MULTIPLE-T NULL RC—NETWORKS

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A study of double T-networks, triple T-networks and quadruple T-networks has been made. These have been obtained by taking two double T-networks of different resonant frequencies and putting them in parallel in all possible combinations. The resonant frequencies of the combinations are obtained from the formula derived. The nine combinations that are obtained can be used by means of the given switching system for producing a variable slope filter with a variable pass band useful in narrow band transmitters and high fidelity preamplifiers.

Twin-T RC-networks are useful in narrow band selective amplifiers and low frequency stable oscillators. Considerable amount of work has been done on these networks by several workers. Cowels[1] derived a relationship in terms of the input and output impedances, the resonant frequency of parallel T RC-networks for equal losses at low as well as high frequencies and showed that the symmetrical parallel T has characteristics similar to a single series resonant circuit. Wolf[2] showed that by introducing symmetry in a parallel T RC-network operating with zero source and infinite load impedances, a better discrimination could be obtained. By varying the shunt resister in a parallel twin-T RC-network, a variable resonant frequency can be obtained and this arrangement was suggested by Hastings[3] for having a stable RC-oscillator or an amplifier of variable resonant frequency. Bowers[4] found that by varying both the shunt arms in the Ts, keeping RC constant, the resonant frequency remains constant. By varying RC, not only a shift in frequency is obtained but also the regeneration can be controlled with a shift in phase which becomes less than 90° for an increased product RC. Frequency selective twin RC-networks have been used by Stanton[5] in feedback amplifiers to improve Q of the filter. The amplifier has a gain characteristic which is approximately the inverse of the equivalent series resonant circuit. A method of analysing three terminal RC-networks has been given by O'Dell[6] by drawing the simple equivalent circuit and calculating the value in terms of circuit constants.

Miller[7] has used a twin-T producing a variable slope low pass filter which is very useful in narrow band transmitters because it attenuates very much all frequencies above a specified value to avoid excessive bandwidth. High fidelity preamplifiers also require high attenuation for frequencies beyond the pass band above a specified frequency. Filters of this type make use of inductances with their disadvantages. Miller uses an RC filter with the same results. A filter with a pass band below a particular frequency f_r corresponding to the resonant frequency of the twin-T can only be obtained. For filters with different pass bands different twin-Ts will have to be used. A method is given in this article by which two twin-Ts of different resonant frequencies can be used to obtain nine combinations covering a wide range of frequencies, the individual frequency of any such combination being calculated from the formulae given. A common relationship for any number of T-networks in parallel is given in Appendix 1 and its application to various combinations in Appendix 2.

EXPERIMENTAL PROCEDURE

The values of C_k , C_o and R_g will have to be altered in Miller's circuit for the desired response. For convenience, the filter circuit used by Miller is reproduced along with th two twin Ts and the switching circuit for producing nine different combinations in Fig. 1.

Function of the Switching Circuit—T_1 and T_2 are the resistance and capacitance arms of the first twin-T, and T_3 and T_4 are the resistance and the capacitance arms of the second twin-T. T_1 is connected between terminals 1 and 5, T_2 between 2 and 6, T_3 between 3 and 7 and T_4 between 4 and 8. All the four wafers containing the various contacts are mounted on a common shaft. The connections are made in such a way that for a clockwise rotation

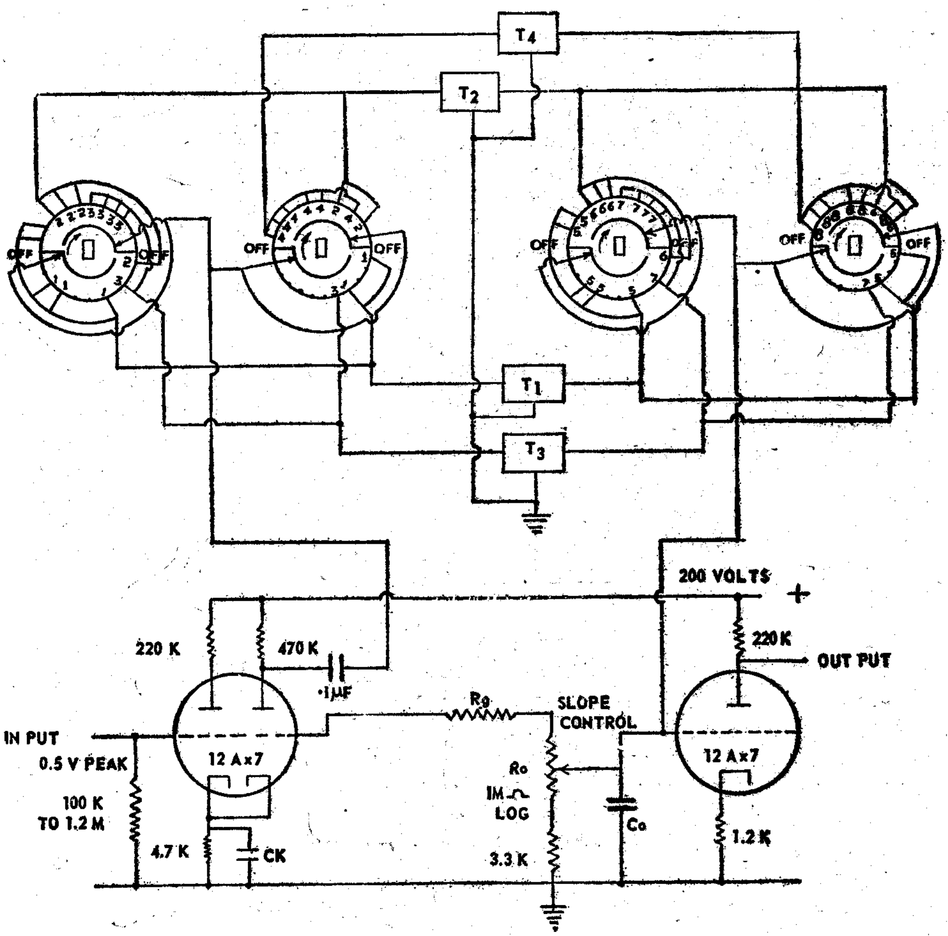

FIG. 1—Variable slope filter circuit with variable pass band.

