OPTIMUM ALLOCATION OF INVENTORY TO A CHAIN OF INTERCONNECTED WAREHOUSES

R. L. Hasija

Defence Science Laboratory, Delhi

(*Received* 28 *June* 66; *Revised* 17 *Jan.* 67)

Optimum allocation of inventory level for a system of n warehouses which are interconnected with each other has been studied. The analysis has been conducted for the deterministic demands only. The case of instantaneous delivery as well as lead time delivery has been treated.

Considerable analytic work has been done for the control of inventory in the case of a single warehouse [1,2,3] but very little work has been done in the case where the management maintains a series of warehouses. The case of a number of warehouses inter-connected with each other is discussed in this paper. These types of problems arise in defence and in fact most of the problems of inventory control are of this kind.

Two cases—one involving lead time for delivery and the other with instantaneous arrivals—have been studied. For instantaneous arrival we assume that there is a chain of n warehouses from which demands are met. Each warehouse has got its own source of demands which are independent of one another. When the stock at the first warehouse finishes then the demands are met from the second warehouse, and when the stock at this warehouse also finishes then all the demands on the warehouses one to three are met from the third warehouse and so on. We assume here that the stock at the ith warehouse will finish only after the stock at all the preceeding $(i - 1)$ warehouses has already finished. We place an order whenever the stock at the last warehouse has also finished and the orders arrive immediately.

In the second case we take a lead time of T units and as soon as the stock at the first warehouse finishes we place an order and in the mean time the demands are met from the second warehouse and when the inventory level at this warehouse reaches zero then all the demands for the warehouse one to three are met from the third warehouse and so on. We assume that the stock at (n-1) warehouses finishes during the lead time whereas the stock at the nth warehouse does not finish during the lead time. As soon as the order arrives the inventory situation reaches to the original position.

We assume that the demands from the ith warehouse can be met from $(i + 1)$ th but not vice versa.

We determine the Total Variable Cost (T.V.C.) per unit time and then minimize it with respect to the inventory at the warehouse to determine the optimum inventory level at the various warehouse of the chain.

FORMULATION OF THE PROBLEM

Q_i = Initial stock at the ith warehouse ($i = 1, 2, \ldots\ldots n$)

λ_i = Demand per unit time at the ith warehouse ($i = 1, 2, \ldots\ldots n$)

In the second case where we place the order when the stock at the 1st warehouse finishes we have $Q_1 = \lambda_1 \tau$ where τ is the ordering interval. And $\lambda_i < Q_i / \tau$ for all $i \neq 1$

C_i = The cost of the product (materials and labour and other fixed costs) at the ith warehouse ($i = 1, 2 \ldots\ldots n$)

I = Interest rate per unit time.

C_{ij} = Transportation cost per unit item from ith warehouse to the jth warehouse

$i = 1, 2 \ldots\ldots (n-1)$

$j = 2, 3 \ldots\ldots n$ $\quad i \neq j \quad i < j$

T = The lead time

A = Ordering cost

$(\text{T.V.C.})_w$ = Total variable cost without lead time.

$(\text{T.V.C.})_l$ = Total variable cost with lead time.

We now proceed to find the T.V.C. for keeping the inventory at various warehouses in the chain for the case of instantaneous delivery and for the case with lead time.

Without lead time

If we assume that the delivery of orders is instantaneous we find that T.V.C. for n warehouses is

$$(\text{T.V.C.})_w = A + \frac{IC_1 Q_1^2}{2\lambda_1} + \frac{IC_2}{2}\left(2Q_2 - \frac{Q_1}{\lambda_1}\lambda_2\right)\frac{Q_1}{\lambda_1} + \frac{IC_2\left(Q_2 - \frac{Q_1}{\lambda_1}\lambda_2\right)^2}{2(\lambda_1+\lambda_2)}$$

