ELECTRO-MAGNETIC WAVE PRPAOGATION IN A DISSIPATIVE MEDIUM (EARTH)

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This paper deals with a brief review of the work carried out in 'Sub-Surface Communication', by various investigators. The feasibility of underground communication using low-conducting granite layer as a medium has been discussed. The effect of ground-constants on the propagation constant 'v', and the dependence of attenuation on the frequency of operation and the conductivity of the medium, are dealt with.

Propagation of electromagnetic waves from a transmitting antenna to a receiving one depends on the distance between the two points, the wave-length and the characteristics of the medium. The velocity of propagation and also the attenuation suffered by the propagating wave are determined by the characteristics of the medium. For a lossless medium (perfect di-electric) there is no attenuation, whereas a dissipative medium gives attenuation which reduces the field strength exponentially with distance (in addition to that caused by divergence).

Dissipative media are of many kinds but the medium of our interest is the earth. Since the soil behaves like a lossy di-electric, it should be specially treated while considering the wave propagation through it. The conductivity and di-electric constant of the earth vary with the frequency of operation as well as the depth below the surface of the earth. The exponential attenuation inherent in the medium of finite conductivity limits the distance of transmission. In sub-surface communication the earth plays the part of the medium. Here, both the transmitting and the receiving antennas are placed underground. There are broadly two methods by which communication with such antennas is possible. One is known as up-over-down system in which the wave from the transmitting antenna travels to the surface of the earth and over the surface it propagates as a ground wave. The receiving antenna placed underneath will receive the energy which leaks through this surface wave. Thus, the wave propagating in this manner will experience twice the attenuation due to propagation through the lossy medium (earth). While coming to the surface the receiving antenna gets the signal after being attenuated by the layer of earth above the receiving antenna. In addition to these losses the loss suffered by the surface wave over the ground is also present. Obviously, for such a communication system to be practicable, the depth below which the transmitting and receiving antennas could be kept will be limited to the skin depth which is dependent on the conductivity of the earth and the frequency of operation.

Another mode of propagation of a sub-surface wave is through the low-conducting granite layer which is supposed to exist at a particular depth below the surface of the earth. It has been argued that a layer of low-conducting granite exists deep (about 10 km) within the earth's crust bounded on both sides by regions of highly conducting layers. The upper layer has a high conductivity due to (*i*) the presence of electrolytic solutions and (*ii*) the porous upper layer surface through which water sets in. At greater depths, the increase in conductivity can be attributed to the presence of high temperature. The experimental data of G. V. Keller support this view. If both antenna and the receiver are placed under ground in the low-conducting layer below the surface of the earth, propagation can take place in more or less the same way as tropospheric duct propagation above the surface of the earth.

PROPAGATION THROUGH DISSIPATIVE MEDIA

Mathematical expressions can be derived for the attenuation, velocity of propagation and their dependence on frequency and the values of ground constants. Any electromagnetic wave propagating through a homogeneous medium will satisfy the four field equations:

$$\begin{aligned} \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times H &= i + \frac{\partial D}{\partial t} \\ \nabla . D &= \rho \\ \nabla . B &= 0 \end{aligned} \tag{1}$$

The quantities in (1) have their original meanings. In addition to the above equations there are three relations that concern the characteristics of the medium in which the fields exist. These are :

$$\left.\begin{aligned} D &= \epsilon E \\ B &= \mu H \\ i &= \sigma E \end{aligned}\right\} \tag{2}$$

where ϵ, μ and σ are permitivity, permeability and conductivity of the medium.

The region is assumed to be source-free. Also assuming sinusoidal time-variance of the field components, one arrives at the most general vector wave-equation of the type :

$$\nabla^2 E - j \omega \mu (\sigma + j \omega \epsilon) E = 0 \tag{3}$$

Let

$$v^2 = j \omega \mu (\sigma + j \omega \epsilon) \tag{4}$$

then, (3) can be written as

$$\triangle^2 E - v^2 E = 0 \tag{5}$$

The quantity "v" is referred to as propagation constant and also as wave-numbee

The above derivation[1—4] will be found in any text book on "Electromagnetic Wave Propagation". v , in general, is a complex quantity and may be represented as

$$v = \alpha + j \beta \tag{6}$$

where α is the attenuation constant and β , the phase constant

Again consider a uniform plane wave travelling in the x direction. For this case (5) becomes

$$\frac{\partial^2 E}{\partial x^2} = v^2 E \tag{7}$$

A possible solution of (5) would be

$$E = E' e^{-vx} \tag{8}$$

where E' is expressed as

$$E' = E_0 e^{j\omega t} \tag{9}$$

(8) now becomes

$$E = E_0 e^{j\omega t - \gamma x} \tag{10}$$

Or

$$E = E_0 e^{-\alpha x} e^{j(\omega t - \beta x)} \tag{11}$$

(11) is the equation of wave moving in the x direction with a velocity ω/β. The wave is attenuated by the factor $e^{-\alpha x}$

The expressions for the attenuation factor α, and the phase factor β, can be derived from the complex propagation-constant by separating it into real and imaginary parts. The values of α and β are,

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} - 1 \right)} \tag{12}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} + 1 \right)} \tag{13}$$

These expressions can be simplified by making some approximations depending on the medium. $\sigma/\omega\epsilon$ gives the ratio of conduction current to displacement current; when $\sigma/\omega\epsilon >> 1$ for the entire radio frequency spectrum, the substance is considered to be a good conductor, whereas if $\sigma/\omega\epsilon << 1$, it is classified as a good dielectric (insulator). Most materials met with in practice fall into either 'good conductor' or 'good insulator' class. But the case of the earth is typical. The earth occupies an intermediary position and thus acts like a lossy di-electric.