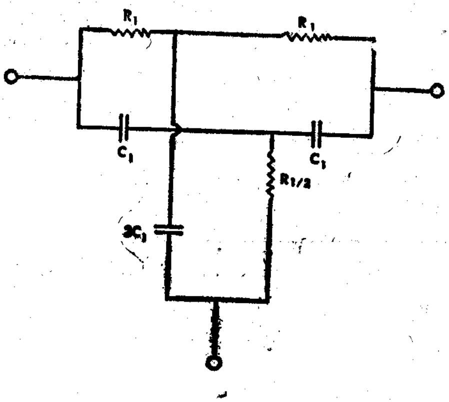

FIG. 2—Twin-T network.

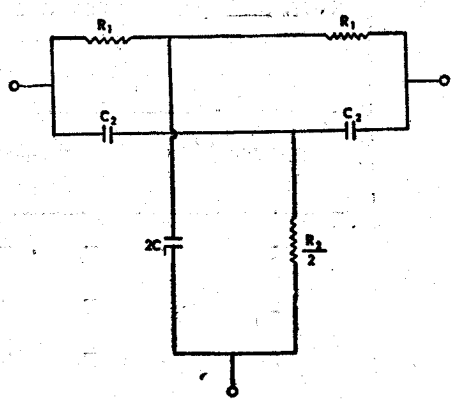

FIG. 3—Twin-T network.

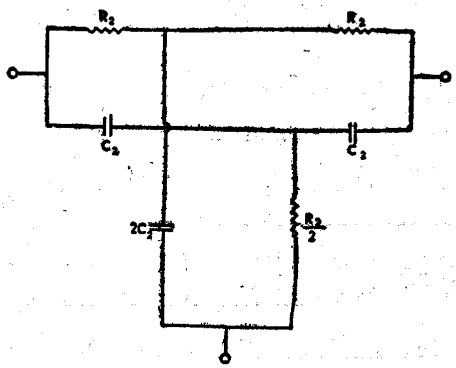

FIG. 4—Second twin-T network.

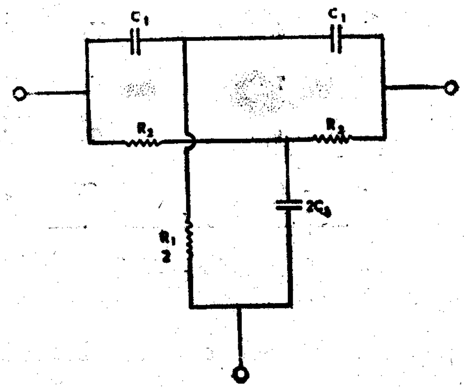

FIG. 5—Twin-T network.

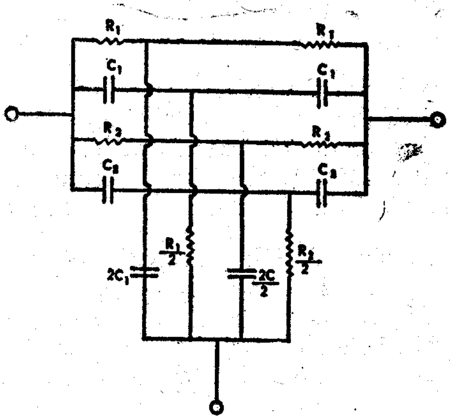

FIG. 6—Double twin-T network.

of the switch from its off position, various *Ts* are included giving resonant frequencies in the increasing order. The combinations and their resonant frequencies are given in Table 1.

TABLE 1

Serial No.	T—combination				Resonant frequency
1	T_1	T_2	T_4		10·07 Kc/s
2	T_1	T_4			13·38 ,,
3	T_2	T_3	T_4		13·66 ,,
4	T_1	T_2			15·92 ,,
5	T_1	T_2	T_3	T_4	16·80 ,,
6	T_3	T_4			17·85 ,,
7	T_2	T_3			21·24 ,,
8	T_1	T_3	T_4		22·31 ,,
9	T_1	T_2	T_3		22·56 ,,

Resonant frequencies of different combinations of T-networks—Twin-Ts—The case of a simple RC-twin-T, shown in Fig. 2, is considered and its resonant frequency is given by $W_{01} = \frac{1}{R_1 C_1}$ which is the same as obtained by any other method. Substituting the values for R_1=100 k and C_1=100 pF, f_{01} = 15·92 Kc/s and the experimental value for f_{01} is 16 Kc/s. A second twin-T, with values of R_2=59·5 k and C_2=150 pF, gives a resonant frequency f_{02} = 17·85 Kc/s and the corresponding experimental value is 17·75 Kc/s. In both the cases, the amplitude and the phase minima occur at the same frequency.

A twin-T (Fig. 3) formed with the resistance arm of the first (Fig. 2) and the capacitance arm of the second twin-T (Fig 4) has two nulls corresponding to its phase and amplitude. Both the nulls though near to each other in value, occur much below the resonant values of either of the individual twin-Ts. The phase null occurs at 13·00 Kc/s and the amplitude null at 13·77 Kc/s for the values of the components used.

Similarly, twin-T (Fig. 5) formed with the capacitance arm of the first and the resistance arm of the second twin-T, has also two nulls as before. Though, the nulls occur very near to each other, they are higher than either of the resonant values of the individual twin-Ts. The phase null occurs at 21·86 Kc/s and the amplitude null at 20·64 Kc/s. It is seen that the separation between these two nulls is much more than in the previous case.

