

FORMFUNCTION OF A PARTIALLY INHIBITED CYLINDRICAL CHARGE WITH THE DISTRIBUTION OF HOLES OVER n -RINGS IN THE 6-FOLD AXIS OF SYMMETRY

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This paper studies the formfunction of a charge with the distribution of holes along the sides of n similar and similarly situated hexagons called rings in the 6-fold axis of symmetry. The centres of the holes are taken on the vertices of exactly alike hexagons.

It has been shown¹ that the symmetrical distribution of points in a 2-dimensional space is only possible about 1-fold, 2-fold, 3-fold, 4-fold and 6-fold axes of symmetry. The cases for 1-fold and 2-fold axes being trivial, are left out; the cases for 3-fold and 4-fold axes have already been discussed by Patni *et al.*^{1,2}. In this paper, we have obtained the formfunction of a cylindrical charge with the distribution of holes over n rings about the 6-fold axis of symmetry. The centres of the holes are taken on the vertices of exactly alike hexagons. The holes lie along the sides of similar and similarly situated hexagons called rings and there is an extra hole at the centre of the innermost hexagon. The number of stages of burning has been reduced to two by inhibiting that portion of the charge which lies outside the outermost ring at the end of the first phase of combustion.

NOTATIONS

D	—	diameter of the charge grain.
d	—	diameter of the holes in the charge.
e	—	distance between any two adjacent holes or between any exterior hole and the curved surface of the grain, <i>i. e.</i> , the web size of the grain.
L	—	length of the charge.
m	—	ratio of the diameter of the charge grain to the diameter of any hole.
ρ	—	ratio of the length of the charge grain to the diameter of the charge grain.
z	—	fraction of the charge burnt at any instant t .
S_0	—	initial surface of the charge.
S	—	surface of the charge at any instant t .
ξ	—	density of the charge grain.
V_0	—	initial volume of the charge grain.
V	—	volume of the charge at any instant t .
n	—	number of rings.

FORMFUNCTION FOR THE FIRST PHASE OF COMBUSTION

The number of holes on a side of the n th ring

$$= n + 1 \tag{1}$$

and the total number of holes in the charge (including one hole at the centre) is

$$N = 3n(n + 1) + 1 \tag{2}$$

Let $2a$ (see Fig 1a) be the side of the innermost hexagon (*i e*, 1st ring) and $2b$ that of the outermost hexagon (*i e*, n th ring),

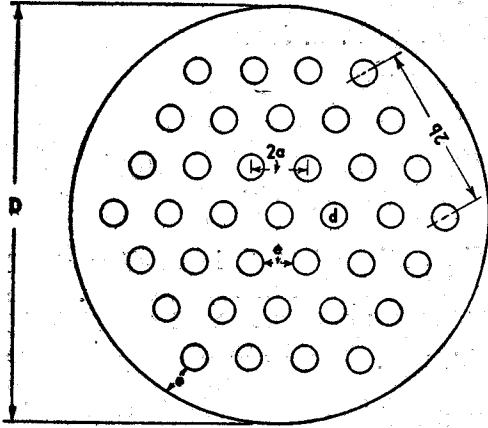
then $b = na$ (3)

Now $D = 4(n + 1)a - d = md$

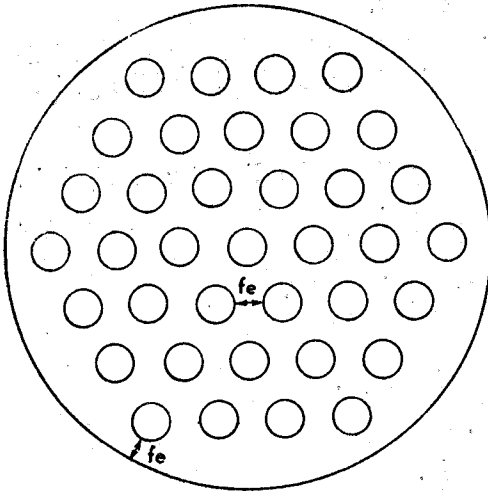
$$\therefore a = \frac{(m + 1)}{4(n + 1)} d \tag{4}$$

