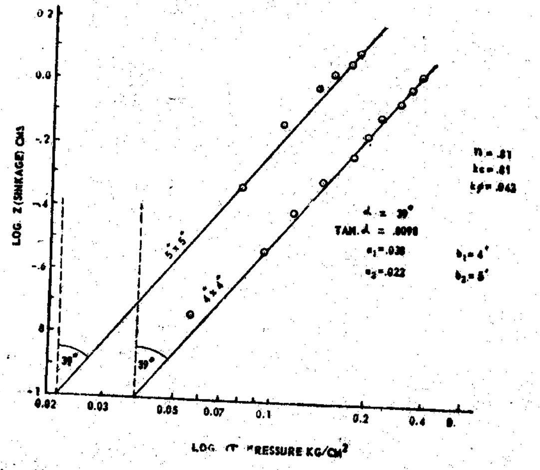

Fig 12—Variation of Log Z (sinkage) cm *vs* Log σ pressure kg/cm² for different plates shown on the curves

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