Double twin-T—The two twin-Ts are paralleled as shown in (Fig. 6) and its resonant frequency corresponding to the phase minimum is given by

$$\omega = \left[\omega_{01} \ \omega_{02} \frac{\left(\dfrac{1}{R_1 \, \omega_{02}} + \dfrac{1}{R_2 \, \omega_{01}} \right)}{C_1 + C_2} \right]^{\frac{1}{2}} \qquad (1)$$

The resonant frequency corresponding to amplitude minimum is given by

$$\omega = \sqrt{\omega_{01} \, \omega_{02} \frac{(R_1 + R_2)}{R_1 \, R_2 \, (C_1 \, \omega_{02} + C_2 \, \omega_{01})}} \qquad (2)$$

In this case, there are two frequencies at which the phase and the amplitude nulls occur whereas in the case of twin-Ts (Fig. 2 & 4) the phase and the amplitude nulls occur at the same frequency.

The resonant frequency calculated by substituting the values of components is equal to 17·09 Kc/s in the first case and 17·02 Kc/s in the second case. The experimental value which corresponds to the amplitude null is 17·00 Kc/s. Since the two frequencies are close to each other, not much error will be introduced by taking either the geometric mean or the arithmetic mean. The values of the two means are the same correct to the second decimal place. For convenience of calculation, the geometric mean is taken as the resonant frequency and is given by

$$\omega = \left\{ \frac{\omega_{01} \, \omega_{02} \, (R_1 \, \omega_{02} + R_2 \, \omega_{01}) \, (R_1 + R_2)}{R^2_1 \, R^2_2 \, (C_1 \, \omega_{02} + C_2 \, \omega_{01}) \, (C_1 + C_2)} \right\}^{\frac{1}{4}}$$

Using the component values, the value obtained for f is 17·06 Kc/s.

Triple-Ts.—In the case of the triple-T considered in Fig. 7 there are again two frequencies at which the phase and amplitude nulls occur and they are given by

$$\omega = \omega_{01} \left[1 + \left(\frac{\omega_{02}}{\omega_{01}} \right)^2 \frac{C_2}{C_1} \right]^{\frac{1}{2}} \qquad (3)$$

$$\omega = \omega_{01} \left[1 + \left(\frac{R_1}{R_2} \right)^2 \right]^{\frac{1}{2}} \qquad (4)$$

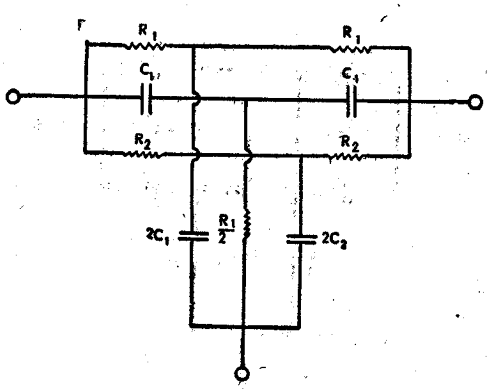

FIG. 7—Triple-T network.

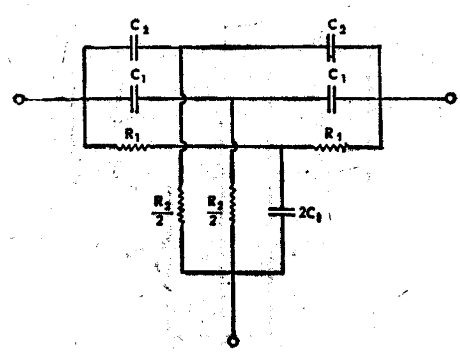

FIG. 8—Triple-T network.

The resonant frequencies are 26·08 Kc/s and 27·02 Kc/s respectively. The experimental value corresponding to the amplitude minimum is 27·00 Kc/s.

The triple-T (Fig 8) formed with the first twin-T and the capacitance section of the second twin-T also gives two resonant frequencies given by

$$\omega = \frac{\omega_{01}}{\left(1 + \frac{C_2}{C_1}\right)^{\frac{1}{2}}} \tag{5}$$

and

$$\omega = \frac{\omega_{01}}{\left[1 + \left(\frac{\omega_{01}}{\omega_{02}}\right)^2 \frac{R_1}{R_2}\right]^{\frac{1}{2}}} \tag{6}$$

The calculated values are the same and equal to 10·07 Kc/s. This shows that the phase and the amplitude minima occur at the same frequency as in a twin-T. The experimental value of resonant frequency corresponding to the amplitude minimum is 10·20 Kc/s.

The resistance arm of the first twin-T is paralleled with the second twin-T to form the triple-T as shown in Fig. 9. This network also gives two nulls at two different frequencies given by

$$\omega = \omega_{01}\left[\frac{C_1}{C_2} + \left(\frac{\omega_{02}}{\omega_{01}}\right)^2\right]^{\frac{1}{2}} \tag{7}$$

and

$$\omega = \omega_{02}\left[1 + \frac{R_2}{R_1}\right]^{\frac{1}{2}} \tag{8}$$

The resonant frequencies corresponding to phase and amplitude minima are 22·09 Kc/s and 22·54 Kc/s respectively. The experimental value for the amplitude minimum is 22·5 Kc/s.

The capacitance arm of the first twin-T with the second twin-T gives again a triple-T as shown in Fig. 10 whose phase and amplitude minima correspond to frequencies given by

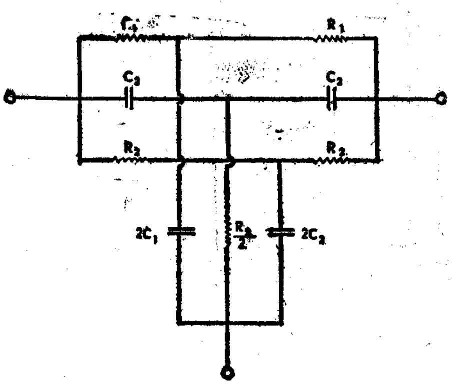

FIG. 9—Triple-T network.

