HEAT AND MASS TRANSFER IN A SEMI-INFINITE POROUS CYLINDER

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Unsteady axially symmetric transfer of heat and mass in a semi-infinite porous circular cylinder initially at a constant temperature and mass transfer potential has been considered. The circular boundary of the porous cylinder is maintained at temperature and mass transfer potential which are functions of both axial co-ordinate and time, whereas the plane end is impervious to heat and mass transfer. Both the axial and radial components of heat and diffusive mass transfer have been taken into account. A particular case when the temperature and mass transfer potential are unit step functions has been discussed in detail and some results have been exhibited graphically.

In the present paper, the transfer of heat and moisture in a semi-infinite porous cylinder has been discussed. The temperature and mass transfer potential at the cylindrical surface are assumed to be some known functions of both axial coordinate and time, whereas the plane end is impervious to heat and mass transfer. This general problem is of interest in the transpiration cooling technique. The solution has been obtained by the application of finite Hankel, cosine and Laplace transforms. The solutions in the general problem have been expressed in terms of infinite integrals which are difficult to solve analytically because of the general nature of the functions assumed at the cylindrical surface. However a particular case of interest has been worked out by choosing a step function for both temperature and mass transfer potential at the surface. With this choice of the functions we have been able to represent the solutions in closed analytical forms and the results have been depicted numerically.

NOMENCLATURE

t	—Temperature
θ	—moisture transfer potential
r	—radial coordinate
R	— boundary surface radius
τ	— time
a_m	— moisture diffusivity coefficient
a_q	— thermal diffusivity coefficient
ρ	— specific heat of evaporation
δ_s	—Soret coefficient
ϵ	— coefficient of moisture (internal evaporation)
C_m	— specific mass capacity

C_q		—specific heat capacity
Θ	$= (\theta_o - \theta)/\theta_o$	— non-dimensional mass transfer potential
T	$= (t - t_o)/t_o$	— non-dimensional temperature
X	$= r/R$	— non-dimensional radial coordinate
Z	$= z/R$	—non-dimensional axial coordinate
F_o	$= a_q\, \tau/R^2$	—non-dimensional time
Lu	$= a_m/a_q$	—Luikov Number
P_n	$= \delta_s\, t_o/\theta_o$	— Posnov criterion
K_o	$= \rho C_m\, \theta_o/C_q\, t_o$	— Kossovich criterion
P_e	$= uR/a_q$	— Peclet Number
U		—constant velocity of the fluid
$J_n(x)$		—Bessel function of *nth* order and of first kind
$H(P_e F_o - Z)$		— Unit step function defined as

$$H(P_e F_o - Z) = 1 \qquad Z < P_e F_o$$
$$= 0 \qquad Z > P_e F_o$$

STATEMENT OF THE PROBLEM

The porous circular cylinder is initially at constant temperature t_o and mass transfer potential θ_o. The circular boundary is kept at temperature $t = f(z, \tau)$ and mass transfer potential $\theta = g(z, \tau)$. The end $z = o$ is assumed to be impervious to heat and mass transfer. The distributions of temperature and mass transfer potential in the cylinder are to be determined at any time τ. Mathematically

$$\frac{\partial t}{\partial \tau} = a_q \left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2} \right) + \frac{\epsilon \rho}{C_q} C_m \frac{\partial \theta}{\partial \tau} \tag{1}$$

$$\frac{\partial \theta}{\partial \tau} = a_m \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right) + a_m\, \delta_s \left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2} \right)$$

$$o < r < R,\ o < z < \infty,\ \tau > o \tag{2}$$

$$\left.\begin{aligned} t &= t_o \\ \theta &= \theta_o \end{aligned}\right] \tau = o,\ o < z < \infty,\ o < r < R \tag{3, 4}$$

$$\left.\begin{aligned} t &= f(z,\tau) \\ \theta &= g(z,\tau) \end{aligned}\right] \begin{aligned} r &= R \qquad \tau > o \\ o &< z < \infty \end{aligned} \tag{5, 6}$$

$$\left.\begin{aligned} \frac{\partial t}{\partial z} &= o \quad (7) \\ \frac{\partial \theta}{\partial z} &= o \end{aligned}\right] z = o, \quad o < r < R, \quad \tau > o \tag{8}$$