$$+ \frac{IC_3}{2}\left(2Q_3 - \frac{Q_1+Q_2}{\lambda_1+\lambda_2}\lambda_3\right)\frac{Q_1+Q_2}{\lambda_1+\lambda_2} + \frac{IC_3}{2}\frac{\left(Q_3 - \frac{Q_1+Q_2}{\lambda_1+\lambda_2}\lambda_3\right)^2}{\lambda_1+\lambda_2+\lambda_3} + \ldots\ldots$$

$$+ \frac{IC_n}{2}\left[2Q_n - \frac{\sum_1^{n-1} Q_i}{\sum_1^{n-1}\lambda_i}\lambda_n\right] \times \frac{\sum_1^{n-1} Q_i}{\sum_1^{n-1}\lambda_i}$$

$$+ \frac{IC_n\left[Q_n - \frac{\sum_1^{n-1} Q_i}{\sum_1^{n-1}\lambda_i}\lambda_n\right]^2}{2\sum_1^n \lambda_i} + \lambda_1 C_{12}\frac{Q_2 - \frac{Q_1}{1}\lambda_2}{\lambda_1+\lambda_2} +$$

$$\frac{Q_3 - \frac{Q_1 + Q_2}{\lambda_1 + \lambda_2}\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} (\lambda_1 C_{13} + \lambda_2 C_{23}) + \ldots\ldots$$

$$+ \frac{\sum_1^{n-1} C_{in}\lambda_i}{\sum_1^{n} \lambda_i} \left[Q_n - \frac{\sum_1^{n-1} Q_i}{\sum_1^{n-1} \lambda_i} \right] \tag{1}$$

Here the length of the cycle is $\dfrac{\sum_1^n Q_i}{\sum_1^n \lambda_i}$ therefore

$$\text{T.V.C. per unit time} = \frac{(\text{T.V.C.})_w \sum_1^n \lambda_i}{\sum_1^n Q_i}$$

With lead time

We place an order when the inventory level at the first warehouse reaches the level zero and it arrives after a lead time T. We assume that here the stock at the last warehouse does not finish during the lead time.

We find that the total variable cost for n warehouses is

$$(\text{T.V.C.})_l = A + \frac{IC_1 Q_1^2}{2\lambda_1} + \frac{IC_2}{2}\left(2Q_2 - \frac{Q_1}{\lambda_1}\lambda_2\right)\frac{Q_1}{\lambda_1} + \frac{IC_2\left(Q_2 - \frac{Q_1}{\lambda_1}\lambda_2\right)^2}{2(\lambda_1 + \lambda_2)} + \ldots$$

$$+ \frac{IC_n}{2}\left\{2Q_n - \frac{\sum_1^{n-1} Q_i \lambda_n}{\sum_1^{n-1}\lambda_i}\right\} \times \frac{\sum_1^{n-1} Q_i}{\sum_1^{n-1}\lambda_i} + \frac{IC_n}{2} T_1$$

$$\times \left[2Q_n - \frac{2\sum_1^{n-1} Q_i}{\sum_1^{n-1}\lambda_i}\lambda_n - T_1 \sum_1^n \lambda_i\right]$$

$$+ \lambda_1 C_{12} \frac{Q_2 - \frac{Q_1}{\lambda_1}\lambda_2}{\lambda_1 + \lambda_2} + \frac{Q_3 - \frac{Q_1 + Q_2}{\lambda_1 + \lambda_2}\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}(C_{13}\lambda_1 + C_{23}\lambda_2) + \ldots$$

$$+ \sum_1^{n-1} C_{in}\lambda_i [T_1] \tag{2}$$

Where $$T_1 = T - \sum_1^{n-1} Q_i \Big/ \sum_1^{n-1} \lambda_i + \frac{Q_1}{\lambda_1}$$

T.V.C. per unit time $= (\text{T.V.C.})_l / (Q_1/\lambda_1 + T)$ ......

