INTERNAL BALLISTICS OF RECOILLESS GUNS

ASIM RAY

Calcutta Technical School, Calcutta

(*Received* 8 *Jan.* 65; *revised* 18 *Jan.* 66)

A new method for calculating the ballistics of recoilless guns during the period of burning of the propellant has been obtained. Ballistics have also been calculated by exact numerical integration in a few cases and these results have been compared with those obtained by the method described in this paper. It has been found that the results obtained by these two methods agree satisfactorily.

The internal ballistics of recoilless guns was first discussed by Corner [1,2] who suggested a semi-empirical method—based on a good number of accurate numerical integrations—for the calculation of ballistics. Later Thiruvenkatachar & Venkatesan[3] suggested a method in which the ballistic variables were assumed to be expansible in powers of the leakage parameter Ψ. They calculated the ballistics retaining only upto the first power of Ψ. Jain[4] extended their work by calculating the ballistics retaining upto the second power of Ψ-and pointed out that for accurate calculation contribution of the Ψ^2-term should be taken into consideration in case of leaking guns while for recoilless guns solution even upto Ψ^2-term was not reliable. Jain[4] did not discuss how these solutions compared with exact numerical integration. Because of this we have obtained the ballistics by exact numerical integration in a few arbitrary cases. Next we have developed an approximate solution of the ballistic equations valid during the period of burning of propellants in the specific form of tube. Since for the period of burning our main interest is to calculate the burnt-values of the ballistic variables and since the difference between our approximate solution and exact numerical solution is greatest—discussed later—for the burnt values, we have obtained these values by our approximate solution, by Jain's solution and by exact numerical integration. It has been found that for the calculation of the burnt-values of the ballistic variables, our approximate solution agrees very satisfactorily with the exact numerical solution at least for practical values of M. But the solution suggested by Jain[4] in this respect differs considerably from the exact numerical solution. Furthermore calculation by our method requires evaluation of certain powers of different numbers in a particular interval and so it is expected to be quite easy and quick. Calculation of burnt-values involves practically no labour.

BASIC EQUATIONS AND APPROXIMATE SOLUTION

With notations used by Jain[4] and Corner[2] the basic equations giving the interna ballistics of recoilless and leaking guns using tubular propellants and with linear rate of burning can be written as follows :

$$\zeta\xi = N\,T'\left(1 + \frac{K\,C\,N}{6\,W}\right) \tag{1}$$

$$\frac{d\eta}{d\tau} = \eta\,\frac{d\eta}{d\xi} = \frac{M}{1 + \dfrac{K\,C\,N}{2\,W}}\,\zeta \tag{2}$$

$$\frac{dZ}{d\tau} = \zeta \tag{3}$$

$$\frac{dN}{d\tau} = \frac{dZ}{d\tau} - \Psi \zeta (T')^{-\frac{1}{2}} \tag{4}$$

$$\frac{d(N T')}{d\tau} = -\left(\bar{\gamma} - 1\right) \zeta \frac{d\xi}{d\tau} + \frac{dZ}{d\tau} - \gamma \Psi \zeta (T')^{\frac{1}{2}} \tag{5}$$

where

$$Al = U - \frac{C}{\delta} \tag{6}$$

$$\xi = 1 + \frac{x}{l} \tag{7}$$

$$\tau = \left(\frac{\beta C R T_o}{A D l}\right) t \tag{8}$$

$$\zeta = \left(\frac{Al}{C R T_o}\right) p \tag{9}$$

$$\eta = \left(\frac{A D}{C \beta R T_o}\right) v \tag{10}$$

$$T' = T/T_o \tag{11}$$

$$M = \frac{A^2 D^2}{\beta^2 C R T_o W} \tag{12}$$

$$\Psi = \frac{\Psi S D}{\beta C (R T_o)^{\gamma_2}} \tag{13}$$

Let us denote the values of Z at short-start and nozzle-start by $Z_0 \neq 0$ and $Z_N \neq 0$ respectively. The equation (2) holds good from the instant of short-start, (4) from the instant of nozzle-start and (5) when the shot is in motion and the nozzle is in operation. To be specific let us assume that the nozzle-start occurs before shot-start. In the intervening period from nozzle-start to shot-srart the gun behaves like a rocket and during this period the internal ballistics is obtained by integrating (14), (3), (4) and (15) [obtained from (1) to (5) by putting $\xi = 1, \eta = 0, W = \infty$, since the shot is at rest].