The non-dimensional form of the equations (1) to (8) is

$$\frac{\partial T}{\partial F_o} = \left(\frac{\partial^2 T}{\partial X^2} + \frac{1}{X}\frac{\partial T}{\partial X} + \frac{\partial^2 T}{\partial Z^2} \right) - \epsilon K_o \frac{\partial \Theta}{\partial F_o} \tag{9}$$

$$\frac{\partial\Theta}{F_o} = L_u \left(\frac{\partial^2\Theta}{\partial X^2} + \frac{1}{X}\frac{\partial\Theta}{\partial X} + \frac{\partial^2\Theta}{\partial Z^2}\right) - L_u P_n \left(\frac{\partial^2 T}{\partial X^2} + \frac{1}{X}\frac{\partial T}{X} + \frac{\partial^2 T}{\partial Z^2}\right)$$

$$o \leqslant X < 1, \quad o < Z < \infty, F_o > o \qquad (10)$$

$$\left.\begin{array}{l} T = 0 \\ \Theta = 0 \end{array}\right\} F_o = 0, \quad o < Z < \infty, o \leqslant X < 1 \qquad (11)\ (12)$$

$$\left.\begin{array}{l} T = F\,(Z, F_o) \\ \Theta = G\,(Z. F_o) \end{array}\right\} X = 1, \quad o < Z < \infty, F_o > o \qquad (13)\ (14)$$

$$\left.\begin{array}{l} \dfrac{\partial T}{\partial Z} = 0 \\ \dfrac{\varepsilon\Theta}{\partial Z} = 0 \end{array}\right\} Z = 0, \quad o \leqslant X < 1, \quad F_o > o \qquad (15)\ (15a)$$

where $F(Z, F_o)$, $G(Z, F_o)$ are assumed to be the non-dimensional forms of $f(z, \tau)$ $g(z, \tau)$ respectively.

Performing the finite Hankel, Fourier Cosine and Lalpace Transforms[1] with respect to the variables X, Z, F_o on (9), (10) and using the conditions (11—15), we get

$$\overline{\overset{*}{T}}_c \left[p + (\xi_i^2 + \eta^2)\right] + \epsilon K_o\, p\, \overline{\overset{*}{\Theta}}_c = \xi_i\, J_1(\xi_i)\, \overline{F}_c\,(\eta, p) \qquad (16)$$

$$-\overline{\overset{*}{T}}_c \,.\, L_u P_n \left(\xi_i^2 + \eta^2\right) + \left[p + L_u\left(\xi_i^2 + \eta^2\right)\right]\overline{\overset{*}{\Theta}}_c = L_u\, \xi_i\, J_1(\xi_i) \times$$

$$\left[\overline{G}_c\,(\eta, p) - P_n \overline{F}_c\,(\eta, p)\right] \qquad (17)$$

where $\overline{\overset{*}{T}}_c$, $\overline{\overset{*}{\Theta}}_c$ are the transformed functions of $T(X, Z. F_o)$ and $\Theta(X, Z, F_c)$ by the successive applications of finite Hankel, Cosine and Laplace transforms. Also $\overline{F}_c\,(\eta, p)$ and $\overline{G}_c\,(\eta. p)$ are transformed functions of $F.\,(Z, F_o)$ and $G\,(Z, F_o)$ by the successive applications of Fourer cosine and Laplace Transforms.

On solving for $\overline{\overset{*}{T}}_c$ and $\overline{\overset{*}{\Theta}}_c$ from (16) and (17) we get

$$\overline{\overset{*}{T}}_c = \frac{\xi_i\, J_1(\xi_i)\left[p\left\{\overline{F}_c\,(\eta, p) - \epsilon L_u K_o \overline{G}_c\,(\eta, p) + L_u\, \epsilon K_o P_n \overline{F}_c\,(\eta, p)\right\} + L_u\,(\xi_i^2 + \eta^2)\,\overline{F}_c\,(\eta, p)\right]}{\left[p + L_u\,(\xi_i^2 + \eta^2)\,\nu_1^2\right]\left[p + L_u\,(\xi_i^2 + \eta^2)\,\nu_2^2\right]} \qquad (18)$$