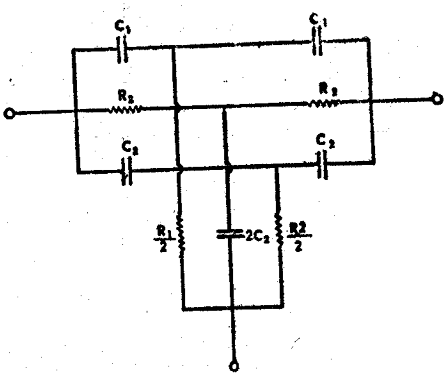

FIG. 10—Triple-T network.

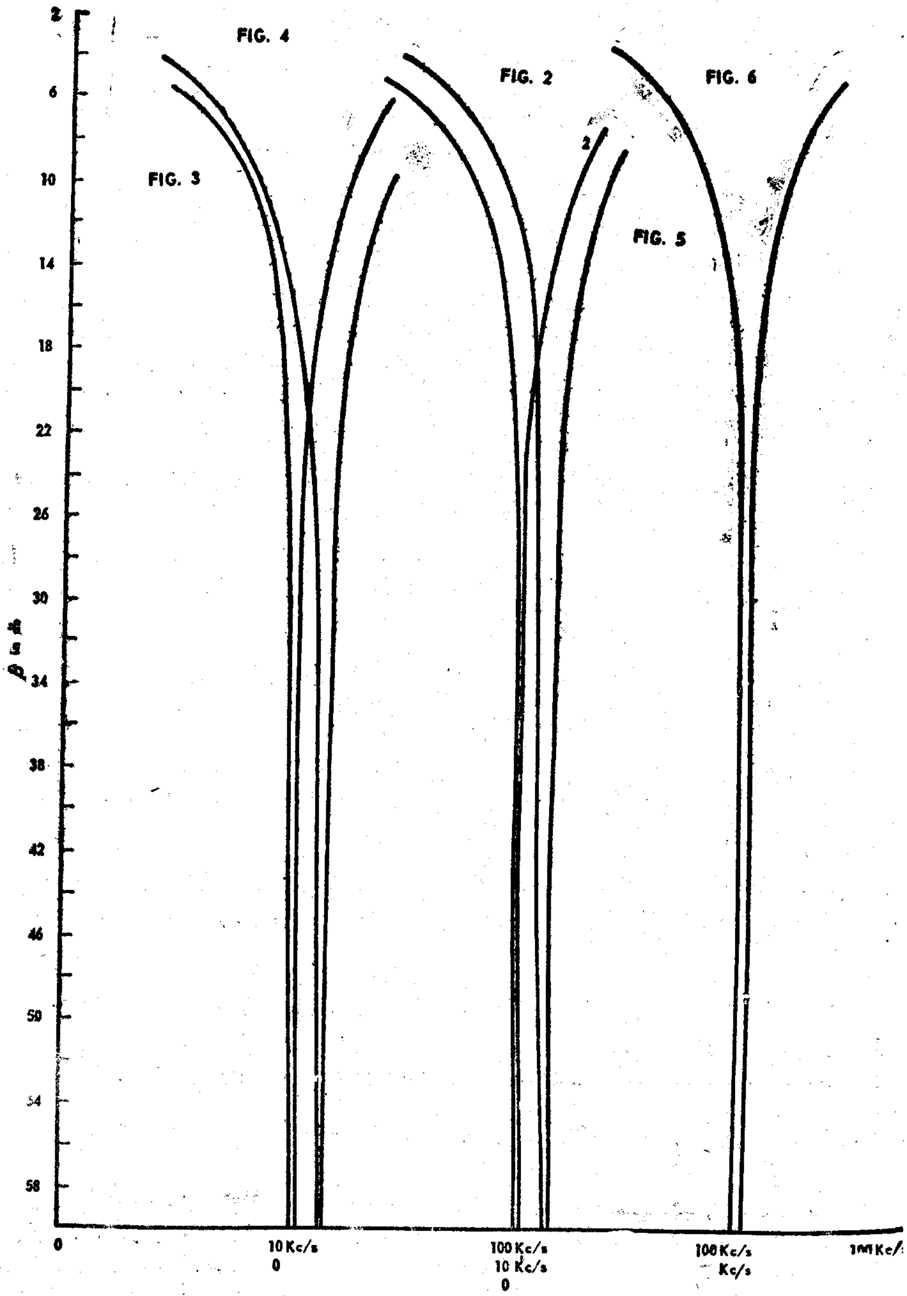

FIG. 11—Respons curves for twin-Ts and double twin-T networks.

$$\omega = \frac{\omega_{02}}{\left(1 + \frac{C_1}{C_2}\right)^{\frac{1}{2}}} \tag{9}$$

and

$$\omega = \frac{\omega_{02}}{\left[1 + \left(\frac{\omega_{02}}{\omega_{01}}\right)^2 \frac{R_2}{R_1}\right]^{\frac{1}{2}}} \tag{10}$$

The calculated values are given by $f = 13 \cdot 82$ Kc/s and $13 \cdot 50$ Kc/s and the experimental value is $13 \cdot 67$ Kc/s.

If two twin-T networks of the same type are paralleled together, the resonant frequency is the same as for single twin-T. But when the capacitance arm of one is paralleled with the other twin-T, the triple-T formed has a resonant frequency $0 \cdot 707$ times of the resonant frequency of the single twin-T. Similarly, when the resistance arm is added to the other twin-T to form the triple-T, the resonant frequency of the triple-T is $1 \cdot 414$ times that of the resonant frequency of the twin-T. It is found that for two similar twin-Ts in parallel or triple-Ts formed out of them, the phase and the amplitude minima occur at the same frequency as in a twin-T.