(1) can be put in a simplified form easy to differentiate thus;

$$T.V.C. = \frac{\sum_1^n \lambda_i}{\sum_1^n Q_i} \left[-\frac{IC_n\lambda_n}{2\sum_1^{n-1}\lambda_i \sum_1^n \lambda_i} - \frac{IC_{n-1}\lambda_{n-1}\left(\sum_1^{n-2} Q_i\right)^2}{2\sum_1^{n-2}\lambda_i \sum_1^{n-1}\lambda_i} - \cdots - \frac{IC_3\lambda_3(Q_1+Q_2)^2}{2(\lambda_1+\lambda_2)\sum_1^3\lambda_i} \right.$$

$$- \frac{IC_2\lambda_2 Q_1^2}{2\lambda_1(\lambda_1+\lambda_2)} + \frac{IC_n Q_n\left[\sum_1^{n-1} Q_i + \frac{1}{2}Q_n\right]}{\sum_1^n \lambda_i} + \cdots\cdots + \frac{IC_2 Q_2(Q_1+\frac{1}{2}Q_2)}{\sum_1^2 \lambda_i}$$

$$+ \frac{IC_1 Q_1(\frac{1}{2}Q_1)}{\lambda_1} + \frac{\lambda_1 C_{12}\left(Q_2 - \frac{Q_1}{\lambda_1}\lambda_2\right)}{\lambda_1+\lambda_2} + \cdots\cdots\cdots$$

$$\left. + \frac{\sum_1^{n-1} C_{in}\lambda_i \left[Q_n - \frac{\sum_1^{n-1} Q_i}{\sum_1^{n-1}\lambda_i}\lambda_n \right]}{\sum_1^n \lambda_i} + A \right]$$

Diff. w.r.t. $Q_1, Q_2, Q_3 \ldots\ldots\ldots\ldots Q_n$ and putting each equal to zero we get the following system of n equations:

$$\sum_1^n Q_i \left[-\sum_{j=2}^n \frac{IC_j\lambda_j \sum_1^{j-1} Q_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i} + \sum_{j=1}^n \frac{IC_j Q_j}{\sum_{i=1}^{j}\lambda_i} - \sum_{j=2}^n \frac{\sum_{i=1}^{j-1} C_{ij}\lambda_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i}\lambda_i \right] = (\text{T.V.C.})\, w \quad (i)$$

$$\left.\begin{aligned} &\sum_1^n Q_i \left[-\sum_{j=3}^n \frac{IC_j\lambda_j \sum_{i=1}^{j-1} Q_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i} + \sum_{j=2}^n \frac{IC_j Q_j}{\sum_{i=1}^{j}\lambda_i} - \sum_{j=3}^n \frac{\sum_{i=1}^{j-1} C_{ij}\lambda_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i}\lambda_j \right] \\ &\qquad + \frac{IC_2 Q_1}{\lambda_1+\lambda_2} + \frac{\lambda_1 C_{12}}{\lambda_1+\lambda_2} = (\text{T. V. C.})\, w \quad (ii) \\ &\sum_1^n Q_i \left[-\sum_{j=4}^n \frac{IC_j\lambda_j \sum_{i=1}^{j-1} Q_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i} + \sum_{j=3}^n \frac{IC_j Q_j}{\sum_{i=1}^{j}\lambda_i} - \sum_{j=4}^n \frac{\sum_{i=1}^{j-1} C_{ij}\lambda_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i} \right. \\ &\qquad \left. + \frac{IC_3(Q_1+Q_2)}{\lambda_1+\lambda_2} + \frac{\lambda_1 C_{13}+\lambda_2 C_{23}}{\lambda_1+\lambda_2+\lambda_3} \right] = (\text{T.V.C.})\, w \quad (iii) \\ &\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right]$$