$$\overset{*}{\bar{\Theta}}_c = \frac{L_u\, \xi_i\, J_1(\xi_i)\left[(\xi_i^2 + \eta^2)\,\bar{G}_c(\eta, p) + p\left\{\bar{G}_c(\eta, p) - P_n\,\bar{F}_c(\eta, p)\right\}\right]}{\left[p + L_u\,(\xi_i^2 + \eta^2)\nu_1^2\right]\left[p + L_u\,(\xi_i^2 + \eta^2)\,\nu_2^2\right]} \tag{19}$$

where

$$\nu_1^2 = \tfrac{1}{2}\left[\left(1 + \frac{1}{L_u} + \epsilon\, K_o\, P_n\right) - \sqrt{\left(1 + \frac{1}{L_u} + \epsilon\, K_o\, P_n\right)^2 - \frac{4}{L_u}}\right] \tag{20}$$

$$\nu_2^2 = \tfrac{1}{2}\left[\left(1 + \frac{1}{L_u} + \epsilon\, K_o\, P_n\right) + \sqrt{\left(1 + \frac{1}{L_u} + \epsilon\, K_o\, P_n\right)^2 - \frac{4}{L_u}}\right] \tag{21}$$

and ξ_i are the roots of $\quad J_o(\xi_i) = o \qquad (22)$

The solution of this problem i.e. non-dimensional temperature and mass transfer potential can be obtained by performing the inverse transformations with respect to finite Hankel, Fourier Cosine and Laplace Transforms.

A special case of the general problem discussed above, which is of practical interest, is the case of unsteady heat and mass transfer in semi-infinite cylinder with the boundary conditions at the cylindrical surface to be step functions of axial co-ordinate. Such boundary conditions are suggested in the case of continuously growing cylinders of porous materials during manufacture and consequent drying.

PARTICULAR CASE

We shall now consider that the functions representing the temperature and mass transfer potential at the surface are step functions of axial coordinate i.e. when $F(Z, F_o) = G(Z, F_o)$ $= H(P_e\, F_o - Z)$ (23)

Physically the problem can be stated as under:

Let us assume that the hot fluid suddently starts flowing around the porous circular cylinder (initially at constant temperature and mass transfer potential) starting from the end $z=o$ with a constant velocity u. The circular boundary of the porous cylinder upto which the fluid surrounds the cylinder at any time τ (i.e. upto $z = u\tau$) is kept at a constant temperature and mass transfer potential (different from the initial values while the rest of the cylindrical surface (i.e. $z > u\tau$) remains at initial values of temperature and mass transfer potential.

The transformed functions $\overset{*}{\bar{T}}_c$ and $\overset{*}{\bar{\Theta}}_c$ can be derived from the equations (18), (19) by substituting the values of $F(Z, F_o)$ and $G(Z, F_o)$ from the equation (23). Thus we get

$$\overset{*}{\bar{T}}_c = \frac{\xi_i\, J_1(\xi_i)\, P_e\,(Ap + B).}{(p^2 + \eta^2 P_e^2)\left[p + L_u\,(\xi_i^2 + \eta^2)\,\nu_1^2\right]\left[p + L_u\,(\xi_i^2 + \eta^2)\,\nu_2^2\right]} \tag{24}$$

and

$$\tilde{\overset{*}{\Theta}}_c = \frac{\xi_i J_1(\xi_i) P_e (A' p + B)}{(p^2 + \eta^2 P_e^2) \left[p + L_u (\xi_i^2 + \eta^2)\nu_1^2 \right] \left[p + L_u (\xi_i^2 + \eta^2)\nu_2^2 \right]} \tag{25}$$

where

$$A = 1 - L_u \epsilon K_o (1 - P_n) \tag{26}$$

$$A' = L_u (1 - P_n) \tag{27}$$

$$B = L_u (\xi_i^2 + \eta^2) \tag{28}$$

The Laplace Inversion of (24) and (25) by the inversion formula

$$\overset{*}{T}_c (\xi_i, \eta, F_o) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \overset{*}{\bar{T}}_c (\xi_i, \eta, p) e^{pF_o} dp \tag{29}$$

gives [3]

