EXPLOSIONS IN RAREFIED ATMOSPHERES

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Cylindrical explosion in cold, homogeneous and rarefied atmosphere is studied in this paper. For different values of γ, the non-dimensional radius, ϕ is plotted against the non-dimensional time S, and a comparison is made with the results obtained by considering the flow representable by simple snowplow. It is found that the rate of change of ϕ withr espect to S is more in the case of cylindrical explosion than in the spherical explosion, whereas the variation of T/E with respect to (M/M_o-1) is same in the both cases.

Stuart[1] has studied the spherical explosions in cold, homogeneous and rarefied atmospheres for which the Taylor's solution[2] is not applicable. To describe the expansion of a spherical shock in a rarefied atmosphere, Stuart has assumed that the flow is representable by a snowplow model. Stuart's study has been extended to cylindrical explosion in the rarefied, homogeneous and cold atmosphere by using the same model.

It is considered that the explosive mass is confined initially in a very thin cylinder of infinite length. It is visualized that the medium is such that the mean free path of particles outside the cylinder is large as compared to that of particles inside the cylinder for a sufficiently long time even after the explosion. With the result, that after the explosion, the particles outside the cylinder rush inside it adding to its mass. The expansion of the cylinder is considered to be uniform and shape preserving throughout. This mode of expansion is given a name 'Snowplow model'. The addition in the mass of the cylinder goes on until the cylinder becomes sufficiently thick for the snowplow to continue.

The thickness of the shock being of the order of a few mean free paths, no appreciable amount of energy and momentum can be transferred in front of the cylinder through the agency of the shock. From a hydrodyamical solution of an expanding cylinderical piston[3], the distance between the shock and piston is obtained as $0 \cdot 07\ R$. Therefore the present explosion is studied only to a stage when the mean free path outside the cylinder is greater than $0 \cdot 07\ R$.

While considering the expansion to be based on snowplow model, the formation of interior shocks inside the cylinder is neglected. Assuming the conservation of Kinetic energy— a property of simple snowplow, we obtain

$$\frac{dR}{dt} = V_o \left(\frac{M_o}{M}\right)^{\frac{1}{2}}, \quad M = M_o + \pi R^2 \hat{\rho} \tag{1}$$

M_0 being the mass of the explosive per unit length of the cylinder, V_o the initial surface velocity and $\hat{\rho}$ the density of the medium under consideration.

Further it will be seen that in terms of the dimensionless radius ϕ and time S, (1) becomes:

$$(1+\phi^2)\left(\frac{d\phi}{dS}\right)^2 = 1 \tag{2}$$

This simple snowplow does not take into account the fact that a part of energy of explosion is utilised in heating the atmosphere. From conservation of momentum and energy, a more rigorous equation is obtained

$$(\phi + \phi^3) \frac{d^2 \phi}{dS^2} + \left[(\gamma - 1) + (\gamma + 1) \phi^2 \right] \left(\frac{d \phi}{d S} \right)^2 = 1 \tag{3}$$

γ being the ratio of specific heats.

SNOWPLOW MODEL

Suppose that as a result of explsion, E (ergs) of energy has been liberated from M_o (gm) of the explosive contained in unit length of the clyinder. The medium outside the explosive mass is supposed to be rerefied, homogeneous and cold as stated before. In order that this explosion may be of the type assumed in the snowplow model, the expansion velocity must be a function of $\frac{r}{R}$. Assuming

$$V = \left(\frac{r}{R} \right) \frac{d R}{dt} \tag{4}$$

let us write

$$\int r^2 \, dm = \alpha \, MR^2 \tag{5}$$

where $dm = \rho \, dv$, α is $\frac{1}{2}$ for a solid cylinder and 1 for a thin cylindrical shell. From (4) and (5) is obtained

$$T = \tfrac{1}{2} \int V^2 \, dm = \tfrac{1}{2} \, \alpha \, M \left(\frac{d R}{dt} \right)^2 \tag{6}$$

T being the Kinetic energy per unit length of the cylinder. Also, the mass sucked inside the cylinder can be written as

$$S = \frac{dm}{dt} = 2 \pi R \left(\frac{d R}{dt} \right) \hat{\rho} \frac{\rho (r)}{M} \tag{7}$$

Integrating it over the surface of the cylinder per unit length, the rate of mass flow through a unit length of the cylinder is obtained.