$$\sum_1^n Q_i \left[-\sum_{j=n-1}^{n} \frac{IC_j \lambda_j \sum_{i=1}^{j-1} Q_i}{\sum_{i=1}^{j-1} \lambda_i \sum_{i=1}^{j} \lambda_i} + \sum_{j=n-2}^{n} \frac{IC_j Q_j}{\sum_{i=1}^{j} \lambda_i} - \sum_{j=n-1}^{n} \frac{\sum_{i=1}^{j-1} C_{ij} \lambda_i}{\sum_{i=1}^{j-1} \lambda_i \sum_{i=1}^{j} \lambda_i} \lambda_j + \frac{IC_{n-2} (\sum_1^{n-3} Q_i)}{\sum_1^{n-2} \lambda_i} + \frac{\sum_{i=1}^{n-3} \lambda_i C_{in-2}}{\sum_1^{n-2} \lambda_i} \right] = (\text{T.V.C.})_w \quad \ldots (n-ii)$$

$$\sum_1^n Q_i \left[-\frac{IC_n \lambda_n \sum_{i=1}^{n-1} Q_i}{\sum_{i=1}^{n-1} \lambda_i \sum_{i=1}^{n} \lambda_i} + \sum_{j=n-1}^{n} \frac{IC_j Q_j}{\sum_{i=1}^{j} \lambda_i} - \frac{\sum_{i=1}^{n-1} C_{in} \lambda_i}{\sum_{i=1}^{n-1} \lambda_i \sum_{i=1}^{n} \lambda_i} \lambda_n + \frac{IC_{n-1} \sum_{i=1}^{n-2} Q_i}{\sum_{i=1}^{n-1} \lambda_i} + \frac{\sum_{i=1}^{n-2} \lambda_i C_{in-1}}{\sum_1^{n-1} \lambda_i} \right] = (\text{T.V.C.})_w \quad \ldots (n-i)$$

$$\sum_1^n Q_i \left[\frac{IC_n \sum_1^n Q_i}{\sum_1^n \lambda_i} + \frac{\sum_{i=1}^{n-1} C_{in} \lambda_i}{\sum_1^n \lambda_i} \right] = (\text{T.V.C.})_w \quad \ldots (n) \qquad (3)$$

From 3(i) and 3(ii) we get

$$\frac{Q_1}{\lambda_1} = \frac{C_{12}}{I(C_1 - C_2)} \quad \ldots\ldots\ldots\ldots (i)$$

From 3(ii), 3(iii) we get

$$\frac{Q_1 + Q_2}{\lambda_1 + \lambda_2} = \frac{C_{23}}{I(C_2 - C_3)} \quad \ldots\ldots (ii)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

From 3($n-i$) and 3(n) we get

$$\frac{\sum_1^{n-1} Q_i}{\sum_1^{n-1} \lambda_i} = \frac{C_{n-1\,n}}{I(C_{n-1} - C_n)} \quad \ldots\ldots\ldots (n-i)$$

$$\left[\frac{IC_n \sum_{i=1}^{n} Q_i}{\sum_1^n \lambda_i} + \frac{\sum_1^{n-1} C_{in} \lambda_i}{\sum_1^n \lambda_i} \right] \sum_1^n Q_i = (\text{T.V.C.})_w \quad \ldots (n) \qquad (4)$$

From the system of n equations given in (4) we can get the inventory levels at the different n warehouses in the chain. The values of Q_i $[i = 1, 2, \ldots\ldots (n\ 1)]$ can be easily obtained from the linear relations (4); for the level at the last warehouse Q_n we have to use the quadratic relation given in the equation 4(n). As this relation is of complicated nature it is possible to give explicit value of Q_n only for small n but still this can be used for large n by a series of computation.