$$\begin{aligned}
\overset{*}{T}_c (\xi_i, \eta, F_o) = \xi_i J_1(\xi_i) P_e \Bigg\{ & \frac{(1 - A\nu_1^2) \cdot e^{-L_u (\xi_i^2 + \eta^2) \nu_1^2 F_o}}{(\nu_2^2 - \nu_1^2) L_u^2 \nu_1^4 (\eta^2 + \mu_1) (\eta^2 + \mu_2^2)} + \\
& + \frac{(1 - A\nu_2^2) e^{-L_u (\xi_i^2 + \eta^2) \nu_2^2 F_o}}{(\nu_1^2 - \nu_2^2) L_u^2 \nu_2^4 (\eta^2 + \mu_3^2)(\eta^2 + \mu_4^2)} \\
& + \frac{(1 - A\nu_1^2) \operatorname{Cos} (\eta P_e F_o)}{(\nu_1^2 - \nu_2^2) L_u^2 \nu_1^4 (\eta^2 + \mu_1^2) (\eta^2 + \mu_2^2)} \\
& + \frac{(1 - A\nu_2^2) \operatorname{Cos} (\eta P_e F_o)}{(\nu_2^2 - \nu_1^2) L_u^2 \nu_2^4 (\eta^2 + \mu_3^2) (\eta^2 + \mu_4^2)} \\
& + \frac{(1 - A\nu_1^2)(\xi_i^2 + \eta^2)}{(\nu_2^2 - \nu_1^2) L_u \nu_1^2 (\eta^2 + \mu_1^2) (\eta^2 + \mu_2^2)} \quad \frac{\sin (\eta P_e F_o)}{\eta P_e} \\
& + \frac{(1 - A\nu_2^2)(\xi_i^2 + \eta^4)}{(\nu_1^2 - \nu_2^2) L_u \nu_2^2 (\eta^2 + \mu_3^2) (\eta^2 + \mu_4^2)} \quad \frac{\sin (\eta P_e F_o)}{\eta P_e} \Bigg\}
\end{aligned} \tag{30}$$

where

$$\mu_1^2 = \tfrac{1}{2}\left[\left(2\,\xi_i^2 + \frac{P_e^2}{L_u^2\,\nu_1^4}\right) - \sqrt{\left(2\,\xi_i^2 + \frac{P_e^2}{L_u^2\,\nu_1^4}\right)^2 - 4\,\xi_i^4}\,\right] \tag{31}$$

$$\mu_2^2 = \tfrac{1}{2}\left[\left(2\,\xi_i^2 + \frac{P_e^2}{L_u^2\,\nu_1^4}\right) + \sqrt{\left(2\,\xi_i^2 + \frac{P_e^2}{L_u^2\,\nu_1^4}\right)^2 - 4\,\xi_i^4}\,\right] \tag{32}$$

$$\mu_3^2 = \tfrac{1}{2}\left[\left(2\,\xi_i^2 + \frac{P_e^2}{L_u^2\,\nu_2^4}\right) - \sqrt{\left(2\,\xi_i^2 + \frac{P_e^2}{L_u^2\,\nu_2^4}\right)^2 - 4\,\xi_i^4}\,\right] \tag{33}$$

$$\mu_4^2 = \tfrac{1}{2}\left[\left(2\,\xi_i^2 + \frac{P_e^2}{L_u^2\,\nu_2^4}\right) + \sqrt{\left(2\,\xi_i^2 + \frac{P_e^2}{L_u^2\,\nu_2^4}\right)^2 - 4\,\xi_i^4}\,\right] \tag{34}$$

The inversion of (30) by the Cosine inversion formula

$$\overset{*}{T}\,(\xi_i\,, Z, F_\circ) = 2/\pi \int_0^\infty \overset{*}{T}_c\,(\xi_i, \eta, F_\circ)\cos \eta Z\, d\eta \tag{35}$$

gives two solutions valid for two regions $Z < P_e\ F_\circ$ and $Z > P_e\ F_\circ$ respectively as

$$\overset{*}{T}\,(\xi_i, Z, F_\circ) = 1 + \frac{\xi_i\ J_1(\xi_i\,)\,P_e\ (1 - A\nu_1^2)}{2(\nu_2^2 - \nu_1^2)\,L_u^2\,\nu_1^4(\mu_2^2 - \mu_1^2)}\, e^{-L_u\,\nu_1^2\xi_i^2F_\circ} \times$$