The values of 'conductivity' and 'di-electric constant' for several soil samples (collected in India from different places and at different depths below the surface) are being determined by the author in the laboratory. The effect of frequency, and moisture on conductivity and di-electric constant are also being investigated. The conductivity of soil was found to increase with frequency and moisture content, whereas the di-electric constant decreases with increasing frequency. Results of these measurements will be presented in due course. The basic problems that are peculiar to sub-surface communication are the determination of the propagation characteristics from underground or underwater sources, coupling of energy from the transmitti g antenna to the medium and from the medium to the receiving antenna and also the noise environment.

PROPAGATION THROUGH THE SUB SURFACE

On Sub-surface propagation, various theoretical investigations have been carried out on the assumption that a low-conducting granite layer is existing below surface of the earth. Wheeler[5] has expressed his idea in support of the possibility of a low conducting layer to exist below a depth of 2 and 20 km. In this case the wave-guide mode of transmission is possible between two stations situated beneath the sea or below the earth's crust. The excitation of this waveguide is possible by means of a long pipe of 2 to 3 km kept in a vertical bore and insulated from the ground. The basement rock from 2 to 20 km has conductivity of the order of 10^{-6} to 10^{-11} mho/m. The present plan is useful if the conductivity is around 10^{-8} or lower, and the di-electric constant 6.

Wheeler has worked out the feasibility of this mode of propagation assuming some data for conductivity at an operating frequency of 1·5 kc/s. Inside the earth there is a gradual change in the value of conductivity with depth and since, the tangential electric field vector can not exist inside a conductor, the depths of penetration of electric and magnetic fields inside the earth are different. The location of the boundaries of E wave and H wave depend on the frequency, the conductivity and the rate of change of conductivity with depth. According to Wheeler, the T.E.M Mode (with vertical polarisation) is the one that has the greatest probability of enabling long-range communication.

Burrows[6] has described four possible modes of propagation within the earth's crust Mode I is similar to that of ground wave propagation with rock di-electric substituting for air as the propagating medium and the highly conducting layers acting as ionosphere and the earth's surface. The highly conducting lower surface is often called the inverted ionosphere. Mode II results when the Q of the propagating medium is small so that the antennas must be modified, which also results in the modification of the propagation formulae. Mode III deals with dc signalling. Mode IV indicated when the over-burden is so thin that it does not provide an effective conducting plane. Here the antennas consist of a pair of vertical conductors insulated form the thin layer of high conductivity but making contact with the medium of intermediate conductivity thereby generating a horizontally polarized wave. This also makes use of dc signalling.

M. E. Viggh[7] has, in the same year, contributed a paper on "Modes in lossy stratified media with application to underground propagation of radio waves". He has worked out the problem by stratifying the layers into three different regions of conductivities 10^{-2} mho/m, 10^{-6} mho/m and 10^{-2} mho/m respectively with a di-electric constant of 10. This is a simplified approximation of the actual regions of different conductivities which are supposed to exist inside the earth's crust.

Radio-wave propagation at very low frequencies in the stratified rock below the bottom of the sea is studied by Mott & Biggs[8]. Their paper deals with the structure of the earth's layers and more important attempts to measure and estimate earth's conductivity. Discussing the topic, Biggs describes the experimental results obtained by Schotte and Veldkamp, who arrived at an upper layer conductivity of 2 mho/m (in a 600 m thickness) and a lower layer conductivity of 10^{-1} mho/m. They have shown by calculation that the merit of sub-surface communication is mostly its freedom from external noise. External noise may be atmospheric such as cyclones and tornadoes and hazards resulting from a nuclear bombardment. Under such disturbances, conventional means of communication may be ineffective, if not impossible.

Ghosh[9] in an article on "Sub-Surface Communication for survival", has explained the basic problems that are encountered in sub-surface communication. They are essentially the depths below which the transmitting and receiving antennas have to be kept, the length or sizes of these antennas, the electrical properties of the earth, operating frequency range, noise, receiver bandwidth, ionospheric characteristics and the transmitter power.