and $e = 2a - d$

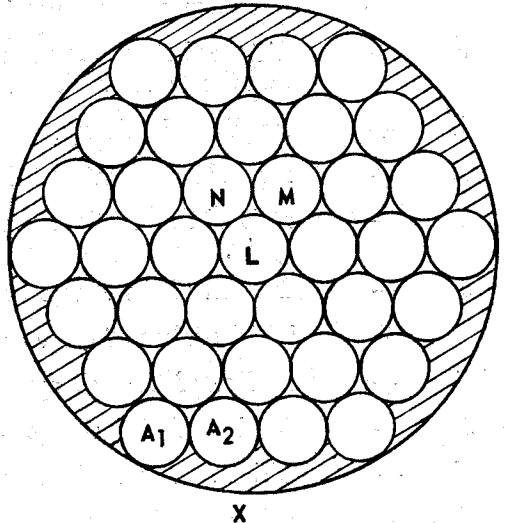
$$= \frac{m - 2n - 1}{2(n + 1)} d \tag{5}$$



(a)—at the beginning of combustion



(b)—during the first phase of combustion



(c)—at the end of the first phase of combustion (the shaded portion has been inhibited)

Fig 1—Section of the charge

At the end of the first phase of combustion when any hole touches all the holes adjacent to it, the charge consists of (i) $6n^2$ prisms whose bases are curvilinear figures like LMN (ii) six prisms having bases curvilinear figures like $A_1 A_2 \dots X$ and lying outside the outermost ring.

The diameter of the charge at this instant

$$\begin{aligned} &= 4na + 2a = 2(2n + 1)a \\ &= \frac{(2n + 1)(m + 1)d}{2(n + 1)} = D' \text{ (say)} \end{aligned} \tag{6}$$

and the diameter of a hole = $2a$ (6a)

The area of the bases of the outer curvilinear prisms

$$\begin{aligned} &= \pi \left(\frac{D'}{2}\right)^2 - N\pi \left(\frac{2a}{2}\right)^2 - 6n^2 \cdot \Delta_{LMN} \\ &= \frac{(m + 1)^2 \Delta}{16(n + 1)^2} d^2 \end{aligned} \tag{7}$$

where $\Delta = \{(4\pi - 6\sqrt{3})n^2 + n\pi\}$ (8)

Inhibiting this portion from burning, we have

$$\begin{aligned} V_0 &= \left[\frac{\pi D^2}{4} - \frac{N\pi d^2}{4} - \frac{(m + 1)^2 \Delta}{16(n + 1)^2} d^2 \right] \times L \\ &= \frac{\rho m d^3}{16(n + 1)^2} \left[4\pi(n + 1)^2(m^2 - N) - (m + 1)^2 \Delta \right] \end{aligned} \tag{9}$$

and

$$\begin{aligned} V &= \left[\pi \left\{ \frac{D}{2} - \frac{e(1-f)}{2} \right\}^2 - \pi N \left\{ \frac{d}{2} + \frac{e(1-f)}{2} \right\}^2 - \frac{(m + 1)^2 \Delta}{16(n + 1)^2} d^2 \right] \times [L - e(1-f)] \\ &= \frac{\rho m d^3}{16(n + 1)^2} \left[4\pi(m^2 - N)(n + 1)^2 - (m + 1)^2 \Delta \right] - \frac{\pi \rho m d}{4} \left[2(m + N)(1-f)ed \right. \\ &\quad \left. + (N - 1)(1-f)^2 e^2 \right] - e(1-f) \left[\frac{\pi}{4}(m^2 - N)d^2 - \frac{(m + 1)^2 \Delta}{16(n + 1)^2} d^2 \right. \\ &\quad \left. - \frac{2\pi}{4}(m + N)(1-f)ed + \frac{\pi}{4}(1 - N)(1-f)^2 e^2 \right] \end{aligned} \tag{10}$$

