EFFECT OF RADIATIVE TRANSFER ON THE THERMAL INSTABILITY OF A ROTATING FLUID SPHERE

S. K. Bahl

Defence Science Laboratory, Delhi

(*Received* 25 *Nov.*, 1966)

This paper deals with the effect of thermal radiative transfer on the stability of a rotating fluid sphere of constant density and heated within. The problem has been examined for two asymptotic approximations of the radiative transfer equation. For the transparent case the variational principle for solving the relevant equations has been established by assuming the temperature gradient β to be constant. It is observed that radiative transfer has a stabilising effect on the fluid motion.

Goody[1] was first to introduce the thermal radiative transfer effects in the classical problem of Rayleigh[2] or Pellew & Southwell[3] in which they had studied the behaviour of a fluid enclosed between two parallel plates heated from below. Recently Khosla & Murgai[4] extended the earlier works of Goody[1] and Chandrasekhar[5] for plane geometry by studying the effect of radiative transfer on the thermal instability when a Coriolis force is also acting.

Because of its astrophysical significance Chandrasekhar[6] studied the problem of the onset of thermal instability in a rotating fluid sphere heated within for a specific set of boundary conditions. Later Bisshopp[7] studied the same problem by a method which takes into account the full set of boundary conditions.

Following the work of Khosla & Murgai on plane geometries, the work of Bisshopp has been extended here to include the combined effects of radiative transfer and rotation on the thermal stability of an incompressible fluid sphere under Bisshopp's assumptions that the rotational flattening of the sphere is neglected and that the fluid motions are all symmetrical about the axis of rotation. The problem has been examined for two asymptotic cases of the radiative transfer equation namely when the fluide is optically thin and when it is optically thick. The analysis is confined to the case when the bounding surface is free. The temperatue gradient β has been assumed[8] to be constant the case of transparent approximation.

In the opaque case calculation of the critical Rayleigh number reduces to changing thermal diffusivity K into $K\ (1+\chi)$ whereas in the transparent case, the solution is more complicated and we have confined our attention for first approximation only by setting (1, 1) element of the resulting determinant equal to zero. The numerical results show that radiative transfer has an inhibiting influence on the thermal instability of the fluid.

EQUATIONS OF THE PROBLEMS

Consider an incompressible fluid sphere of radius r_o rotating with an angular velocity Ω about the Z — axis and with a distribution of heat sources ϵ which maintains a radial temperature gradient in the fluid.

The temperature distribution inside the sphere is given by the energy equation for the initial static case, *i.e.*

$$K \nabla^2 T_o + \epsilon + \frac{\Phi_o}{C_p} = 0 \quad (1)$$

where K is the thermal diffusivity, T_o is the temperature, Φ_o the radiative heating per unit volume and C_p the specific heat per unit volume.

The solution of (1) is discussed for two asymptotic approximations of the radiative transfer equation, namely when the fluid is optically thin and when it is optically thick[4]. The mean free path of radiation is k^{-1}, where k is the absorption coefficient. The two cases correspond respectively to $kr_o << \text{ or } >> 1$. The radiative heating Φ_o in the two cases is gien as

$$\Phi_o = \begin{cases} -4\pi k B & kr_o << 1 \quad (2a) \\ \dfrac{4\pi}{3k} \nabla^2 B & kr_o >> 1 \quad (2b) \end{cases}$$

where B is the Plank function. The temperature of the outer surface has been assumed to be zero and does not contribute to Φ_o in (2a). Using (2), equation (1) can be solved as

$$T_o = \left\{ \begin{array}{ll} \dfrac{\epsilon}{\lambda^2} \left\{ \dfrac{I_{\frac{1}{2}}(\lambda r)}{\sqrt{r}\, I_{\frac{1}{2}}(\lambda)} - 1 \right\} & kr_0 << 1 \\ \beta_1 (1 - r^2) & kr_o >> 1 \end{array} \right\} \quad (3)$$

where

$$\lambda^2 = \frac{4\pi S kr_o^2}{KC_p} = 3k^2 r_0^2 \chi\,; \qquad \chi = \frac{4\pi S}{3kKC_p}$$

$$S = \frac{\sigma T_o^3}{\pi} \quad \text{and} \quad \beta_1 = \frac{\epsilon r_0^2}{6K(1+\chi)}$$

and σ is Stefan's constant.