$$\zeta = N T' \tag{14}$$

$$\frac{d(N T')}{d\tau} = \frac{dZ}{d\tau} - \gamma \Psi \zeta (T')^{\frac{1}{2}} \tag{15}$$

With the initial conditions at $\tau = 0, T' = 1, Z = N = \zeta = Z_N \neq 0$. These four equations are valid till the shot-start. Since Z_o will, in general, differ slightly from Z_N these four equations are integrated numerically by say Runge-Kutta's method with the above initial conditions to obtain the values of T', Z, N and ζ $(= N T')$ at shot-start. Let us denote these by the suffix zero. After shot-start (1) to (5) are to be integrated with the initial conditions:

$$\zeta = \zeta_o \pm o,\ Z = Z_o \pm o,\ N = N_o \pm o,\ T' = T'_o,\ \xi = 1,\ \eta = 0,\ \text{at } \tau = 0 \text{ (say).}$$

We now develop an approximate solution of the equations. Since the factor $1 + \frac{KCN}{2W}$ deviates little from its mean value during the period of burning it is permissible to replace this by its mean value σ which is close to unity as observed by Corner[2]. So (1) and (2) can be rewritten in the forms:

$$\zeta \xi = \frac{2 + \sigma}{3} N T' \tag{16}$$

$$\frac{d\eta}{d\tau} = \eta \frac{d\eta}{d\xi} = \frac{M}{\sigma}\zeta \tag{17}$$

From (2) and (3) we get

$$\frac{d\eta}{dZ} = \frac{M}{\sigma}$$

which on integration (subject to the condition: $\eta = 0,\ Z = Z_\circ$) gives

$$\eta = \frac{M}{\sigma}(Z - Z_\circ) \tag{18}$$

Eliminating τ from (3) and (4) one has

$$\frac{dN}{dZ} = 1 - \Psi (T')^{-\frac{1}{2}} \tag{19}$$

Again, eliminating τ from (3) and (5), remembering that $\frac{d\xi}{d\tau} = \eta$, we obtain after using (18):

$$\frac{d\,(N\,T')}{dZ} = 1 - \frac{M\,(\bar{\gamma} - 1)\,(Z - Z_\circ)}{\sigma} - \gamma\,\Psi\,(T')^{\frac{1}{2}} \tag{20}$$

(20) can be rewritten in the form [by (19)]

$$N\,\frac{dT'}{dZ} = 1 - T' - \Psi\,(\gamma - 1)\,(T')^{\frac{1}{2}} - \frac{(\bar{\gamma} - 1)\,M\,(Z - Z_\circ)}{\sigma} \tag{21}$$

with the help of (19) which along with (21) will determine N and T' as a function of Z. We now give an approximate solution for N and T' from (19) and (21). In (19) let us replace T' by $T'_\circ$, the value of T' at short-start. On integrating the resulting equation; subject to the initial condition $N = N_\circ,\ Z = Z_\circ$; we obtain

$$N = N_1 \tag{22}$$

where

$$N_1 = N_\circ + K_\circ\,(Z - Z_\circ) \tag{23}$$

$$K_\circ = 1 - \Psi\,(T_\circ')^{\frac{1}{2}} \tag{24}$$

In (21) let us replace N by N_1 and then change the independent variable Z by the variable N_1 as given by (23) and thus get

$$K_\circ N_1\,\frac{dT'}{dN_1} = 1 - T' - \Psi\,(\gamma - 1)\,(T')^{\frac{1}{2}} - \frac{(\bar{\gamma} - 1)\,M}{K_\circ\,\sigma}\,(N_1 - N_\circ)$$