$$\begin{aligned} \therefore z &= \frac{V_0 \delta - V \delta}{V_0 \delta} = 1 - \frac{V}{V_0} \\ &= (1-f)(A - Bf - Cf^2) \end{aligned} \tag{11}$$

where

$$A = \frac{(m - 2n - 1)[2\pi(n + 1)(4n + N + 3)m^2 \rho + 2\pi(n + 1)\{2n(N + 1) + (3N + 1)\}m\rho + (m + 1)^2[\pi\{2n + 1\}^2 - N - \Delta]]}{2m\rho(n + 1)[4\pi(n + 1)^2(m^2 - N) - (m + 1)^2 \Delta]} \tag{12}$$

$$B = \frac{2\pi (m - 2n - 1)^2 \{ (n+1) (N-1) m\rho - (m+1) (2n+N+1) \}}{2 m\rho (n+1) [4\pi (n+1)^2 (m^2 - N) - (m+1)^2 \Delta]} \quad (13)$$

$$C = \frac{\pi (N-1) (m - 2n - 1)^3}{2m\rho (n+1) [4\pi (n+1)^2 (m^2 - N) - (m+1)^2 \Delta]} \quad (14)$$

Now

$$L = \rho D = m\rho d \\ = \frac{2 (n+1) m\rho}{m - 2n - 1} e. \quad (15)$$

In order that the all-burnt position of the charge may not occur before the rupture of the grain we must have

$$L > e$$

$$\text{or } \rho > \frac{m - 2n - 1}{2 (n+1) m} = \rho_{min} \quad (16)$$

where

$$\rho_{min} = \frac{m - 2n - 1}{2 (n+1) m} \quad (17)$$

Now (12) can be written as

$$A = A_0 (m, n) + \frac{A_1 (m, n)}{\rho} \quad (18)$$

where

$$A_0 (m, n) = \frac{(m - 2n - 1) [2\pi (n+1) (4n+N+3) m + 2\pi (n+1) \{2n (N+1) + (3N+1)\}]}{2 (n+1) [4\pi (n+1)^2 (m^2 - N) - (m+1)^2 \Delta]} \quad (19)$$

and

$$A_1 (m, n) = \frac{(m - 2n - 1) [\pi \{ (2n+1)^2 - N \} - \Delta] (m+1)^2}{2m (n+1) [4\pi (n+1)^2 (m^2 - N) - (m+1)^2 \Delta]} \quad (20)$$

Hence for a given value of n

$$A_{max} = A_0 (m, n) + \frac{A_1 (m, n)}{\rho_{min}} \\ = 1, \text{ which is a value independent of } m \text{ and } n. \quad (21)$$

For a given value of m and n , A is minimum when $\rho = \infty$, so that from (18)

$$A_{min} = A_0 (m, n) \quad (22)$$

When

$$m = 2n + 1, \quad A_0 (m, n) = 0$$

and when

$$m = \infty, \quad A_0 (m, n) = \frac{\pi (4n + N + 3)}{[4\pi (n+1)^2 - \Delta]}$$

which shows that in this case $A_0(m, n)$ is a function of n only. Taking the minimum value of $n = 1$, we have

$$N = 7, \quad \Delta = 5.32.$$

Hence
$$A_0(m, n) = \frac{14\pi}{(16\pi - 5.32)} = 0.91.$$

Also when $n = \infty$, $A_0(m, n) = \infty$

Now, as usual,
$$\frac{S}{S_0} = \frac{dz/df}{(dz/df)_{f=1}} \tag{23}$$

which with the help of (11) gives

$$\frac{S}{S_0} = \alpha - \beta f - \gamma f^2 \tag{24}$$

where

$$\begin{aligned} \alpha &= \frac{A + B}{A - B - C} \\ &= \frac{(m+1) [2\pi(n+1)(4n+2N+2)m\rho + \pi(4n^2 - 3N - 1)m + \pi(12n^2 + 12n + 3 + 4Nn + N) - (m+1)\Delta]}{[4\pi(n+1)^2(2m^2\rho + 2m\rho N + m^2 - N) - (m+1)^2\Delta]} \end{aligned} \tag{25}$$