In evaluating the temperature distribution, the two constants of integration have been determined from the condition that T_o is zero at the surface and is finite at the centre. T_o^3 has been assumed to be constant[1].

Applying Bossinesq approximation *i.e.* allowing the variation of density only in the external force the linearised equations of the problem may be written as

$$div\, \vec{U} = 0 \quad (4)$$

$$\frac{\partial \vec{U}}{\partial t} = -\, grad \left(\frac{p}{\rho_o} \right) + \gamma\, \theta\, \vec{r} - \nu\, Curl^2 \vec{U} + 2\,\Omega\, \vec{U} \times \vec{I_z} \quad (5)$$

$$\frac{\partial \theta}{\partial t} = K \nabla^2 \theta + 2\beta\, \vec{U}.\, \vec{r} + \frac{\phi}{C_p} \quad (6)$$

where θ, p, ϕ and $\vec{U}$ are the small perturbations in temperature, pressure, radiative heating and velocity respectively; ν is the kinematic viscosity and ρ_o the density of the fluid in the static case

and
$$\gamma = \frac{4\pi}{3} \rho G \alpha$$

where G is the gravitational constant and α the coefficient of volume expansion.

MARGINAL STABILITY

Now $\left(\frac{p}{\rho_0}\right)$ can be eliminated from (5) by taking its curl. Further we make use of the following expression[5] for a solenoidal vector field as the superposition of a poloidal and a toroidal field in terms of the two azimuth independent scalars U and V in the spherical coordinates $(r, \mu = \cos\theta, \phi)$

$$\vec{U} = -\frac{\partial}{\partial \mu}\left[(1-\mu^2)U\right]\vec{I}_r - \frac{\sqrt{1-\mu^2}}{r}\frac{\partial}{\partial r}(r^2 U)\vec{I}_\theta + r\sqrt{(1-\mu^2)}\, V \vec{I}_\phi \quad (7)$$

The equations governing marginal stability are characterised by $\frac{\partial}{\partial t} = 0$. The dimensionless forms of these will be

$$\triangle_5^2 U - T\frac{\partial V}{\partial Z} = R\frac{1}{r}\frac{\partial \theta}{\partial \mu} \quad (8)$$

$$\triangle_5 V + \frac{\partial U}{\partial z} = 0 \quad (9)$$

$$(\triangle_3 - \lambda^2)\theta = r\frac{\partial}{\partial \mu}\left[(1-\mu^2)U\right], \qquad k r_0 << 1 \quad (10a)$$

$$\triangle_3 \theta = \frac{r}{(1+\chi)}\frac{\partial}{\partial \mu}\left[(1-\mu^2)U\right], \qquad k r_0 >> 1 \quad (10b)$$

where $R = \frac{2\beta\gamma}{K\nu} r_0^6$ and $T = \frac{4\Omega^2}{\nu^2} r_0^4$ are the Rayleigh and Taylor numbers respectively;

$$\triangle_5 = \frac{\partial^2}{\partial r^2} + \frac{4}{r}\frac{\partial}{\partial r} + \frac{(1-\mu^2)}{r^2}\frac{\partial^2}{\partial \mu^2} - \frac{4\mu}{r^2}\frac{\partial}{\partial \mu}$$

and

$$\triangle_3 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{(1-\mu^2)}{r^2}\frac{\partial^2}{\partial \mu^2} - \frac{2\mu}{r^2}\frac{\partial}{\partial \mu}$$

are the five and three dimensional Laplacian operators for axisymmetrical function.

BOUNDARY CONDITIONS

Assuming a free surface at $r = 1$, the boundary conditions will be

$$U = 0,\ \frac{\partial^2}{\partial r^2}(rU) = 0,\ \frac{\partial \theta}{\partial \mu} = 0,\ \frac{\partial V}{\partial r} = 0 \text{ at } r = 1 \quad (11)$$

THE VARIATIONAL PRINCIPLE.