The equation of motion for a line source can be written as

$$\rho \frac{d V_i}{dt} = - \frac{\partial P}{\partial x_i} - V_i \, S \tag{8}$$

P being the Pressure.

Multiply (8) by x_i taking into consideration the summation convention, and integrating it over dV, the equation becomes

$$\int x_i \frac{d^2 x_i}{dt^2} \, dm = - \int x_i \frac{\partial P}{\partial x_i} \, dV - \int x_i \, V_i \, S dV$$

By using the substitution $r = R \, (t) \eta$ and integrating the L.H.S. of the above equation between the limits $\eta = 0$, to $\eta = 1$, the integrant is

$$\int x_i \frac{d^2 x_i}{dt^2} \, dm = \frac{\alpha M}{2} \frac{d^2}{dt^2} (R^2) - \alpha M \left(\frac{d R}{dt} \right)^2$$

Similarly integrating the pressure term by parts (The surface integral being zero for an unconfined system) the integrant is

$$\int x_i \frac{\partial P}{\partial x_i} dV = -2\int P\, dV = -2(\gamma-1)U$$

where U is the internal energy per unit length of the cylinder.

From the conservation of energy is obtained

$$E = T + U \tag{9}$$

The source term becomes

$$-\int x_i V_i S dV = -\int (\vec{r}\cdot\vec{V})\, 2\pi R \frac{dR}{dt} \hat{\rho} \frac{\rho dV}{M}$$

$$= -\frac{2\pi\left(\frac{dR}{dt}\right)^2}{M} \hat{\rho} \int r^2\, dm$$

$$= -2\pi \hat{\rho}\, \alpha R^2 \left(\frac{dR}{dt}\right)^2$$

Making the transformation

$$R = \left(\frac{M_o}{\pi\hat{\rho}}\right)^{\frac{1}{2}} \phi,\ t = \left(\frac{M_o^2\alpha}{2\pi \hat{\rho}(\gamma-1)E}\right)^{\frac{1}{2}} S \tag{10}$$

M_o being the mass of the explosive and E the energy liberated, per unit length of the cylinder.

The dimensionless equation is

$$(\phi + \phi^3)\frac{d^2\phi}{dS^2} + \left[(\gamma-1) + (\gamma+1)\phi^2\right]\left(\frac{d\phi}{dS}\right)^2 = 1$$

Performing the transformation (10) on (1) and putting $\gamma = 2$, $E = T$, we arrive at the Eqn. (2).

Studying some properties of (3), for small φ, (3) can be written as

$$(\gamma-1)\left(\frac{d\varphi}{dS}\right)^2 = 1 \tag{11}$$

On integration it becomes

$$\varphi = \left(\frac{1}{(\gamma-1)}\right)^{\frac{1}{2}} S \tag{12}$$

Thus the expansion is linear in time in this case.

For large φ, (3) becomes

$$\frac{dU}{d\varphi} + 2(\gamma+1)\frac{U}{\varphi} = \frac{2}{\varphi^3} \tag{13}$$

where

$$U = \left(\frac{d\varphi}{dS}\right)^2$$

After integrating

$$\frac{d\varphi}{dS} = \left(\frac{1}{\gamma}\right)^{\frac{1}{2}} \frac{1}{\varphi} \qquad (14)$$

Further integration gives

$$\varphi = \left(\frac{4}{\gamma}\right)^{\frac{1}{4}} S^{\frac{1}{2}}$$

Thus the radius of the shock R, for large t behaves as

$$R = \left[\frac{8}{\pi} \; \frac{\gamma - 1}{\gamma} \; \frac{E}{\hat{\rho}\,\alpha}\right]^{\frac{1}{4}} t^{\frac{1}{2}}$$

Obeying the familiar $t^{\frac{1}{2}}$ law[4].