$$\begin{aligned} \beta &= \frac{2(B - C)}{A - B - C} \\ &= \frac{2\pi(m-2n-1) [2(n+1)(N-1)m\rho - (4n+3N+1)m - (N-2Nn+6n+3)]}{[4\pi(n+1)^2(2m^2\rho + 2m\rho N + m^2 - N) - (m+1)^2\Delta]} \end{aligned} \tag{26}$$

$$\begin{aligned} \gamma &= \frac{3C}{A - B - C} \\ &= \frac{3\pi(N-1)(m-2n-1)^2}{[4\pi(n+1)^2(2m^2\rho + 2m\rho N + m^2 - N) - (m+1)^2\Delta]} \end{aligned} \tag{27}$$

(24) gives a relation between $\frac{S}{S_0}$ and f for the first phase of combustion. Putting $f = 0$,

we get
$$\left(\frac{S}{S_0}\right)_{f=0} = \alpha$$

which is the ratio of the surface at the end of the first phase to the initial surface.

Again from (24), we get

$$\frac{d}{df} \left(\frac{S}{S_0}\right) = -\beta - 2\gamma f \tag{28}$$

and

$$\frac{d^2}{df^2} \left(\frac{S}{S_0}\right) = -2\gamma \tag{29}$$

As is clear from (27), γ is always positive so that $\frac{d^2}{df^2} \left(\frac{S}{S_0} \right)$ is always negative.

This shows that $\frac{S}{S_0}$ can have only a maximum value for some value of f .

Now, for a maximum

$$\frac{d}{df} \left(\frac{S}{S_0} \right) = -\beta - 2\gamma f = 0$$

so that

$$f = -\frac{\beta}{2\gamma} \quad (30)$$

Since

$$1 \geq f \geq 0,$$

we have

$$\frac{\beta}{2\gamma} \leq 1 \quad (31)$$

and

$$\frac{\beta}{2\gamma} \geq 0 \quad (32)$$

From (26) and (27), we get

$$\frac{\beta}{2\gamma} = \frac{[(4n+3N+1)m + (N-2nN+6n+3) - 2(n+1)(N-1)m\rho]}{3(N-1)(m-2n-1)} \quad (33)$$

so that (31) gives

$$\frac{(4n+3N+1)m + (N-2nN+6n+3) - 2(n+1)(N-1)m\rho}{3(N-1)(m-2n-1)} \leq 1$$

or

$$\rho \geq \frac{2(m+N)}{(N-1)m} = \rho_1, \text{ (say)} \quad (34)$$

where

$$\rho_1 = \frac{2(m+N)}{(N-1)m} \quad (35)$$

Similarly from (32),

$$\frac{(4n+3N+1)m + (N-2nN+6n+3) - 2(n+1)(N-1)m\rho}{3(N-1)(m-2n-1)} \geq 0$$

or

$$\rho \leq \frac{(4n+3N+1)m + (N-2nN+6n+3)}{2(n+1)(N-1)m} = \rho_2, \text{ (say)} \quad (36)$$

where

$$\rho_2 = \frac{(4n+3N+1)m + (N-2nN+6n+3)}{2(n+1)(N-1)m} \quad (37)$$

(35) and (37) give the least and the greatest values of ρ for a maximum value of $\frac{S}{S_0}$.

If $\rho = \rho_1$, the maximum occurs at the beginning of combustion; and when $\rho = \rho_2$, it occurs at the end of the first phase. In order that the maximum may occur during the first phase, the value of ρ should lie between ρ_1 and ρ_2 .

Using (17), (33), (35) and (37) in

$$f = - \frac{\beta}{2\gamma}$$

we have

$$f = 1 - \frac{\rho - \rho_1}{\rho_{min}} = \frac{\rho_2 - \rho}{\rho_{min}} \tag{38}$$

which gives the value of f for a maximum of $\frac{S}{S_0}$.