Response Curves: The response curves for the various combinations are given in Figs. 11 and 12. These are drawn by using a simple circuit shown in Fig. 13 which does not load the T-network. Constant output from a signal generator is fed to the input side of the T-network at various frequencies ranging from 4 Kc/s to 40 Kc/s in small steps and the corresponding deflection in the micro-ammeter are noted. Curves (2) and (4) are for the first and the second twin-Ts and curve (6) is for the double twin-T. Curves (3) and (5) are for the resistance arm of the first twin-T with the capacitance arm of the second twin-T, and the resistance arm of the second twin-T with the capacitance arm of the first twin-T, respectively. Curves (7), (8), (9) and (10) are for the four triple-Ts formed with all possible combinations of the T-arms in the two twin-Ts. For obtaining the output of the T-network in volts, the signal generator output is directly fed to the grid of the cathode follower at various frequencies and the corresponding micro-ammeter deflections are noted. For any value of deflection in the micro-ammeter, the corresponding voltage output from the T-network can be found out.

RESULTS

Table 2 gives the frequencies at which the amplitude null and the phase null occur, their geometric mean, the arithmetic mean, the practical values and the amplitude and the phase factors of the numerator of β in each case.

CONCLUSIONS

In all these cases, except the twin-Ts and the triple-T, shown in Fig. 8, there are two frequencies at which the null condition is satisfied. One frequency corresponds to an amplitude minimum and the other corresponds to a phase minimum. These two frequencies are close to each other, and no appreciable error will be introduced by taking either the geometric mean or the arithmetic mean. The two means are the same in the range of values used. In the case of twin-Ts (Figs. 2 & 4), the Triple-T (Fig. 8) the amplitude and the phase minima occur at the same frequency.