$$\times \left\{ \frac{e^{\mu_1^2 L_u\,\nu_1^2 F_\circ - \mu_1 Z}}{\mu_1}\, erf_c\left[\frac{2\mu_1 L_u\,\nu_1^2 F_\circ - Z}{2\sqrt{L_u\,F_\circ}\,\nu_1}\right] + \frac{e^{\mu_1^2 L_u\,\nu_1^2 F_\circ + \mu_1 Z}}{\mu_1}\, erf_c\left[\frac{2\mu_1 L_u\,\nu_1^2 F_\circ + Z}{2\sqrt{L_u\,F_\circ}\,\nu_1}\right] - \frac{e^{\mu_2^2 L_u\,\nu_1^2 F_\circ - \mu_2 Z}}{\mu_2}\,.\, erf_c\left[\frac{2\mu_2 L_u\,\nu_1^2 F_\circ - Z}{2\sqrt{L_u\,F_\circ}\,\nu_1}\right] - \frac{e^{\mu_2^2 L_u\,\nu_1^2 F_\circ + \mu_2 Z}}{\mu_2}\,.\, erf_c\left[\frac{2\mu_2 L_u\,\nu_1^2 F_\circ + Z}{2\sqrt{L_u\,F_\circ}\,\nu_1}\right] \right\}$$

$$+ \frac{\xi_i\ J_1(\xi_i\,)P_e\ (1 - A\nu_2^2)}{2(\nu_1^2 - \nu_2^2)\,L_u^2\nu_2^4\,(\mu_4^2 - \mu_3^2)}\, e^{-L_u\,\nu_2^2\xi_i^2F_\circ} \times$$

$$\times \left\{ \begin{array}{l} \dfrac{e^{\mu_3^2 L_u \nu_2^2 F_\circ - \mu_3 Z}}{\mu_3} \cdot erf_c \left[\dfrac{2\mu_3 L_u \nu_2^2 F_\circ - Z}{2\sqrt{L_u F_\circ}\, \nu_2} \right] \\ + \dfrac{e^{\mu_3^2 L_u \nu_2^2 F_\circ + \mu_3 Z}}{\mu_3} \cdot erf_c \left[\dfrac{2\mu_3 L_u \nu_2^2 F_\circ + Z}{2\sqrt{L_u F_\circ}\, \nu_2} \right] \\ - \dfrac{e^{\mu_4^2 L_u \nu_2^2 F_\circ - \mu_4 Z}}{\mu_4} \cdot erf_c \left[\dfrac{2\mu_4 L_u \nu_2^2 F_\circ - Z}{2\sqrt{L_u F_\circ}\, \nu_2} \right] \\ - \dfrac{e^{\mu_4^2 L_u \nu_2^2 F_\circ + \mu_4 Z}}{\mu_4} \cdot erf_c \left[\dfrac{2\mu_4 L_u \nu_2^2 F_\circ + Z}{2\sqrt{L_u F_\circ}\, \nu_2} \right] \end{array} \right\}$$

$$+ \frac{\xi_i\ J_1(\xi_i)P_e\ (1 - A\nu_1^2)}{(\nu_1^2 - \nu_2^2)L_u^2\ \nu_1^4(\mu_2^2 - \mu_1^2)} \left[\frac{e^{-P_e F_\circ \mu_1}}{\mu_1} \cosh(\mu_1 Z) - \frac{e^{-P_e F_\circ \mu_2}}{\mu_2} \cosh(\mu_2 Z) \right]$$

$$+ \frac{\xi_i\ J_1(\xi_i)P_e\ (1 - A\nu_2^2)}{(\nu_2^2 - \nu_1^2)L_u^2\ \nu_2^4(\mu_4^2 - \mu_3^2)} \left[\frac{e^{-P_e F_\circ \mu_3}}{\mu_3} \cosh(\mu_3 Z) - \frac{e^{-P_e F_\circ \mu_4}}{\mu_4} \cosh(\mu_4 Z) \right]$$