VERIFICATION

We now proceed to derive Wilson formula from the general model treated here by setting

$$Q_1 = Q_2 = Q_3 = \ldots\ldots\ldots\ldots\ldots\ldots Q_{n-1} = 0$$

$$\lambda_1 = \lambda_2 = \lambda_3 = \ldots\ldots\ldots\ldots \lambda_n = \frac{\lambda}{n}$$

and $\quad Q_n = Q \quad$ and $C_N = C$

We easily obtain from 4(n) that

$$\frac{IC\,Q^2}{2\ \lambda} = A$$

Therefore

$$Q = \sqrt{\frac{2\,\lambda\,A}{IC}} \tag{5}$$

This is Wilson formula[1]

With lead Time

Putting (2) in the simplified form which is easy to differentiate we get

$$\frac{1}{\frac{Q_1}{\lambda_1} + T} \left\{ -\frac{IC_n \left(\sum_1^{n-1} Q_i\right)^2}{2 \sum_1^{n-1} \lambda_i} - \frac{IC_{n-1}\,\lambda_{n-1} \left(\sum_1^{n-2} Q_i\right)^2}{2 \sum_1^{n-2} \lambda_i \sum_1^{n-1} \lambda_i} \ldots - \frac{IC_3\,\lambda_3 (Q_1 + Q_2)^2}{2 \sum_1^{2} \lambda_i \sum_1^{3} \lambda_i} \right.$$
$$- \frac{I\,C_2\,Q_1^2\,\lambda_2}{2\lambda_1 (\lambda_1 + \lambda_2)} + \frac{IC_{n-1}\,Q_{n-1} \left[\sum_1^{n-2} Q_i + \frac{1}{2} Q_{n-1}\right]}{\sum_1^{n-1} \lambda_i}$$
$$+ \ldots\ldots\ldots\ldots + \frac{IC_2\,Q_2\,(Q_1 + \frac{1}{2} Q_2)}{\lambda_1 + \lambda_2} + \frac{IC_1\,Q_1 (\frac{1}{2} Q_1)}{\lambda_1}$$
$$+ \left(T + \frac{Q_1}{\lambda_1}\right) IC_n \sum_1^{n} Q_i - \frac{IC_n}{2} \left(T + \frac{Q_1}{\lambda_1}\right)^2 \sum_1^{n} \lambda_i$$
$$+ \lambda_1 C_{12} \frac{Q_2 - \frac{Q_1}{\lambda_1} \lambda_2}{\lambda_1 + \lambda_2} + \frac{Q_3 - \frac{Q_1 + Q_2}{\lambda_1 + \lambda_2} \lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} (C_{13} \lambda_1 + C_{23} \lambda_2) + \ldots\ldots$$
$$\left. + \left[\sum_1^{n-1} C_{in} \lambda_i\right] \left[T - \frac{\sum_1^{n-1} Q_i}{\sum_1^{n-1} \lambda_i} + \frac{Q_1}{\lambda_1}\right] + A \right\}$$

Diff. w.r.t. $Q_1, Q_2, Q_3 \ldots\ldots\ldots Q_{n-1}$ and putting each equal to zero we get the following system of $(n-1)$ equations

$$(Q_1+\lambda_1 T)\left[\begin{array}{l} -\sum_{j=2}^{n-1} \dfrac{IC_j\,\lambda_j \sum_{1}^{j-i} Q_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i} + \sum_{j=1}^{n-1}\dfrac{IC_j\,Q_j}{\sum_{i=1}^{j}\lambda_i} - \dfrac{IC_n \sum_{1}^{n-1} Q_i}{\sum_{1}^{n-1}\lambda_i} \\ + IC_n\left(T+\dfrac{Q_1}{\lambda_1}\right) + \dfrac{IC_n}{\lambda_1}\sum_{1}^{n} Q_i - \dfrac{IC_n}{\lambda_1}\left(T+\dfrac{Q_1}{\lambda_1}\right)\sum_{1}^{n}\lambda_i \\ - \sum_{j=2}^{n-1}\dfrac{\sum_{i=1}^{j-1} C_{ij}\,\lambda_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i}\lambda_j - \dfrac{\sum_{i=1}^{n-1} C_{in}\,\lambda_i}{\sum_{1}^{n-1}\lambda_i} + \dfrac{\sum_{1}^{n-1} C_{in}\,\lambda_i}{\lambda_1} \end{array}\right] = (T.V.C.)_l \quad (i)$$