The values of R can be determined in terms of the parameter T with the help of a variational principle. Multiplying (9) by $U\, r^4\, (1-\mu^2)\, dr\, d\mu$ and integrating over the five dimensional sphere, we get the following formula[7] which provides the basis for this method.

$$R = \frac{\int_0^1 r^4\, dr \int_{-1}^{1} (1-\mu^2) \left[U \triangle^2_5 U - TU \left(\frac{\partial V}{\partial z} \right) \right] d\mu}{\int_0^1 r^4\, dr \int_{-1}^{1} (1-\mu^2)\, U \left(\frac{1}{r} \frac{\partial \theta}{\partial \mu} \right) d\mu} \tag{12}$$

The solution of the problem requires the construction of a trial function involving several arbitrary parameters. We will have to find the values of θ and V for some assumed form of U and the chosen form must satisfy the boundary conditions. These conditions can be met by functions generated as solutions by the equation.

$$\triangle^2_5 U = \alpha^4 U \tag{13}$$

Since radiation does not affect the boundary conditions of the problem and the form of (9) in this case agrees with the non-radiative stability problem studied by Bisshopp, we can use the same general solution of (13) without any loss of generality for the radiative case as well. So we may put

$$U = \sum_{n=0}^{\infty} \sum_{j=1}^{\infty} A_{nj}\, f\,(\alpha_{nj}\, r)\, C_n^{\frac{3}{2}}(\mu) \Big/ r^{\frac{3}{2}} \tag{14}$$

where

$$f\,(\alpha_{nj}\, r) \equiv f_{\frac{n}{n}}\,(\alpha_{nj}\, r) = \frac{J_{n+\frac{3}{2}}(\alpha_{nj}\, r)}{J_{n+\frac{3}{2}}(\alpha_{nj})} - \frac{I_{n+\frac{3}{2}}(\alpha_{nj}\, r)}{I_{n+\frac{3}{2}}(\alpha_{nj})} \tag{15}$$

and α_{nj} is the jth root of

$$\frac{J'_{n+\frac{3}{2}}(\alpha)}{J_{n+\frac{3}{2}}(\alpha)} - \frac{I'_{n+\frac{3}{2}}(\alpha)}{I_{n+\frac{3}{2}}(\alpha)} = -\alpha \tag{16}$$

$C_n^{\frac{3}{2}}$ are the Gegenbauer polynomials and $J_{n+\frac{3}{2}}$ and $I_{n+\frac{3}{2}}$ are the Bessel functions of order $(n+\frac{3}{2})$ for real and imaginary arguments.

Now we shall solve (10) by making use of the above mentioned form of U.

By taking into account the identity

$$\frac{1}{r} \frac{\partial}{\partial \mu} (\triangle_3 - \lambda^2)\, \theta = (\triangle_5 - \lambda^2) \left(\frac{1}{r} \frac{\partial \theta}{\partial \mu} \right) \tag{17}$$

and expression (14), we can rewrite equation [10a] as

$$(\triangle_5 - \lambda^2)\left(\frac{1}{r}\ \frac{\partial\theta}{\partial\mu}\right) = \frac{\partial^2}{\partial\mu^2}\left[\sum_{n=0}^{\infty}\sum_{j=1}^{\infty}(1-\mu^2)\ A_{nj}\, f(\alpha_{nj}\, r)\ C_n^{3/2}(\mu)/r^{3/2}\right] \quad (18)$$

Since $\frac{d^2}{d\mu^2}\left[(1-\mu^2)\ C_n^{3/2}(\mu)\right] = -(n+1)(n+2)\ C_n^{3/2}(\mu)$

(18) can be written as

$$\left(\triangle_5 - \lambda^2\right)\left(\frac{1}{r}\ \frac{\partial\theta}{\partial\mu}\right) = -\sum_{n=0}^{\infty}\sum_{j=1}^{\infty}(n+1)(n+2)\ A_{nj}\, f(\alpha_{nj}\, r)\ C_n^{3/2}(\mu)/r^{3/2} \quad (19)$$

$$\text{Let}\left(\frac{1}{r}\ \frac{\partial\theta}{\partial\mu}\right) = \sum_{n=0}^{\infty}\sum_{j=1}^{\infty}\left[B_{nj}\, f(\alpha_{nj}\, r) + C_{nj}\, g(\alpha_{nj}\, r)\right] C_n^{\frac{3}{2}}(\mu)/r^{\frac{3}{2}} \quad (20)$$

be the particular solution of (19)

where $g(\alpha_{nj}\, r) = \dfrac{J_{n+3/2}(\alpha_{nj}\, r)}{J_{n+3/2}(\alpha_{nj})} + \dfrac{I_{n+3/2}(\alpha_{nj}\, r)}{I_{n+3/2}(\alpha_{nj})}$