Let us now change the dependent variable T' by T_1 given by the relation

$$T' = 1 - T_1 \tag{25}$$

and retain only upto the first power of T_1 in the binomial expansion of $(1 - T_1)$ corresponding to the term $(T')^{\frac{1}{2}}$ and obtain the following linear differential equation:

$$\frac{dT_1}{dN_1} + \frac{a}{N_1}\,T_1 = b + \frac{C}{N_1} \tag{26}$$

where

$$a = \frac{1 + \frac{1}{2}\,\Psi\,(\gamma - 1)}{K_\circ} \tag{27}$$

$$b = \frac{(\bar{\gamma} - 1)\, M}{K_o^2\, \sigma} \tag{28}$$

and

$$c = \frac{\Psi\,(\gamma - 1) - \dfrac{(\bar{\gamma} - 1)\, M\, N_o}{K_o\, \sigma}}{K_o} \tag{29}$$

Equation (26) when integrated subject to the initial condition
$T_1 = T_{10} = 1 - T'_0$ at $N_1 = N_0$ ($\because$ at $Z = Z_0$
$N_1 = N_0$, $T' = T_0$) yeilds

$$T_1 = \frac{bN_1}{a+1} + \frac{c}{a} + \left[T_{10} - \frac{bN_0}{a+1} - \frac{c}{a} \right] \left(\frac{N_0}{N_1} \right)^a \tag{30}$$

We may regard (22) and (23) together and (25), (30) and (23) together to give first approximation value of N and T' respectively as a function of Z. Now we proceed to find a second approximation value of N. Let us combine (19), (23) and (25) and retain only upto the first power of T_1 in the binomial expansion of $(1 - T_1)^{-\frac{1}{2}}$ corresponding to $(T')^{-\frac{1}{2}}$ and thus get

$$\frac{dN}{dN_1} = \frac{1 - \Psi}{K_o} + \frac{\Psi}{2K_o}\, T_1$$

Replacing T_1 by its value given by (30) and integrating with the initial conditions $N = N_0$, $N_1 = N_0$ ($\because$ at $Z = Z_0$, $N_1 = N_0$) we obtain

$$N = N_0 + \frac{1 - \Psi}{K_0}\,(N_1 - N_0) - \frac{\Psi}{2\,K_0} \left[\frac{b}{2\,(a+1)}\,(N_1^2 - N_0^2) + \frac{c}{a}\,(N_1 - N_0) + \frac{N_0}{1-a} \left(T_{10} - \frac{b\, N_0}{a+1} - \frac{c}{a} \right) \left\{ \left(\frac{N_0}{N_1} \right)^{a-1} - 1 \right\} \right] \tag{31}$$

(31) gives a second approximation value of N. We cannot, however, similarly find a second approximation for T'. Therefore (30) and (31) give an approximate solution for N and T'. Now with this solution we find solution for other ballistic variables. Remembering that

$1 + \dfrac{K\,C\,N}{2\,W} \cdot \sigma$ we have from (1) and (2)

$$\eta\, \frac{d\eta}{d\xi} = \frac{M\,(2 + \sigma)}{3\,\sigma}\; \frac{N\,T'}{\xi} \tag{32}$$