With the help of (26), (27) and (35), we get from (28)

$$\left[\frac{d}{df} \left(\frac{S}{S_0} \right) \right]_{f=1} = -\beta - 2\gamma$$

$$= \frac{4\pi (m - 2n - 1) (n + 1) (N - 1) (\rho_1 - \rho) m}{[4\pi (n + 1)^2 (2m^2 \rho + 2m\rho N + m^2 - N) - (m + 1)^2 \Delta]} \tag{39}$$

and

$$\left[\frac{d}{df} \left(\frac{S}{S_0} \right) \right]_{f=0} = -\beta = \frac{4\pi (m - 2n - 1) (n + 1) (N - 1) (\rho_2 - \rho) m}{[4\pi (n + 1)^2 (2m^2 \rho + 2m\rho N + m^2 - N) - (m + 1)^2 \Delta]} \tag{40}$$

Hence in general, for any given value of n , if

(i) $\rho_{min} \leq \rho \leq \rho_1$, $\frac{d}{df} \left(\frac{S}{S_0} \right)$ is always positive right from the beginning and the charge is degressive throughout.

(ii) $\rho_1 < \rho \leq \rho_2$, $\frac{d}{df} \left(\frac{S}{S_0} \right)$ is negative in the beginning and then positive. Hence the charge is first progressive and then degressive.

(iii) $\rho > \rho_2$, $\frac{d}{df} \left(\frac{S}{S_0} \right)$ is always negative and the charge is progressive throughout.

These results have been summarised in Table 1.

TABLE 1

$\rho_{min} \leq \rho \leq \rho_1$	$\rho_1 < \rho \leq \rho_2$	$\rho > \rho_2$
$\frac{S}{S_0}$ Decreasing function of f and the charge is throughout degressive.	Increasing function of f in the beginning and then decreasing function. The charge is first progressive and then degressive.	Increasing function of f throughout. The charge is always progressive.

FORMFUNCTION FOR THE SECOND PHASE OF COMBUSTION

At the beginning of the second phase of combustion, the charge consists of exactly alike prisms whose bases are curvilinear triangles like LMN (Fig 2). The combustion of these triangles is similar to that of the inner prisms in a heptatubular charge as discussed in H.M.S.O. (1951)³ or of the central prism in the tri-tubular charge discussed by Jain (1962)⁴. But, on account of partial inhibition and characteristic arrangement of holes in the present case, the results obtained for the two aforesaid charges are different from those obtained for our charge, as shown below.

The number of such triangular prisms = $6n^2$ (41)

and the length of the charge at this instant

$$\begin{aligned} &= L - e \\ &= \frac{2(n+1)\rho m - (m - 2n - 1)}{2(n+1)} d \end{aligned} \quad (42)$$

Also the radius of the arc like $LN = a$ (43)

and the side of a triangle like $A' B' C' = 2a$ (44)

Let R be the circum-radius of a triangle like $A' B' C'$ and R' the radius of the arc of a curvilinear triangle DEF to which the curvilinear triangle LMN shrinks in time t during the second phase of combustion. If the bounding radii of the arc DF make angle ω each with the sides $B' C', B' A'$ of the triangle $A' B' C'$, we have

$$R \cos \frac{\pi}{6} = R' \cos \omega = a \quad (45)$$

$$\therefore R = \frac{2a}{\sqrt{3}} \quad \text{and} \quad R' = a \sec \omega \quad (46)$$