and B_{nj} and C_{nj} are the constants to be determined. Substituting the value of $\frac{1}{r}\ \frac{\partial\theta}{\partial\mu}$ from (20) in (19) and using the relevant relations[7] among the various functions used, we get

$$\sum_{n=0}^{\infty}\sum_{j=1}^{\infty}\left[B_{nj}\ \alpha^2_{nj}\ g(\alpha_{nj}\, r) + C_{nj}\ \alpha^2_{nj}\, f(\alpha_{nj}\, r)\right] C_n^{\frac{3}{2}}(\mu)/r^{\frac{3}{2}}$$

$$+\ \lambda^2\sum_{n=0}^{\infty}\sum_{j=1}^{\infty}\left[B_{nj}\ f(\alpha_{nj}\, r) + C_{nj}\ g(\alpha_{nj} r)\right] C_n^{\frac{3}{2}}(\mu)/r^{\frac{3}{2}}$$

$$= \sum_{n=0}^{\infty}\sum_{j=1}^{\infty} a_{nj}\, f(\alpha_{nj}\, r)\ C_n^{\frac{3}{2}}(\mu)/r^{\frac{3}{2}} \quad (21)$$

where $a_{nj} = (n+1)(n+2)\ A_{nj}$

Constants B_{nj} and C_{nj} are determined from (21) by comparing the coefficents of $f(\alpha_{nj}\, r)$ and $g(\alpha_{nj}\, r)$ Substituting these in (20) we get

$$\frac{1}{r}\ \frac{\partial\theta}{\partial\mu} = \sum_{n=0}^{\infty}\sum_{j=1}^{\infty}\left[\lambda^2 f(\alpha_{nj}\, r) - \alpha^2_{nj}\ g(\alpha_{nj} r)\right]\frac{a_{nj}}{\lambda^4 - \alpha_{nj}{}^4}\ \ \frac{C_g(\mu)}{r^{3/2}} \quad (22)$$

The complementary function will be given as

$$\frac{1}{r}\frac{\partial\theta}{\partial\mu}=\sum_{n=0}^{\infty}\left[D_n\, I_{n+\frac{3}{2}}(\lambda r)+E_n\, K_{n+\frac{3}{2}}(\lambda r)\right]\frac{C_n^{\frac{3}{2}}(\mu)}{r^{3/2}} \tag{23}$$

where D_n and E_n are constants.

Since $\frac{1}{r}\frac{\partial\theta}{\partial\mu}$ has no singularity at $r=0$. therefore $E_n=0$.

Thus the complete solution of (19) can be written as

$$\frac{1}{r}\frac{\partial\theta}{\partial\mu}=\sum_{n=o}^{\infty}\sum_{j=1}^{\infty}\left[\lambda^2 f(\alpha_{nj}\, r)-\alpha^2_{nj}\, g(\alpha_{nj}\, r)\right]\frac{a_{nj}}{\lambda^4-\alpha^4_{nj}}C_n^{\frac{3}{2}}(\mu)\Big/ r^{\frac{3}{2}}$$

$$+\sum_{n=o}^{\infty}D_n\, I_{n+\frac{3}{2}}(\lambda r)\, C_n^{\frac{3}{2}}(\mu)\Big/ r^{\frac{3}{2}} \tag{24}$$

D_n is determined by the application of boundary condition $\frac{\partial\theta}{\partial\mu}=0$ at $r=1$.

In more abridged form (24) finally becomes

$$\frac{1}{r}\frac{\partial\theta}{\mu}=\sum_{n=o}^{\infty}\sum_{j=1}^{\infty}A_{nj}\, S_{nj}(r)\, C_n^{\frac{3}{2}}(\mu)\Big/ r^{\frac{3}{2}} \qquad \text{for } k\, r_o \ll 1 \tag{25a}$$