With help of (18) and (23) equation (32) can be rewritten in the form

$$\frac{d\xi}{\xi} = \frac{3\,M}{K_0^2\, \sigma\,(2 + \sigma)} \cdot \frac{N_1 - N_0}{N\,T'}\, d\,N_1$$

which on intergration ; subject to the initial condition $\xi = 1$, $N_1 = N_0\,(Z = Z_0)$; gives

$$\log \xi = \frac{3\,M}{K_0^2\, \sigma\,(2 + \sigma)} \times I \tag{33}$$

where

$$I = \int_{N_0}^{N_1} \frac{N_1 - N_0}{N\,T'}\, d\,N_1 \tag{34}$$

Now we will find an approximate value of the above integral. Since T_0' will be close to unity, $K_0 \sim 1 - \Psi$ so that from (31) we can write

$$N = N_1 - n \tag{35}$$

where

$$n = \frac{\Psi}{2\,K_0}\left[\frac{b\,(N_1^2 - N_0^2)}{2\,(a+1)} + \frac{c}{a}\,(N_1 - N_0) + \frac{N_0}{1-a}\left(T_{10} - \frac{bN_0}{a+1} - \frac{c}{a}\right)\left\{\left(\frac{N_0}{N_1}\right)^{a-1} - 1\right\}\right] \tag{36}$$

Therefore

$$I = \int_{N_0}^{N_1} \frac{N_1 - N_0}{(N_1 - n)\,(1 - T_1)}\,dN_1 = \int_{N_0}^{N_1} \frac{(N_1 - N_0)\,(1 + T_1)}{N_1}\left(1 + \frac{n}{N_1}\right) dN_1$$

where we have neglected squares and higher power of T_1 and n/N_1 Let us replace n/N_1 by its mean value. How this mean value is chosen will be discussed later in connection with determination of burnt values. Denoting the mean value by bar we have

$$I = \left[1 + \left(\frac{\bar{n}}{N_1}\right)\right] \times \int_{N_0}^{N_1} \frac{(N_1 - N_0)\,(1 + T_1)}{N_1}\,dN_1$$

Let us denote the above integral by I_1 and evaluate it after replacing T_1 by its value given by (30). Thus we obtain,

$$I = \left[1 + \left(\frac{\bar{n}}{N_1}\right)\right] \times I_1 \tag{37}$$

where

$$I_1 = (N_1 - N_0) - N_0 \log\frac{N_1}{N_0} + \frac{b\,(N_1^2 - N_0^2)}{2\,(a+1)} + \frac{c}{a}\,(N_1 - N_0) + \frac{N_0}{1-a}\left(T_{10} - \frac{bN_0}{a+1} - \frac{c}{a}\right)\left\{\left(\frac{N_0}{N_1}\right)^{a-1} - 1\right\} - \frac{bN_0\,(N_1 - N_0)}{a+1} - \frac{c\,N_0}{a}\log\frac{N_1}{N_0} + \frac{N_0}{a}\left(T_{10} - \frac{bN_0}{a+1} - \frac{c}{a}\right)\left\{\left(\frac{N_0}{N_1}\right)^{a} - 1\right\} \tag{38}$$

Equations (33), (37), (35) and (23) together determine ξ as a function of Z. With the determination of N, T', ξ we get the remaining ballistic variable ζ from (1). This completes the calculation for obtaining an approximate solution.

CALCULATION OF BURNT—VALUES

Our main interest is to obtain the burnt-values of the ballistic variables during the period of burning. These can obviously be obtained from the approximate solution analysed above. However, we give below a simpler procedure for the calculation of burnt-values; Table V gives the burnt-values for the following:

$$A_1 = \left(T_{10} - \frac{bN_0}{a+1} - \frac{c}{a}\right)\left(\frac{N_0}{N_1}\right)^{a},$$

$$A_2 = \frac{N_0}{1-a}\left(T_{10} - \frac{bN_0}{a+1} - \frac{c}{a}\right)\left\{\left(\frac{N_0}{N_1}\right)^{a-1} - 1\right\}$$

$$A_3 = \frac{\Psi\ b\ N_0^2}{4\ K_0\,(a+1)}$$

and

$$A_4 = \frac{N_0}{a}\left(T_{10} - \frac{bN_0}{a+1} - \frac{c}{a}\right)\left\{\left(\frac{N_0}{N_1}\right)^{a} - 1\right\}$$