The kinetic energy per unit length of the cylinder is given in dimensionless form as

$$\frac{T}{E} = (\gamma - 1)\,(1 + \varphi^2)\left(\frac{d\varphi}{dS}\right)^2 \qquad (15)$$

when $\phi = o$, (11) and (15) give that initially $\frac{T}{E} = 1$. Therefore, the initial instant after the

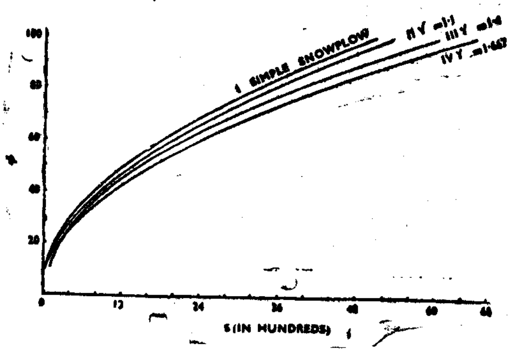

Fig. 1—Graph showing the behaviour of cylinderical explosion.

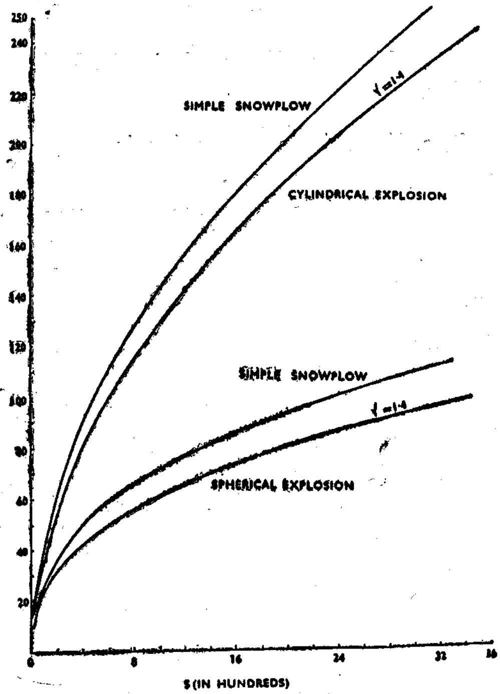

Fig. 2—Comparison of the cylindrical and spherical explosion.

explosion is that time when the internal energy of the explosion has been completely converted into Kinetic energy but M still continues to be nearly equal to M_o.

For large φ (14) and (15) give

$$\frac{T}{E} = \frac{\gamma - 1}{\gamma}$$

Comparing this result with that of Stuart for the expanding sphere it is concluded that same amount of energy is used up in frictional heating of the atmosphere in the case of cylindrical as well as spherical explosion. The solution of (3) can be obtained by numerical integration. The results are plotted in Fig. 1. The simple snowplow based on (2) is also plotted here. Also, it is found that the rate of increase of φ with respcet to S is more in the case of cylindrical explosion than in the case of spherical explosion as is evident from Fig. 2.

The graph between $\frac{T}{E}$ and $\left(\frac{M}{M_o} - 1\right)$ is plotted in Fig. 3 and it is found identical with the spherical case.

SAMPLE APPLICATION

In order to understand the application of (3), consider a long rod of a high explosive having mass 1 kg. per unit length, explodes at an altitude of 150 km. in the earth atmosphere.

Here

$$M_o = 10^3 \text{ gram}, E = 4 \times 10^{13} \text{ erg},$$

$$\hat{\rho} = 10^{-12} \text{ gram/cm}^3$$

$$(N = 10^{10} \text{ atom/cm}^3)$$

Taking $\gamma = 1{\cdot}4$.