Similarly the solution of 10 (b) will give the following expression for $\frac{1}{r}\frac{\partial\theta}{\partial\mu}$

$$\frac{1}{r}\frac{\partial\theta}{\partial\mu}=\sum_{n=o}^{\infty}\sum_{n=j}^{\infty}A_{nj}\, \overset{*}{S}_{nj}(r)\, C_n^{\frac{3}{2}}(\mu)\Big/ r^{\frac{3}{2}} \qquad \text{for } k\, r_o \gg 1 \tag{25b}$$

where

$$S_{nj}(r)=\sum_{n=o}^{\infty}\sum_{j=1}^{\infty}\left[\lambda^2 f(\alpha_{nj}\, r)-\alpha^2_{nj}\, g(\alpha_{nj}\, r)+\frac{2\,\alpha^2_{nj}\, I_{n+\frac{3}{2}}(\lambda r)}{I_{n+\frac{3}{2}}(\lambda)}\right]\times\frac{(n+1)(n+2)}{\lambda^4-\alpha^4_{nj}} \tag{2\ }$$

and

$$\overset{*}{S}_{nj}(r)=\sum_{n=0}^{\infty}\sum_{j=1}^{\infty}\left[g(\alpha_{nj}\, r)-2r^{n+\frac{3}{2}}\right]\frac{(n+1)(n+2)}{(1+\lambda)\,\alpha^2_{nj}} \tag{27}$$

Since (9) remains the same as in Bisshopp's case, we may directly make use of the expression for $\dfrac{\partial V}{\partial Z}$ derived by him.

Now in order to find the critical Rayleigh number, we minimise the right hand side of (12) with respect to A_{mk} and set the determinant of the resulting system of homogeneous linear equations equal to zero. The required determinant is, therefore,

$$\begin{aligned} | \, \delta_{m,n} \, [\alpha^4_{mk} < mj \, | \, F^2 \, | \, mk > - R < mj \, | \, FS \, | \, mk > - T < mj \, | \, FQ \, | \, mk >] \\ - \delta_{m-2,\,n} \, T < mk \, | \, FP \, | nj > - \delta_{m+2,n} \, T < mk \, | \, FR \, | \, nj > | = 0 \end{aligned} \tag{28}$$

$$\text{for } kro << 1$$

Replacing, S by S^* we shall get the corresponding determinant for the opaque case.

On comparison of the above determinant with that of Bisshopp's case (*i.e.* a non radiative case), we find that some of the elements here have been modified due to the introduction of radiative heat transfer. These are detailed below[7]:

$$< nk \, | \, FS \, | \, nk > = \frac{2\,(n+1)\,(n+2)}{2n+3} \int_0^1 r dr \, F_{nj}(r) \, S_{nj}(r)$$

$$\begin{aligned} = \frac{2\,(n+1)^2\,(n+2)^2}{(2n+3)\,(\lambda^4 - \alpha^4_{nk})} \Bigg[- \frac{\lambda^2}{2} \alpha_{nk} \, g'(\alpha_{nk}) - \frac{\alpha^2_{nk}}{4} \Big\{ [f'(\alpha_{nk})]^2 + [g'(\alpha_{nk})]^2 \Big\} \\ + \left(n + \frac{3}{2}\right)^2 + \left(\frac{4\alpha^4_{nk}}{\alpha^4_{nk} - \lambda^4} \right) \lambda \, \frac{I_{n+5/2}(\lambda)}{I_{n+3/2}(\lambda)} + \frac{2\,(2n+3)\,\alpha^4_{nk}}{\alpha^4_{nk} - \lambda^4} \\ - 2\alpha^2_{nk} \left\{ \frac{\alpha_{nk}}{\alpha^2_{nk} + \lambda^2} \, \frac{J'_{n+3/2}(\alpha_{nk})}{J_{n+3/2}(\alpha_{nk})} + \frac{\alpha_{nk}}{\alpha^2_{nk} - \lambda^2} \, \frac{I'_{n+3/2}(\alpha_{nk})}{I_{n+3/2}(\alpha_{nk})} \right\} \Bigg] \end{aligned} \tag{29}$$

and $< nk | FS^* | nk > = \displaystyle\int_0^1 r \, dr \, F_{nj}(r) \, S^*_{nj}(r) \, \frac{2\,(n+1)\,(n+2)}{2n+3}$

$$\begin{aligned} = \frac{2\,(n+1)^2\,(n+2)^2}{(1+\chi)\,(2n+3)\,\alpha^2_{nk}} \Bigg[\tfrac{1}{4} \Big\{ (f'\{\alpha_{nk}\})^2 + (g'\{\alpha_{nk}\})^2 \Big\} - \\ \frac{n^2 + 7n + 33/4}{\alpha^2_{nk}} + \frac{2g'(\alpha_{nk})}{\alpha_{nk}} \Bigg] \end{aligned} \tag{30}$$