It is now evident that the burnt-values of these may even be neglected without introducing significant error. Thus from (23), (30), (31), (33) and (35)—(38) we have the following equations for the burnt-values (denoted by suffix B):

$$N_B = N_{1B} - \frac{\Psi}{2K_o}\left[\frac{bN^2{}_{1B}}{2(a+1)} + \frac{c}{a}(N_{1B} - N_o)\right] \tag{39}$$

$$T_{1B} = \frac{bN_{1B}}{(a+1)} + \frac{c}{a} \tag{40}$$

$$\log \xi_B = \frac{3M}{K_o\sigma(2+\sigma)} \times I_B \tag{41}$$

$$I_B = \left[1 + \tfrac{1}{2}\left(\frac{n}{N_1}\right)_B\right] \times I_{1B} \tag{42}$$

$$I_{1B} = \left(1 + \frac{c}{a} - \frac{bN_o}{a+1}\right)(N_{1B} - N_o) - \left(\frac{c}{a} + 1\right)N_o \log \frac{N_{1B}}{N_o} + \frac{bN_{1B}^2}{2(a+1)} \tag{43}$$

$$N_B = N_{1B} - n_B \tag{44}$$

$$N_{1B} = N_o + K_o\,(1 - Z_o) \tag{45}$$

$$n_B = \frac{\Psi}{2K_o}\left[\frac{bN_{1B}^2}{2(a+1)} + \frac{c}{a}(N_{1B} - N_o)\right] \tag{46}$$

In deducing (42), the mean value $\left(\frac{\overline{n}}{N_1}\right)$ has been taken as $\frac{1}{2}\left(\frac{n}{N_1}\right)_B$. Equations (39) to (46) give the burnt-values.

NUMERICAL INTEGRATION

In four cases, namely, (a) $M = 1, \Psi = 0{\cdot}5$, (b) $M = 2, \Psi = 0{\cdot}5$, (c) $M = 4, \Psi = 0{\cdot}5$ (d) $M = 2, \Psi = 0{\cdot}1$, values of N and T' were obtained by numerical integration according to the method of Runge-Kutta in Steps of $Z = 0{\cdot}05$ and these values are produced in the following Tables in Steps of $Z = 0{\cdot}1$. In all these cases $\gamma = 1{\cdot}25$, $\overline{\gamma} = 1{\cdot}3$, $\sigma = 1{\cdot}1$, $Z_N = 0{\cdot}03$, $Z_o = 0{\cdot}05$. So in all these cases nozzle-start occurs before the shot-start. From Z_N to Z_o equations resulting from (4) and (15) with the elimination of $d\tau$ by (3) were integrated in one step. Then equations (19) and (20) were integrated.

TABLE 1

$M = 1\ \psi, = \cdot 5$

Z	T'	N
·03	1	·03
·05	·946211	·039839
·10	·900802	·063739
·20	·868480	·110511
·30	·847441	·156519
·40	·828409	·201893
·50	·809957	·246649
·60	·791706	·290775
·70	·773524	·334255
·80	·755353	·377067
·90	·737165	·419185
1·00	·718943	·460587

TABLE 2

$M = 2,\ \psi = \cdot 5$

Z	T'	N
·03	1	·03
·05	·946211	·039839
·1	·896210	·063713
·2	·847804	·110107
·3	·809218	·155163
·4	·772252	·198929
·5	·735633	·241341
.6	·699014	·282301
·7	·662261	·321689
·8	·625304	·359369
·9	·588095	·395169
1·0	·550594	·428897

TABLE 3

$M = 4,\ \psi = \cdot 5$

Z	T'	N
·03	1	·03
·05	·946211	·039839
·1	·887023	·063667
·2	·806331	·109260
·3	·732262	·152243
·4	·658659	·192265
·5	·584440	·228821
·6	·509152	·261169
·7	·432415	·288247
·8	·353746	·308417
·9	·272395	·318903
1·0	·186968	·314215