Wait[10] has expressed the possibility of a natural wave-guide to exist within the earth's crust. The existence of a low-conducting granite structure is shown by some available experimental data. The conductivities of granite upto certain depths below the surface of the earth are explored[11]. The effect of temperature on conductivity has also been studied. The data, however, is not sufficient to estimate the depth of this low-conducting granite and its uniformity in distribution throughout the earth's surface. Wait has deduced expressions for the field equations assuming a sharp boundary between highly conducting upper boundary and low-conducting middle layer. The lower boundary of this wave guide is fixed in the region where the conduction current begins to dominate over the displacement current. This is a function of frequency. For the frequencies in the VLF band, the height of this wave guide is of the order of 30 km. The excitation of the waveguide can be carried out either by a vertical electrical antenna which is co-axial with the bore hole or by means of a solenoidal coil whose axis is co-axial with the bore. Wait has deduced expressions for attenuation and other factors for such a mode of propagation and has shown that the attenuation in such an idealized earth-crust wave-guide is very small. Wait[12] has also given a review of the work carried out by many investigators in the field of "Wave propagation in lossy media". It presents seventy useful references.

Time harmonic and transient electromagnetic fields in spherical cavities whose walls are made of any material characterized by σ, μ and ϵ, have been investigated by Butter & Bladel[13]. The purpose of the investigation was to solve the wave-equation exactly for a particular source and geometry and also to study the influence of the electrical properties of the surrounding materials upon the fields. Ample numerical data have been presented in support of this study.

The effect of pulse communication through the conducting medium has also been theoretically investigated. Richards[14] has derived expressions for the electric and magnetic fields of a short pulse of electric or magnetic dipole moment in a conducting medium. In the case of pulse transmission, the amplitude was shown to decay as $\frac{1}{r^3}$ or $\frac{1}{r^4}$ (where r is the distance from the transmitter) rather than exponentially as is expected for a continuous wave in a conducting medium. In addition, it is expected that a pulse lengthens with the distance travelled and the average propagation velocity decreases. Thus, communication by pulses is expected to be inferior to that by low-frequency continuous waves. An almost similar opinion has been given by Anderson & Moore[15]—a pulse spectrum with appreciable high frequency content will be mostly absorbed. Considerations applying to propagation of continuous waves also apply to propagation of transients namely, the range of transmission sets an upper limit to the frequencies which can be sent with the minimum attenuation. Energy put into frequencies above this limit is essentially wasted. Wait's[16] work on "transients in conducting medium" gives information in this regard.

DISCUSSION

Lack of underground noise and effective shielding against atmospheric, noise by the earth's upper layers make an underground system superior to an above-ground system for long-range communication. The factors which limit the maximum possible distance for communication are frequency, ground constants and the power of the transmitter. The power of the transmitter can not be increased very much for abvious reasons. The ground constants being fixed for a particular region, the only factor that can be controlled is frequency. The frequency should be low for the attenuation to be low. But for communication purposes, especially when speech and information have to be transmitted, the carrier frequency should be more than twice the highest modulating frequency. When the carrier frequency is low, the size of the antenna becomes too large for efficient transmission and physical problems like antenna layout etc limit its size. The frequency range 200 Kc/s to 300 kc/s is found to be most convenient. Special antennas have to be designed to make them suitable for working at such low frequencies. To an investigator working in this field, a knowledge of the electrical properties of the ground and thier variation with frequency as also seasonal and diurnal variations, is necessary.

The Table I gives an idea of the attenuation per km of the propagating wave. The dependence of attenuation on frequency and conductivity is noteworthy.

For an efficient long-distance communication, one has to use low frequencies. For investigating the field strengths theoretically, studies on the induction field are as important (if not more) as those on the radiation field. When the distance from the transmitting

TABLE I

DEPENDENCE OF ATTENUATION ON FREQUENCY AND CONDUCTIVITY

Conductivity (mho/m)	Frequency (cps)	attn. (db/m)	Conductivity (mho/m)	Frequency (cps)	attn. (db/m)
10^{-8}	10^2	$6 \cdot 6 \times 10^{-3}$	10^{-6}	10^2	0·17
10^{-8}	10^3	$6 \cdot 7 \times 10^{-3}$	10^{-6}	10^3	0·46
10^{-8}	10^4	$6 \cdot 7 \times 10^{-3}$	10^{-6}	10^4	0·67
10^{-8}	10^5	$4 \cdot 6 \times 10^{-2}$	10^{-6}	10^5	0·66
10^{-7}	10^2	$6 \cdot 6 \times 10^{-2}$	10^{-5}	10^2	0·55
10^{-7}	10^3	$6 \cdot 7 \times 10^{-2}$	10^{-5}	10^3	1·7
10^{-7}	10^4	$6 \cdot 7 \times 10^{-2}$	10^{-5}	10^4	4·6
10^{-7}	10^5	$6 \cdot 7 \times 10^{-2}$	10^{-5}	10^5	6·6

antenna is comparable with wave-length, the field is mostly inductive and its strength varies as $\frac{1}{r^2}$ (instead of $\frac{1}{r}$ as in the case of radiation field) thus complicating the problem.

The existence of a low-conducting granite below the surface of the earth has to be experimentally confirmed and to use this wave-guide for sub-surface propagation, its uniformity over the entire region under consideration and its dimensions are to be determined. The electrical conductivity, thermal and other possible noise sources are to be explored. Once these are determined and suitable antennas are designed, sub-surface propagation can be more effectively put to civilian as well as military use.

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