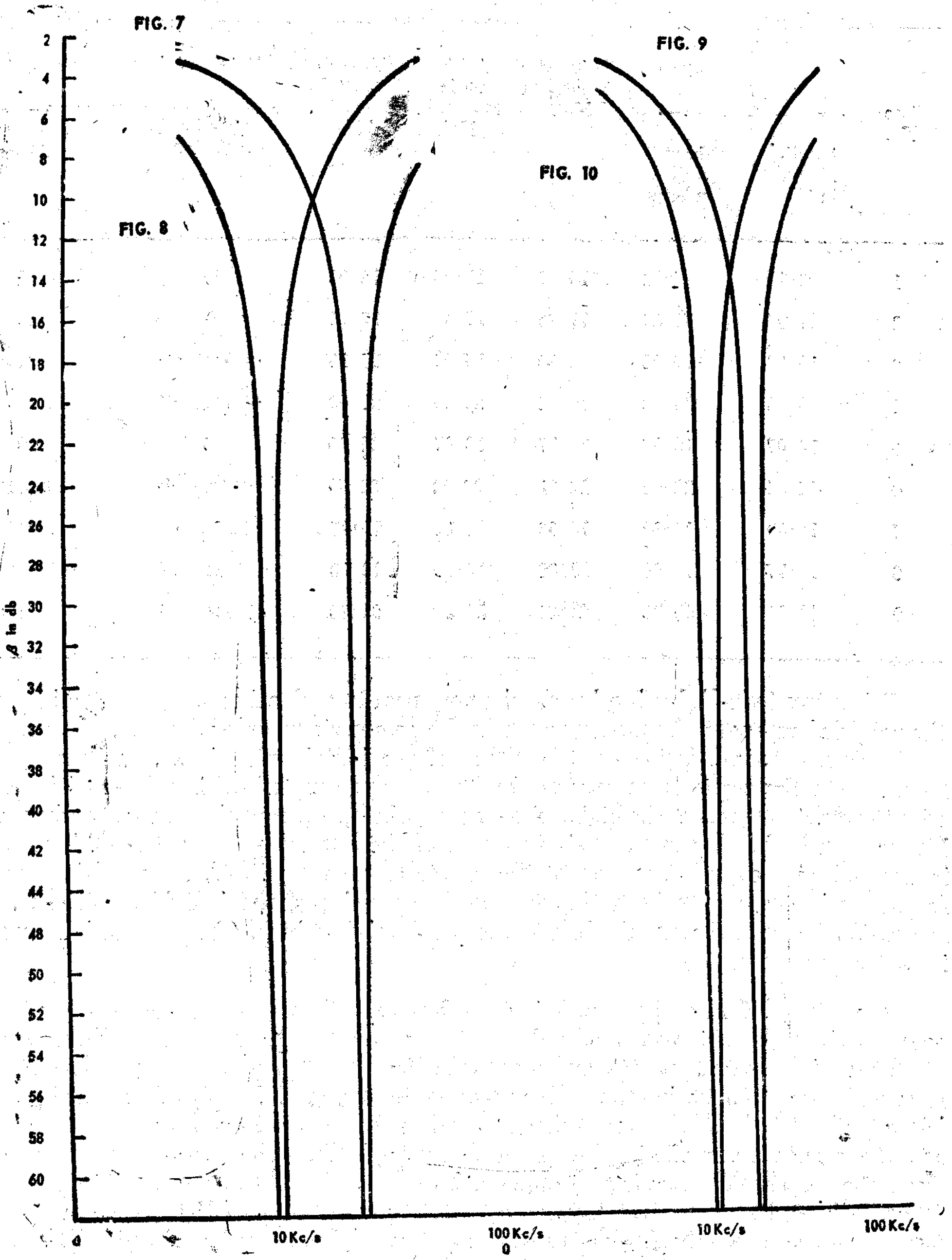

FIG. 12—Response curves for triple-T networks,

TABLE 2

Case No.	corres onding to		Geometric mean f in Kc/s	Arith-metic mean f in Kc/s	Experi-mental value f in Kc/s	Values of the numerator of β at f	
	Ampli-tude null f in Kc/s	Phase null f in Kc/s				Amplitude factor	Phase factor
1	15·92	15·92	15·92	15·92	16·00	0	0
2	17·85	17·85	17·85	17·85	17·75	0	0
3	16·52	17·08	16·80	16·80	17·00	$\cdot 045 \times 10^{-14}$	$\cdot 0046 \times 10^{-14}$
4	26·08	27·02	26·56	26·55	27·00	$\cdot 451 \times 10^{-14}$	$\cdot 041 \times 10^{-14}$
5	10·07	10·07	10·07	10·07	10·20	0	0
6	22·54	22·09	22·31	22·32	22·50	$\cdot 017 \times 10^{-14}$	$\cdot 589 \times 10^{-14}$
7	13·50	13·82	13·66	13·66	13·67	$\cdot 0005 \times 10^{-14}$	$\cdot 474 \times 10^{-14}$
8	13·77	13·00	13·38	13·39	13·30	$\cdot 0212 \times 10^{-14}$	$\cdot 0158 \times 10^{-14}$
9	20·64	21·86	21·24	21·25	21·50	$\cdot 505 \times 10^{-14}$	$\cdot 0459 \times 10^{-14}$

When two twin-Ts having frequencies very near to each other are connected in parallel, the resulting resonant frequency is nearly the geometric mean of the two. The response of the double twin-T is flatter than either of the individual twin-Ts. In the example given, when the capacitance arm of the lower frequency twin-T is connected in parallel with the resistance arm of the higher frequency twin-T, the resulting resonant frequency is higher than both. When the resistance arm of the lower frequency twin-T is connected in parallel with capacitance arm of the higher frequency twin-T, the resulting resonant frequency is lower than both the resonant frequencies of the individual twin-Ts. The separation between the amplitude and the phase minima in the former case is larger than in the latter case.

The triple-T formed by paralleling the first twin-T with the capacitance arm of the seocnd twin-T (Fig. 8), resonates at the least frequency and the response is also the sharpest of all the triple-Ts and the separation between the two frequencies corresponding to amplitude and phase minima is zero. The resonant frequency of the triple-T, formed by paralleling the first twin-T with the resistance arm of the second twin-T (Fig. 7) is the highest and the response curve is the flattest of all the triple-Ts. The frequency separation between the amplitude and phase minima is largest in this case. The two other triple-Ts (Figs. 9 & 10) have resonant frequencies and responses intermediate between the above two. With two exactly similar twin-Ts only two triple-Ts can be formed. The resonant frequency of the triple-T formed with two capacitance arms and one resistance arm has a resonant frequency equal to 0·707 times the resonant frequency of the single twin-T. Similarly, by having two resistance arms and one capacitance arm, a triple-T is formed, whose resonant frequency is 1·414 times that of the resonant frequency of the single twin-T.

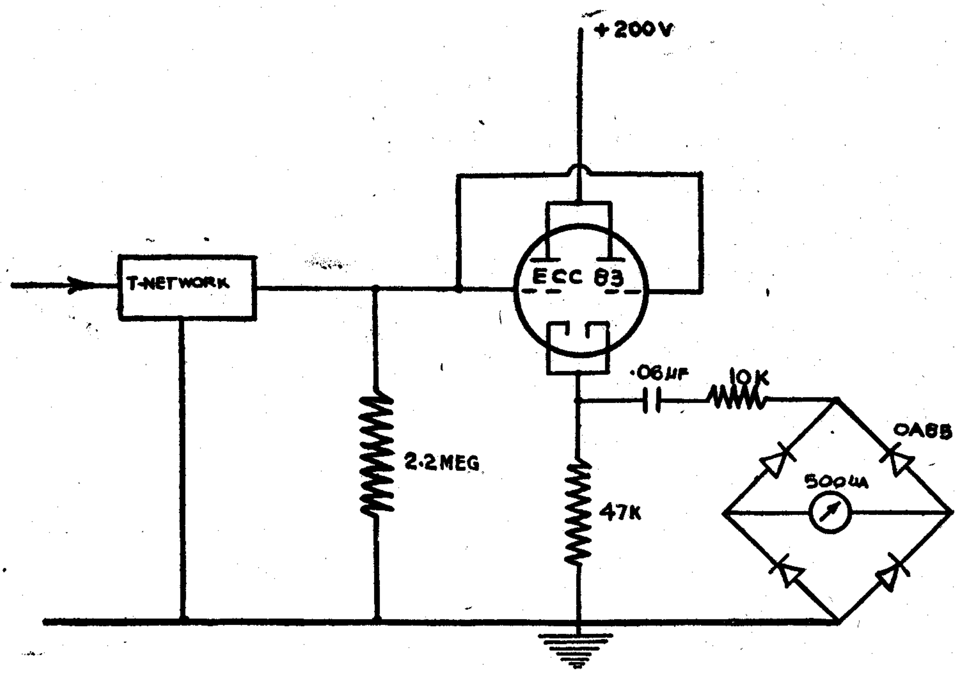

FIG. 13—Cathode follower and detector.

Further, with two twin-Ts of slightly differing frequencies nine combinations of parallel T-networks of different resonant frequencies can be obtained. By having a suitable switch for effecting these combinations, nine spot frequencies can be obtained. In this particular case, it is found that the difference between the lowest and the highest frequencies is more or less equal to the resonant frequency of the double twin-T.

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APPENDIX 1

A general formula relating the attenuation factor β in terms of the admittances of the various components in the arms of Ts, is derived for any number of Ts in parallel.