TABLE 4

$M = 2,\ \psi = \cdot 1$

Z	T'	N
·03	1	·03
·05	·989863	·047995
·1	·975057	·092949
·2	·946258	·182747
·3	·917624	·272387
·4	·889019	·361865
·5	·860416	·451173
·6	·831806	·540301
·7	·803185	·629241
·8	·774551	·717981
·9	·745901	·806511
1·0	·717235	·894819

Combining (18) and (32) one has on integration

$$\log \xi_B = \frac{3M}{\sigma(2+\sigma)} \int_{\cdot 05}^{1} \frac{(Z - Z_o\ )dZ}{NT^{1}}$$

Since $\xi = 1$, when $Z = \cdot 05$

ξ_B was calculated by evaluating the integral by Simpson's rule in the four cases. These values we produce later.

COMPARISON OF VARIOUS SOLUTIONS

Our approximate solution was obtained firstly approximating T' by its initial value T_o, (value at shot-start in the cases considered) which is always close to unity and secondly approximating $(1 - T_1\ )^{\pm\frac{1}{2}}$ linearly. These approximations can give valuable results only if T' diverges little from unity. Now from physical point of view we can easily see that divergence of T' from unity and consequently error in the approximate solution will be increasing with the increase of M and Ψ and also with progress of burning. There fore if for a pair of values M_o and Ψ_o of M and Ψ, the burnt-values obtained by our approximate solution show an error ϵ as compared with the values obtained by exact numerical integration, error in these two solutions will be less than ϵ for the whole period of burning and for any other pair M and Ψ such that $M \leqslant M_o$ and $\Psi \leqslant \Psi_o$. Consequently we have collected below the burnt-values of ballistic variables given by two solutions and also values given by Jains[6] Ψ^2 — solution and Thiruvenkatachar and Venkatesan's[3] Ψ — solution. In the following Table prefixes E, A, Ψ, Ψ^2 will denote respectively exact numerical, approximate analytical, Ψ and Ψ^2 solution.

TABLE 5

BURNT-VALUES

M	ψ	EN	AN	ET'	AT'	$\psi T'$	$\psi^2 T'$	$E\xi$	$A\xi$	A_1	A_2	A_3	A_4
1	·5	·460	·455	·719	·722			5·936	5·923	—0·0002	—0·0018	0·0001	0·0011
2	·5	·428	·437	·551	·561	·573	·651	68·23	61·59	—0·0002	—0·0016	0·0002	0·0010
4	·5	·314	·400	·187	·239			9·53	4·19	—0·0001	—0·0011	0·0005	0·0007
2	·1	·894	·896	·717	·717	·718	·721	6·440	7·280	—0·00003	—0·0001	0·00002	0·00005

Since in practical application of recoiless guns M will never exceed 2 from the table it is clear that we can get useful information by our approximate solution in such cases of recoilless guns. And for leaking guns the approximate solution analysed above is as good as exact solution even when M exceeds 2 except in calculating burnt-value of ξ.

ACKNOWLEDGEMENTS

The author gratefully remembers the encouragement and criticism of Late Prof. N. R.Sen during the preparation of this paper and acknowledges with thanks the facilities enjoyed by him at the seminar organised under the research scheme of National Professor S.N. Bose at the Indian Association for the Cultivation of Science, Jadavpur.

REFERENCES

1. CRONER, J., *Proc. Roy. Soc.*, 188A (1947), 237
2. CORNER, J., "Theory of Internal Ballistics of guns", (JOHN WILLEY & SONS, NEW YORK), 1950.
3. JAIN, B.S., *PRoc. Nat. Inst. Sci. India.*, 22A (1956), 324
4. THIRUVENKATACHAR, V.R. & VENKATESAN, N.S., *Ibid.*, 19 (1953), 829