$$+ \frac{\xi_i^3\ J_1(\xi_i)(1 - A\nu_1^2)}{L_u\ \nu_1^2(\nu_2^2 - \nu_1^2)(\mu_2^2 - \mu_1^2)} \left[\frac{e^{-P_e F_\circ \mu_2}}{\mu_2^2} \cosh(\mu_2 Z) - \frac{e^{-P_e F_\circ \mu_1}}{\mu_1^2} \cosh(\mu_1 Z) \right]$$

$$+ \frac{\xi_i^3\ J_1(\xi_i)(1 - A\nu_2^2)}{L_u\ \nu_2^2(\nu_1^2 - \nu_2^2)(\mu_4^2 - \mu_3^2)} \left[\frac{e^{-P_e F_\circ \mu_4}}{\mu_4^2} \cosh(\mu_4 Z) - \frac{e^{-P_e F_\circ \mu_3}}{\mu_3^2} \cosh(\mu_3 Z) \right]$$

$$+ \frac{\xi_i\ J_1(\xi_i)(1 - A\nu_1^2)}{L_u\ \nu_1^2(\nu_2^2 - \nu_1^2)(\mu_2^2 - \mu_1^2)} \left[e^{-P_e F_\circ \mu_1} \cosh(\mu_1 Z) - e^{-P_e F_\circ \mu_2} \cosh(\mu_2 Z) \right]$$

$$+ \frac{\xi_i\ J_1(\xi_i)(1 - A\nu_2^2)}{L_u\ \nu_2^2(\nu_1^2 - \nu_2^2)(\mu_4^2 - \mu_3^2)} \left[e^{-P_e F_\circ \mu_3} \cosh(\mu_3 Z) - e^{-P_e F_\circ \mu_4} \cosh(\mu_4 Z) \right]$$

$$Z < P_e\ F_\circ \tag{36}$$

and

$$
\begin{aligned}
\overset{*}{T}(\xi_i, Z, F_o) = {} & \frac{\xi_i\, J_1(\xi_i)\, P_e\, (1-A\nu_1^2)}{2(\nu_2^2-\nu_1^2)\, L_u^2\, \nu_1^4\, (\mu_2^2-\mu_1^2)}\, e^{-L_u \xi_i^2 \nu_1^2 F_o} \\
& \times \left\{ \frac{e^{\mu_1^2 L_u \nu_1^2 F_o - \mu_1 Z}}{\mu_1}\, erf_c\left[\frac{2\mu_1 L_u \nu_1^2 F_o - Z}{2\sqrt{L_u F_o}\, \nu_1}\right] \right. \\
& \qquad + \frac{e^{\mu_1^2 L_u \nu_1^2 F_o + \mu_1 Z}}{\mu_1}\, erf_c\left[\frac{2\mu_1 L_u \nu_1^2 F_o + Z}{2\sqrt{L_u F_o}\, \nu_1}\right] \\
& \qquad - \frac{e^{\mu_2^2 L_u \nu_1^2 F_o - \mu_2 Z}}{\mu_2}\, erf_c\left[\frac{2\mu_2 L_u \nu_1^2 F_o - \mu_2 Z}{2\sqrt{L_u F_o}\, \nu_1}\right] \\
& \qquad \left. - \frac{e^{\mu_2^2 L_u \nu_1^2 F_o + \mu_2 Z}}{\mu_2}\, erf_c\left[\frac{2\mu_2 L_u \nu_1^2 F_o + \mu_2 Z}{2\sqrt{L_u F_o}\, \nu_1}\right] \right\} \\
& + \frac{\xi_i\, J_1(\xi_i)\, P_e\, (1-A\nu_2^2)}{2(\nu_1^2-\nu_2^2)\, L_u^2\, \nu_2^4\, (\mu_4^2-\mu_3^2)}\, e^{-L_u \nu_2^2 \xi_i^2 F_o} \\
& \times \left\{ \frac{e^{\mu_3^2 L_u \nu_2^2 F_o - \mu_3 Z}}{\mu_3}\, erf_c\left[\frac{2\mu_3 L_u \nu_2^2 F_o - Z}{2\sqrt{L_u F_o}\, \nu_2}\right] \right. \\
& \qquad + \frac{e^{\mu_3^2 L_u \nu_2^2 F_o + \mu_3 Z}}{\mu_3}\, erf_c\left[\frac{2\mu_3 L_u \nu_2^2 F_o + Z}{2\sqrt{L_u F_o}\, \nu_2}\right] \\
& \qquad - \frac{e^{\mu_4^2 L_u \nu_2^2 F_o - \mu_4 Z}}{\mu_4}\, erf_c\left[\frac{2\mu_4 L_u \nu_2^2 F_o - Z}{2\sqrt{L_u F_o}\, \nu_2}\right] \\
& \qquad \left. - \frac{e^{\mu_4^2 L_u \nu_2^2 F_o + \mu_4 Z}}{\mu_4}\, erf_c\left[\frac{2\mu_4 L_u \nu_2^2 F_o + Z}{2\sqrt{L_u F_o}\, \nu_2}\right] \right\} \\
& + \frac{\xi_i\, J_1(\xi_i)\, P_e\, (1-A\nu_1^2)}{(\nu_1^2-\nu_2^2)\, L_u^2\, \nu_1^4\, (\mu_2^2-\mu_1^2)} \left[\frac{e^{-\mu_1 Z}}{\mu_1} \cosh(\mu_1 P_e F_o) - \frac{e^{-\mu_2 Z}}{\mu_2} \cosh(\mu_2 P_e F_o) \right] \\
& + \frac{\xi_i\, J_1(\xi_i)\, P_e\, (1-A\nu_2^2)}{(\nu_2^2-\nu_1^2)\, L_u^2\, \nu_2^4\, (\mu_4^2-\mu_3^2)} \left[\frac{e^{-\mu_3 Z}}{\mu_3} \cosh(\mu_3 P_e F_o) - \frac{e^{-\mu_4 Z}}{\mu_4} \cosh(\mu_4 P_e F_o) \right] \\
& + \frac{\xi_i^3 J_1(\xi_i)\, (1-A\nu_1^2)}{L_u\, \nu_1^2\, (\nu_2^2-\nu_1^2)\, (\mu_2^2-\mu_1^2)} \left[\frac{e^{-\mu_1 Z}}{\mu_1^2} \sinh(\mu_1 P_e F_o) - \frac{e^{-\mu_2 Z}}{\mu_2^2} \sinh(\mu_2 P_e F_o) \right]
\end{aligned}
$$