Derivation of the general formula for a multiple—T :

Let A, B, C be the admittances in the arms of a T. (Fig. 14) A_1, B_1, C_1 correspond to the first T, A_2, B_2, C_2 to the second and Am, Bm, Cm to the mth T. There are three nodes for any T, two being common to all, since they are connected in parallel. The number of nodes for a combination of the Ts, as shown in Fig. 14, is two more then the number of Ts in the combination. Hence, if there are 'm' Ts, there are (m+2) nodes and an equal number of nodal equations. Nodal analysis of the multiple T-network is resorted to because of its simplicity in this particular case compared to the mesh analysis. For simplicity, the input and output impedances are taken as zero and infinity respectively. Writing down the equations.

$$(V_i - V_1) A_1 + (V_i - V_2) A_2 + \ldots\ldots\ldots\ldots + (V_i - V_m) A_m = i \qquad (1)$$

where 'i' is the input current passing through the node at the input terminal having a potential V_i .

This equation can be written as

$$\sum_{1}^{n} (V_i - V_m) A_m = i \qquad (2)$$

where 'm' has values ranging from 1 to n.

For node 1

$$(V_i - V_1) A + (V_0 - V_1) B - V_1 C = 0$$

Hence for node 'm'

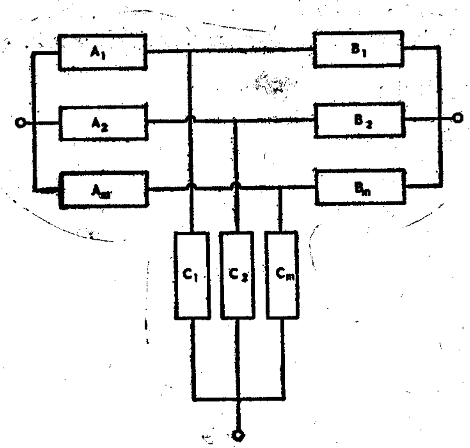

FIG. 14—Multiple-T network.

$$(V_i - V_m) A_m + (V_0 - V_m) B_m - V_m C_m = 0 \qquad (3)$$

For the output node—

$$(V_1 - V_0) B_1 + (V_2 - V_0) B_2 + \ldots\ldots\ldots + (V_m - V_0) B_n = 0 \qquad (4)$$

This equation can be written as

$$\sum_{1}^{n} (V_m - V_0) B_m = 0 \qquad (5)$$

From equation (3),

$$V_m = \frac{V_i A_m + V_o B_m}{A_m + B_m + C_m} \qquad (6)$$

From equation (5), $V_0 \sum_1^n B_m = \sum_1^n V_m B_m$ (7)

Substituting the value of V_m from (6) we get,

$$V_0 \sum_1^n B_m = \sum_1^n B_m \frac{V_i A_m + V_0 B_m}{A_m + B_m + C_m} \tag{8}$$

$$V_0 \left[\sum_1^n B_m - \sum_1^n \frac{B_m^2}{A_m + B_m + C_m} \right] = V_i \sum_1^n \frac{A_m B_m}{A_m + B_m + C_m} \tag{9}$$

$$V_0 \sum_1^n \left[B_m - \frac{B_m^2}{A_m + B_m + C_m} \right] = V_i \sum_1^n \frac{A_m B_m}{A_m + B_m + C_m} \tag{10}$$

$$\therefore \frac{1}{\beta} = \frac{V_i}{V_0} = \frac{\sum_1^n \frac{B_m (A_m + C_m)}{A_m + B_m + C_m}}{\sum_1^n \frac{A_m B_m}{A_m + B_m + C_m}} \tag{11}$$

$$= 1 + \left\{ \frac{\sum_1^n \frac{B_m C_m}{A_m + B_m + C_m}}{\sum_1^n \frac{A_m B_m}{A_m + B_m + C_m}} \right\} \tag{12}$$

This relationship holds good for any number of *Ts* in parallel. The resonant frequency of a multiple-T connected in parallel can be calculated.

APPENDIX 2

A study of-RC double twin-Ts in parallel, triple-Ts obtained by combination of any number of *Ts* out of the four from the RC double twin-Ts, in terms of the individual twin-Ts, is made. Using this relationship, the resonant frequencies are evaluated and their relationship correlated in terms of the resonant frequencies of the basic twin-Ts used.

Application of the general formula to various combinations of Ts : *Case* 1 : A simple RC twin-T as shown in Fig. 2, is considered. '*m*' in this case has values equal to 1 and 2.

$$\therefore \frac{1}{\beta} = 1 + \frac{\frac{B_1 C_1}{A_1 + B_1 + C_1} + \frac{B_2 C_2}{A_2 + B_2 + C_2}}{\frac{A_1 B_1}{A_1 + B_1 + C_1} + \frac{A_2 B_2}{A_2 + B_2 + C_2}}$$

Substituting the values for

$$A_1 = B_1 = \frac{1}{R_1} \text{ and } C_1 = 2jwC_1$$

$$A_2 = B_2 = jw\,C_1 \text{ and } C_2 = \frac{2}{R_1}$$

we get
$$\frac{1}{\beta} = 1 + \frac{\dfrac{\dfrac{2jwC_1}{R_1}}{\dfrac{2}{R_1}+2jwC_1} + \dfrac{\dfrac{2jwC_1}{R_1}}{2jwC_1 + \dfrac{2}{R_1}}}{\dfrac{\dfrac{1}{R^2_1}}{\dfrac{2}{R_1}+2jwC_1} + \dfrac{-w^2 C_1^2}{2jwC_1 + \dfrac{2}{R_1}}} \qquad (13)$$

$$= 1 + \frac{\dfrac{4jwC_1}{R_1}}{\left(\dfrac{1}{R^2_1} - w^2C^2_1\right)} \qquad (14)$$