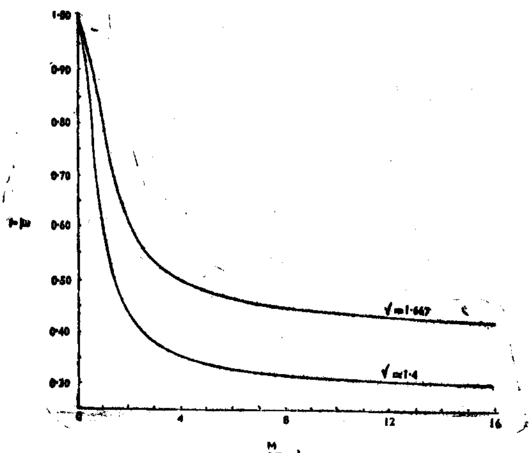

Fig. 3—Variation of the relative kinetic energy T/E with relative mass swept up (M/Mo—1).

two limitations in our problem are (*i*) the mean free path outside the cylinder should be greater than $0{\cdot}07R$ and (*ii*) the mean free path inside the cylinder is less than R. To understand the validity of these limitations, we have to know the relevant cross section, which in general depends on V and atomic specie. For simplicity, assuming $\sigma = 10^{-15}$ cm^2, the mean free path outside the cylinder is $\lambda = (N\sigma)^{-1} = 1$ km. Hence the first condition is valid upto $R < 14$ km.

To know the mean free path inside the cylinder consider the density to be uniform inside the cylinder so that it gives

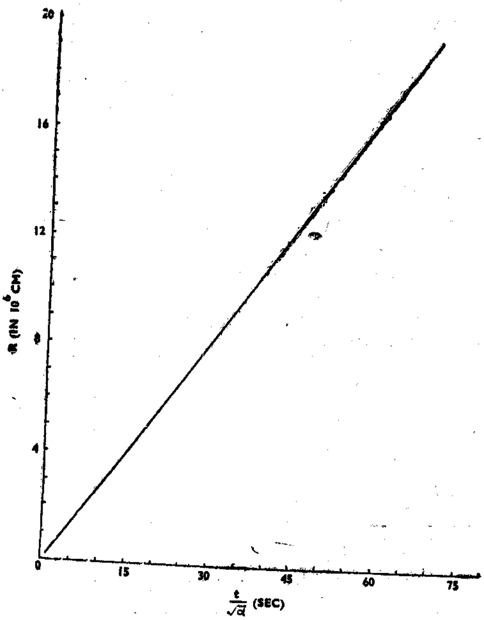

Fig. 4—Graph showing the history of chemical explosion at 150 Km. altitude in earth's atmosphere.

$$\frac{R}{\lambda} = N \sigma R \left[1 + \frac{M_o}{\pi R^2 \hat{\rho}} \right] \qquad (16)$$

From this is obtained

$$\left(\frac{R}{\lambda}\right)_{min} = 2 N \sigma \left(\frac{M_o}{\pi \hat{\rho}}\right)^{\frac{1}{2}}$$

Putting the given values the equation becomes

$$\left(\frac{R}{\lambda}\right)_{min} > 1$$

Thus the snowplow continues throughout the expansion. But for masses per unit length of the rod less than $M_o = \frac{\pi}{400} = 8 \times 10^{-3}$ grams, $\left(\frac{R}{\lambda}\right)_{min} < 1$ and hence air can pass through the cylinder after some critical time.

In the present case the decay of the internal energy U of the explosive is given by the law[5] $U \sim R^{-2\gamma}$ where $\gamma = 2 \cdot 77$. The radius R of the solid cylindrical explosive is a few centimeters, whereas the radius of the cylinder of surrounding air having mass $M_o = 10^3$ gm. per unit length is 178 Km. Hence the conditions $\frac{T}{E} = 1$, $\frac{U}{E} = 0$ initially, can be fulfilled.

After showing the application of (3) to the example, Fig. 4 is plotted by transforming Fig. 1 to dimensional form.

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