$$+\frac{\xi_i^3 J_1(\xi_i)(1-A\nu_2^2)}{L_u \nu_2^2(\nu_1^2-\nu_2^2)(\mu_4^2-\mu_3^2)}\left[\frac{e^{-\mu_3 Z}}{\mu_3^2}\sinh(\mu_3 P_e F_o) - \frac{e^{-\mu_4 Z}}{\mu_4^2}\sinh(\mu_4 P_e F_o)\right]$$

$$+\frac{\xi_i J_1(\xi_i)(1-A\nu_1^2)}{L_u \nu_1^2(\nu_2^2-\nu_1^2)(\mu_2^2-\mu_1^2)}\left[e^{-\mu_2 Z}\sinh(\mu_2 P_e F_o) - e^{-\mu_1 Z}\sinh(\mu_1 P_e F_o)\right]$$

$$+\frac{\xi_i J_1(\xi_i)(1-A\nu_2^2)}{L_u \nu_2^2(\nu_1^2-\nu_2^2)(\mu_4^2-\mu_3^2)}\left[e^{-\mu_4 Z}\sinh(\mu_4 P_e F_o) - e^{-\mu_3 Z}\sinh(\mu_3 P_e F_o)\right]$$

$$Z > P_e F_o \tag{37}$$

Inversion of the equations (36) and (37) by the inversion formula for Hankel transform