For the complete combustion of the prism, $R = R'$, so that (46) gives

$$\cos \omega = \frac{\sqrt{3}}{2}$$

$$\text{or} \quad \omega = \frac{\pi}{6} \quad (47)$$

Thus the complete combustion takes place

when $\omega = \frac{\pi}{6}$. Now, at any time t during

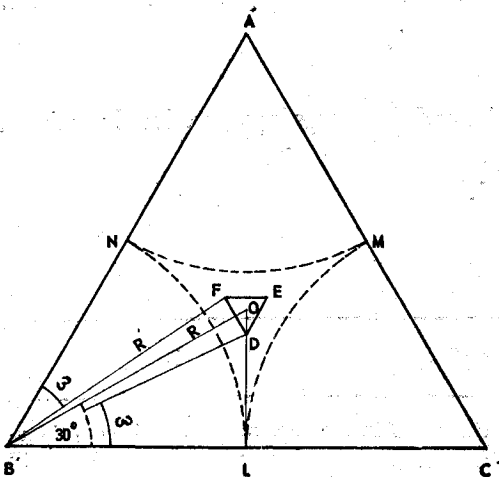


Fig 2—Burning of silver during the second phase of combustion

the second phase of combustion, when a curvilinear triangle LMN shrinks to the curvilinear triangle DEF , the length of the grain is given by

$$\begin{aligned} L' &= L - e - 2(R' - a) \\ &= \left[(\rho m + 1) - \frac{(m+1)}{2(n+1)} \sec \omega \right] d \end{aligned} \quad (48)$$

At the instant of complete combustion, i.e., when $\omega = \frac{\pi}{6}$, this length should remain positive, which is so if

$$(\rho m + 1) - \frac{(m+1)}{\sqrt{3}(n+1)} \geq 0$$

or

$$\rho \geq \frac{1 + \frac{1}{m}}{\sqrt{3}(n+1)} - \frac{1}{m} = \rho_3$$

where

$$\rho_3 = \left[\frac{m+1}{\sqrt{3}(n+1)} - 1 \right] \frac{1}{m} \quad (49)$$

Now area of a curvilinear triangle like DEF

$$\begin{aligned} &= \triangle A'B'C' - 6 \triangle LB'D - 3 \text{ sector } FB'D \\ &= \sqrt{3}a^2 - 3a^2 \tan \omega - \frac{3}{2} \left(\frac{\pi}{3} - 2\omega \right) a^2 \sec^2 \omega \\ &= \frac{(m+1)^2 d^2}{96(n+1)^2} \{ 6\sqrt{3} - 18 \tan \omega - 3(\pi - 6\omega) \sec^2 \omega \} \end{aligned}$$

Hence the area of all the curvilinear triangles

$$= \frac{n^2 (m+1)^2 d^2}{16(n+1)^2} \{ 6\sqrt{3} - 18 \tan \omega - 3(\pi - 6\omega) \sec^2 \omega \} \quad (50)$$

$$= \frac{3n^2 (m+1)^2 d^2}{16(n+1)^2} F(\omega) \quad (51)$$

where

$$F(\omega) = \{ 2\sqrt{3} - 6 \tan \omega - (\pi - 6\omega) \sec^2 \omega \} \quad (52)$$

Denoting the volume of a prism having a base DEF by $V(DEF)$, the volume of the charge at any instant t during the second phase of combustion

$$= 6n^2 V(DEF)$$

$$= \frac{3n^2 (m+1)^2 d^3}{32 (n+1)^3} [2\sqrt{3} - 6 \tan \omega - (\pi - 6\omega) \sec^2 \omega] [2(\rho m + 1)(n+1) - (m+1) \sec \omega]$$

$$= \frac{3n^2 (m+1)^2 d^3}{32 (n+1)^3} G(\omega) \quad (53)$$

where $G(\omega) = [2(\rho m + 1)(n+1) - (m+1) \sec \omega] F(\omega)$ (54)

$$\therefore z = 1 - \frac{6 n^2 V(DEF) \delta}{V_0 \delta} \quad (55)$$

$$= 1 - \frac{3n^2 (m+1)^2 G(\omega)}{2 \rho m (n+1) [4\pi (n+1)^2 (m^2 - N) - (m+1)^2 \Delta]} \quad (56)$$