$$\left[\begin{array}{l} -\sum_{j=3}^{n-1}\dfrac{IC_j\,\lambda_j \sum_{1}^{j-1} Q_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i} + \sum_{j=2}^{n-1}\dfrac{IC_j\,Q_j}{\sum_{i=1}^{j}\lambda_i} - \dfrac{IC_n \sum_{1}^{n-1} Q}{\sum_{1}^{n-1}\lambda_i} + \dfrac{IC_2\,Q_1}{\lambda_1+\lambda_2} \\ + IC_n\left(T+\dfrac{Q_1}{\lambda_1}\right) + \dfrac{\lambda_1 C_{12}}{\lambda_1+\lambda_2} - \sum_{j=3}^{n-1}\dfrac{\sum_{i=1}^{j-1} C_{ij}\,\lambda_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i}\lambda_j - \dfrac{\sum_{i=1}^{n-1} C_{in}\,\lambda_i}{\sum_{1}^{n}\lambda_i} \end{array}\right] = 0 \quad (ii)$$

$$\left[\begin{array}{l} -\sum_{j=4}^{n-1}\dfrac{IC_j\,\lambda_j \sum_{1}^{j-1} Q_i}{\sum_{1}^{j-1}\lambda_i \sum_{1}^{j}\lambda_i} + \sum_{j=3}^{n-1}\dfrac{IC_j\,Q_j}{\sum_{1}^{j} Q_i} - \dfrac{IC_n \sum_{1}^{n-1} Q_i}{\sum_{1}^{n-1}\lambda_i} + \dfrac{IC_3\,(Q_1+Q_2)}{\lambda_1+\lambda_2+\lambda_3} \\ + IC_n\left(T+\dfrac{Q_1}{\lambda_1}\right) + \dfrac{\lambda_1 C_{13}+\lambda_2 C_{23}}{\lambda_1+\lambda_2+\lambda_3} - \sum_{j=4}^{j-1}\dfrac{\sum_{i=1}^{j-1} C_{ij}\,\lambda_i}{\sum_{i=1}^{j-1}\lambda_i \sum_{i=1}^{j}\lambda_i}\lambda_j - \dfrac{\sum_{1}^{n-1} C_{in}\,\lambda_i}{\sum_{1}^{n-1}\lambda_i} \end{array}\right] = 0 \ldots (iii)$$

.. ..

.. ..

.. ..