For deriving (29) we have used the following relation

$$\begin{aligned} \int_0^1 r dr f(\alpha_{nk} r) \, \frac{I_{n+3/2}(\lambda r)}{I_{n+3/2}(\lambda)} = \left(\frac{2\alpha^2_{nk}}{\alpha^4_{nk} - \lambda^4} \right) \lambda \, \frac{I_{n+5/2}(\lambda)}{I_{n+3/2}(\lambda)} + \frac{(2n+3)\,\alpha^2_{nk}}{\alpha^4_{nk} - \lambda^4} - \\ \left\{ \frac{\alpha_{nk}}{\alpha^2_{nk} + \lambda^2} \, \frac{J'_{n+3/2}(\alpha_{nk})}{J_{n+3/2}(\alpha_{nk})} + \frac{\alpha_{nk}}{\alpha^2_{nk} - \lambda^2} \, \frac{I'_{n+3/2}(\alpha_{nk})}{I_{n+3/2}(\alpha_{nk})} \right\} \end{aligned} \tag{31}$$

A first approximation to the value of R is obtained by setting (1,1) element of the determinant (28) equal to zero. Thus we obtain

$$R = \frac{\alpha^4_{01} < 01|F^2|01> - T < 01|FQ|01>}{< 01|FS|01>} \quad \text{for } kr_o << 1 \qquad (32)$$

and for $kr_o >> 1$, S is to be replaced by S^* in (32).

TABLE 1

VALUE OF R FOR DIFFERENT VALUES OF λ & T

λ	T = 0	T = 10^3	T = 10^4
10	$2 \cdot 0061 \times 10^4$	$6 \cdot 1459 \times 10^4$	$4 \cdot 3404 \times 10^5$
10^2	$1 \cdot 5182 \times 10^6$	$4 \cdot 6513 \times 10^6$	$3 \cdot 2846 \times 10^7$

TABLE 2

THE RATIO R/Ro FOR DIFFERENT VALUE OF λ & T

λ	T = 0	T = 10^3
10	$6 \cdot 4901$	$6 \cdot 7891$
10^2	$4 \cdot 9118 \times 10^2$	$4 \cdot 9181 \times 10^2$

Table 1 indicates the values of R for $\lambda = 10$ and 10^2 and $T = 0$, 10^3 and 10^4 for the transparent case. The values of the ratio R/R_o have been given in Table 2, where R_o is the values of the critical Rayleigh number without radiation obtained by Bisshopp. The calculations have been performed for the transparent case only. For the opaque case the values of the Rayleigh number can easily be obtained from those given by Bisshopp by multiplying them with $(1 + \chi)$. It is quite apparent from Tables 1and2 that as λ is increased from 10 to 10^2, Rayleigh number increases much faster, thus increasing the stabilising effect on the disturbance of the fluid.

ACKNOWLEDGEMENTS

The author is grateful to Dr. R.B. Aggarwal, Assistant Director, Applied Mathematics Division and Dr. C.D. Ghildyal for suggestions and guidance. He is also thankful to the Director, Defence Science Laboratory, Ministry of Defence for permission to publish this paper and Shri K. Vishwanathan for useful discussion during the course of this study.

REFERENCES

1. GOODY, R. M., *J. Fluid Mech.*, **1** (1956), 424.
2. RAYLEIGH, LORD, *Phil. Mag.*, (1916), 529.
3. PELLEW & SOUTHWELL, R. V., *Proc. Roy. Soc.*, A, 176 (1940), 312.
4. KHOSLA, P. K. & MURGAI, M. P., *J. Fluid Mech.* **16** (1963), 97.
5. CHANDRASEKHAR, S., "Hydrodynamic and Hydromagnetic stability" (Clarendan Press, Oxford), 1961.
6. ———*Phil. Mag*, Ser 8, **2** (1957), 1282.
7. BISSHOPP., F. E., *Phil. Mag.*, Ser. 8, **3** (1958), 1342, 1347.
8. BAHL, S. K. & AHUJA, K. L., *Def. Sci. J.*, **16** (1966), 223.