Initially when $\omega = 0$

$$F(\omega) = 2\sqrt{3} - \pi \quad (57)$$

and $G(\omega) = \{2(\rho m + 1)(n+1) - (m+1)\} (2\sqrt{3} - \pi)$ (58)

so that from (56) and (58) we get the value of z at the beginning of the second phase of combustion as

$$z = 1 - \frac{3n^2 (m+1)^2 (2\sqrt{3} - \pi) \{2(\rho m + 1)(n+1) - (m+1)\}}{2\rho m (n+1) [4\pi (n+1)^2 (m^2 - N) - (m+1)^2 \Delta]} \quad (59)$$

$$= \frac{(m-2n-1) [2\pi (n+1) (4n+3+N) m^2 \rho + 2\pi (n+1) \{2n(N+1) + (3N+1)\} m\rho + (m+1)^2 [\pi \{(2n+1)^2 - N\} - \Delta]]}{2m\rho (n+1) [4\pi (n+1)^2 (m^2 - N) - (m+1)^2 \Delta]}$$

$= A$

which is the same as the value of z at the end of the first phase of combustion.

At the time of complete combustion of the grain, $\omega = \frac{\pi}{6}$ and $G(\omega) = 0$,

hence $z = 1$,

a value which z should attain when combustion is complete.

Now, let us define f as the ratio of the distance receded (from the beginning of the second phase of combustion upto the instant considered) to the initial thickness e ,

then
$$f = \frac{2(a - R')}{e}$$

$$= \frac{m+1}{m-2n-1} (1 - \sec \omega). \quad (60)$$

Initially when

$$\omega = 0, f = 0.$$

For $\omega = \frac{\pi}{6}$,

$$[f]_{\omega=\frac{\pi}{6}} = \frac{m+1}{m-2n-1} \left(1 - \frac{2}{\sqrt{3}} \right). \quad (61)$$

Hence with the help of (60) and (54) we get from (56)

$$z = 1 - \frac{3n^2 (m+1)^2 [2(n+1)\rho m - (m-2n-1)(1-f)] F(\omega)}{2\rho m(n+1) [4\pi(n+1)^2(m^2-N) - (m+1)^2 \Delta]} \quad (62)$$

We shall now find $\frac{S}{S_0}$ for the second phase of combustion in terms of

ω and n .

$$\begin{aligned} \frac{dz}{df} &= \frac{dz/d\omega}{df/d\omega} \\ &= \frac{3n^2 (m+1) (m-2n-1) [4(6\omega-\pi)(n+1)(\rho m+1) \sec \omega - (m+1) \{2\sqrt{3}-6 \tan \omega - 3(\pi-6\omega) \sec^2 \omega\}]}{2\rho m(n+1) [4\pi(n+1)^2(m^2-N) - (m+1)^2 \Delta]} \end{aligned} \quad (63)$$

and $\left(\frac{dz}{df} \right)_{f=1} = -(A - B - C)$

which with the help of (12), (13) and (14) gives

$$\left(\frac{dz}{df} \right)_{f=1} = \frac{-(m-2n-1) [4\pi(n+1)^2 \{2m^2\rho + 2m\rho N + m^2 - N\} - (m+1)^2 \Delta]}{2m\rho(n+1) [4\pi(n+1)^2(m^2-N) - (m+1)^2 \Delta]} \quad (64)$$

Hence $\frac{S}{S_0} = \frac{dz/df}{(dz/df)_{f=1}}$

$$= \frac{3n^2 (m+1) [(m+1) \{2\sqrt{3}-6 \tan \omega - 3(\pi-6\omega) \sec^2 \omega\} + 4(\pi-6\omega)(n+1)(m\rho+1) \sec \omega] \div [4\pi(n+1)^2 \{2m^2\rho + 2m\rho N + m^2 - N\} - (m+1)^2 \Delta]}{(dz/df)_{f=1}} \quad (65)$$

Initially when $\omega = 0$

$$\frac{S}{S_0} = \frac{3n^2 (m+1) [4\pi(n+1)(m\rho+1) + (m+1)(2\sqrt{3}-3\pi)]}{4\pi(n+1)^2 \{2m^2\rho + 2m\rho N + m^2 - N\} - (m+1)^2 \Delta} \quad (66)$$

At the instant of complete combustion $\omega = \frac{\pi}{6}$, so that from (65)

$$\frac{S}{S_0} = 0.$$

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