$$\left.\begin{aligned}
&\left[-\sum_{j=n-2}^{n-1}\frac{IC_j\,\lambda_j\sum_1^{j-1}Q_i}{\sum_1^{j-1}\lambda_i\;\sum_1^{j}\lambda_i}+\sum_{j=n-3}^{n-1}\frac{IC_j\;Q_j}{\sum_1^{j}\lambda}-\frac{IC_n\sum_1^{n-1}Q_i}{\sum_1^{n-1}\lambda_i}+IC_{n-3}\frac{\sum_1^{n-4}Q_i}{\sum_1^{n-3}\lambda_i}\right.\\
&\left.+IC_n\left(T+\frac{Q_1}{\lambda_1}\right)+\frac{\sum_1^{n-4}C_{in-3}\,\lambda_i}{\sum_1^{n-3}\lambda_i}-\sum_{j=n-2}^{n-1}\frac{\sum_{i=1}^{j-1}C_{ij}\,\lambda_i}{\sum_{i=1}^{j-1}\lambda_i\;\sum_{i=1}^{j}\lambda_i}\lambda_j-\frac{\sum_1^{n-1}C_{in}\,\lambda_i}{\sum_1^{n-1}\lambda_i}\right]\\
&\qquad\qquad\qquad\qquad=0\ldots\ldots(n-iii)\\
&\left[-\frac{IC_{n-1}\,\lambda_{n-1}\sum_1^{n-2}Q_i}{\sum_1^{n-2}\lambda_i\sum_1^{n-1}\lambda_i}+\sum_{j=n-2}^{n-1}\frac{IC_j\,Q_j}{\sum_1^{j}\lambda_i}-\frac{IC_n\sum_1^{n-1}Q_i}{\sum_1^{n-1}\lambda_i}+\frac{IC_{n-2}\sum_1^{n-3}Q_i}{\sum_1^{n-2}\lambda_i}\right.\\
&\left.+\left(T+\frac{Q_1}{\lambda_1}\right)IC_n+\frac{\sum_1^{n-3}C_{in-2}\,\lambda_i}{\sum_1^{n-2}\lambda_i}-\frac{\sum_1^{n-2}C_{in-1}\,\lambda_i}{\sum_1^{n-2}\lambda_i\sum_1^{n-1}\lambda_i}\lambda_{n-1}-\frac{\sum_1^{n-1}C_{in}\lambda_i}{\sum_1^{n-1}\lambda}\right]\\
&\qquad\qquad\qquad\qquad=0\ldots(n-ii)\\
&\left[\frac{IC_{n-1}\,Q_{n-1}}{\sum_1^{n-1}\lambda_i}-\frac{IC_n\sum_1^{n-1}Q_i}{\sum_1^{n-1}\lambda_i}+\frac{IC_{n-1}\sum_1^{n-2}Q_i}{\sum_1^{n-1}\lambda_i}\right.\\
&\left.+IC_n\left(T+\frac{Q_1}{\lambda_1}\right)+\frac{\sum_1^{n-1}C_{in-1}\,\lambda_i}{\sum_1^{n-1}\lambda_i}-\frac{\sum_1^{n-1}C_{in}\,\lambda_i}{\sum_1^{n-1}\lambda_i}\right]\\
&\qquad\qquad\qquad\qquad=0\ldots\ldots(n-i)
\end{aligned}\right\}\quad(6)$$

From 6(*i*) and 6(*ii*) we get

$$Q_1=\frac{\lambda_1\;C_{12}}{I(C_1-C_2)}\quad\ldots\ldots\ldots\ldots(i)$$

From 6(*ii*) and 6(*iii*) we get

$$\frac{Q_1+Q_2}{\lambda_1+\lambda_2}=\frac{C_{23}}{I(C_2-C_3)}\quad\ldots\ldots\ldots\ldots(ii)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Similarly we get

$$\frac{\sum_1^{n-2}Q_i}{\sum_1^{n-2}\lambda_i}=\frac{C_{n-2\;\;n-1}}{I(C_{n-2}-C_{n-1})}\quad\ldots\ldots\ldots\ldots(n-ii)$$

(7)

Diff. (2) *w.r.t.* T and putting equal to zero we get

$$\left(\frac{Q_1}{\lambda_1}+T\right)\left[IC_n\sum_1^n Q_i-\frac{IC_n}{\lambda_1}\left(T+\frac{Q_1}{\lambda_1}\right)\left(\sum_1^{n-1}\lambda_i\right)+\sum_1^{n-1}C_{in}\lambda_i\right]=(\text{T.V.C.})_l \quad \ldots\ldots (8)$$

The value of Q_{n-1} and T can be cbtained from 6(i) and (8) by putting the values of $Q_1, Q_2, \ldots\ldots\ldots\ldots\ldots\ldots Q_{n-2}$ from equations (7)

ACKNOWLEDGEMENTS

The author is grateful to Dr. Kartar Singh, Director and Dr. N. K. Chakravarti, Assistant Director, Dcfence Science Laboratory, Delhi fcr permission to publish this paper.

REFERENCES

1. SAIENI, M. YASPAN, A. & FRIEDMAN, L., 'Operations Research Methods and Problems,' Chapter IV (John Wiley and Sons Inc. New York).

2. ARROW, K.J., KARLIN, S. & SCARF, H., 'Studies in the Mathematical Theory of the Inventory and Production . (Standford University Press, Stanford California) 1958.

3. DVORETSKY, A., Kiefer, J.S. & WOLFWITZ, J., *Econometrice.*, **20** (1952), 187.