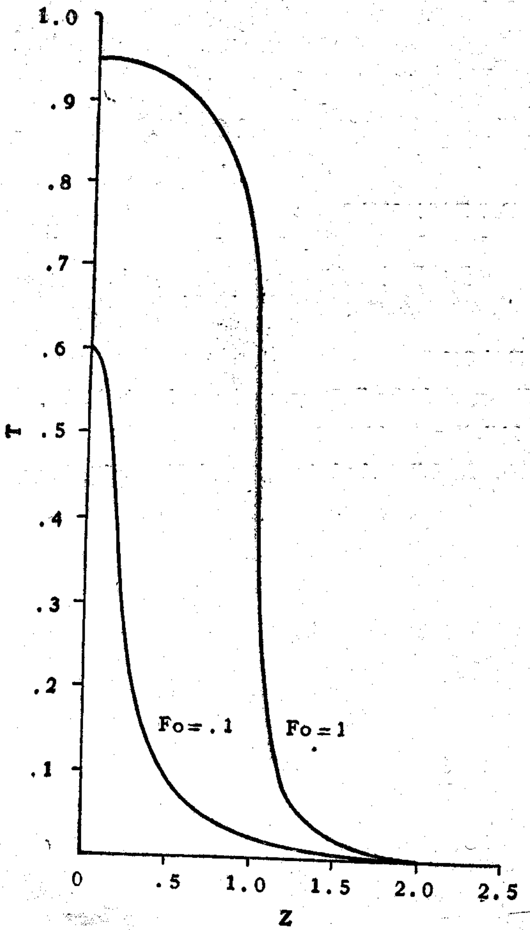

Fig. 1—T versus Z at X=0·9 and Pe=1 for two different values of F_o.

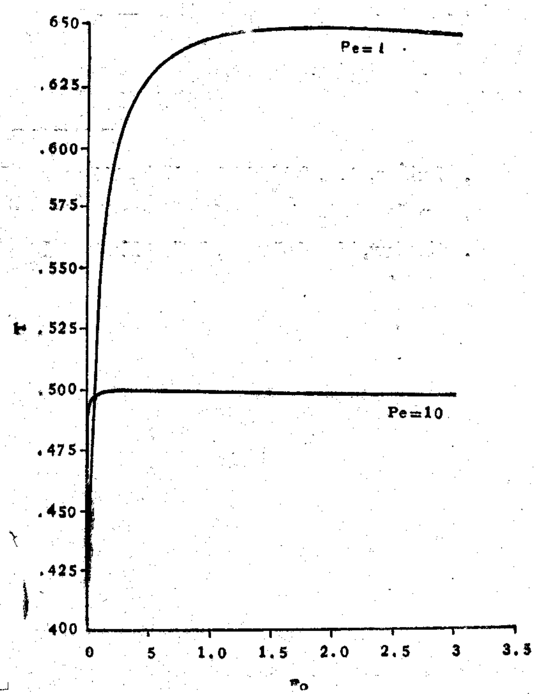

Fig 2—T versus F_0 at Z=Pe F_o and X=0·9 for two different values of Pe.

$$T(X, Z, F_o) = 2 \sum_{\xi_i} \frac{\overset{*}{T}(\xi_i, Z, F_o)\, J_o(X\xi_i)}{J_1^2(\xi_i)} \tag{38}$$

gives the required function $T(X, Z, F_o)$ to be determined, where the summation is taken over all the positive roots of equation (22).

The expressions of $\Theta(X, Z, F_o)$ for $Z < P_e F_o$ and $Z > P_e F_o$ are similar to those $T(X, Z, F_o)$ except that A in T has to be replaced by A' [given by (27)].

NUMERICAL RESULTS FOR PARTICULAR CASE

At Fig 1 the non-dimensional temperature T has been plotted against non-dimensional distance Z for $L_u = 0 \cdot 3$, $\epsilon = 0 \cdot 5$, $K_o = 1 \cdot 2$, $P_n = 0 \cdot 5$, $P_e = 1$, $F_o = 0 \cdot 1$ & $1 \cdot 0$. The value of non-dimensional temperature shows a rapid fall beyond $Z = P_e F_o$ ($z = u\tau$). This can be easily explained on account of the fact that the cylindrical surface in this region is in contact with the atmosphere at initial temperature and the only source of heating is axial conduction beyond the surface $Z = P_e F_o$. The non-dimensional temperature has also been plotted (see Fig 2) against non-dimensional time F_o at $Z = P_e F_o$ for two different values of Peclet number $P_e = 1$ and 10. It appears that with the increase of P_e the steady state value of the non-dimensional temperature T decreases. The graphs of Θ vs Z and Θ vs F_o are expected, to be of the same nature as those of T vs Z and T vs F_o respectively.

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