The attenuation is infinite when $\beta = 0$ and hence, the resonant frequency $w_{01} = 1/R_1C_1$

$$\beta = \frac{1}{1 + j\dfrac{4wC_1}{R_1\left(\dfrac{1}{R^2_1} - w^2C^2_1\right)}} \qquad (15)$$

$$= \frac{1}{1 + j\dfrac{4w/w_{01}}{\left[1 - \left(\dfrac{w}{w_{01}}\right)^2\right]}} \qquad \text{Since } w_{01} = \frac{1}{R_1C_1}$$

$$= \frac{1 - j\dfrac{4r}{1-r^2}}{1 + \dfrac{16r^2}{(1-r^2)^2}} \qquad \text{where } r = \frac{w}{w_{01}}$$

This is of the form $\beta = |\beta|\, e^{j\theta}$ (16)

from which $\tan\theta = -\dfrac{4r}{1-r^2}$

For a series tuned circuit $\text{Cot}\ \theta = Q\left[r - \dfrac{1}{r}\right]$ (17)

$$\tan\theta = \frac{r}{Q\,(r^2-1)} = -\frac{4r}{1-r^2}$$

$$\therefore Q = \tfrac{1}{4}$$

Case 2 : A twin-T is formed by combining the resistance arm of the first twin-T with the capacitance of the second twin-T (Fig. 3).

$$\frac{1}{\beta} = 1 + \frac{\dfrac{\dfrac{2\,jwC_1}{R_1}}{\dfrac{2}{R_1} + 2\,jwC_1} + \dfrac{\dfrac{2\,jwC_2}{R_2}}{\dfrac{2}{R_2} + 2\,jwC_2}}{\dfrac{\dfrac{1}{R_1^2}}{\dfrac{2}{R_1} + 2\,jwC_1} - \dfrac{w^2C_2^2}{\dfrac{2}{R_2} + 2\,jwC_2}}$$

$$\therefore \beta = \frac{\dfrac{1}{R_1^2}\left(\dfrac{2}{R_2} + 2\,jwC_2\right) - w^2C_2^2\left(\dfrac{2}{R_1} + 2\,jwC_1\right)}{\left(\dfrac{1}{R_1^2} + \dfrac{2\,jwC_1}{R_1}\right)\left(\dfrac{2}{R_2} + 2\,jwC_2\right) + \left(\dfrac{2\,jwC_2}{R_2} - w^2C_2^2\right)\left(\dfrac{2}{R_1} + 2\,jwC_1\right)} \tag{18}$$

Equating the imaginary terms of the numerator to zero, we get

$$\frac{2\,jwC_2}{R_1^2} - 2\,jw^3C_1C_2^2 = 0$$

$$w^2 = \frac{1}{C_1C_2R_1^2}$$

$$w = w_{01}\sqrt{\frac{C_1}{C_2}} \tag{19}$$

Equating the real terms to zero, we get

$$\frac{2}{R_1^2R_2} = \frac{2\,w^2C_2^2}{R_1}$$

$$w = w_{02}\sqrt{\frac{R_2}{R_1}} \tag{20}$$

Combining the two values of w, we get,

$$w = \sqrt{w_{01}\ w_{02}\left(\frac{C_1R_2}{C_2R_1}\right)^{\frac{1}{2}}} \tag{21}$$

Case 3 : The network is similar to the previous one except that the values of the resistances and condensers are different (See Fig. 4).

The resonant frequency $w_{02} = \dfrac{1}{R_2C_2}$

and

$$\beta = \frac{1}{1 + j \dfrac{4wC_2}{R_2\left(\dfrac{1}{R^2_2} - w^2C^2_2\right)}} \tag{22}$$

Case 4 : In this case, the twin-T is formed by combining the capacitance arm of the first twin-T with the resistance arm of the second twin-T.

$$\frac{1}{\beta} = 1 + \frac{\dfrac{\dfrac{2jwC_1}{R_1}}{\dfrac{2}{R_1} + 2jwC_1} + \dfrac{\dfrac{2jwC_2}{R_2}}{\dfrac{2}{R_2} + 2jwC_2}}{\dfrac{-w^2C_1^2}{\dfrac{2}{R_1} + 2jwC_1} + \dfrac{\dfrac{1}{R_2^2}}{\dfrac{2}{R_2} + 2jwC_2}}$$

$$\beta = \frac{\dfrac{1}{R_2^2}\left(\dfrac{2}{R_1} + 2jwC_1\right) - w^2C_1^2\left(\dfrac{2}{R_2} + 2jwC_2\right)}{\left(\dfrac{2jwC_1}{R_1} - w^2C_1^2\right)\left(\dfrac{2}{R_2} + 2jwC_2\right) + \left(\dfrac{1}{R_2^2} + \dfrac{2jwC_2}{R_2}\right)\left(\dfrac{2}{R_1} + 2jwC_1\right)} \tag{23}$$

Equating the imaginary terms of the numerator to zero, we get

$$\frac{2jwC_1}{R_2^2} - 2jw^3C_1^2C_2 = 0$$

$$w^2 = \frac{1}{C_1C_2R_2^2} = w_{02}^2\frac{C_2}{C_1}$$

$$\therefore w = w_{02}\sqrt{\frac{C_2}{C_1}} \tag{24}$$

Equating the real terms to zero, we get

$$\frac{2}{R_1R_2^2} - \frac{2w^2C_1^2}{R_2} = 0$$

$$w^2 = \frac{1}{R_1R_2C_1^2} = w_{01}^2\frac{R_1}{R_2} \text{ and hence } w = w_{01}\sqrt{\frac{R_1}{R_2}} \tag{25}$$

Combining the values of w, we get

$$w = \sqrt{w_{01}w_0^2}\left(\frac{C_2R_1}{C_1R_2}\right)^{\frac{1}{2}} \tag{26}$$

Case 5 : The twin-Ts given in case 1 and case 2 are paralleled as shown in Fig. 6. Taking the general formula for $1/\beta$ and substituting the values for $A_1B_1C_1$, $A_2B_2C_2$,

$A_3B_3C_3$ and $A_4B_4C_4$, we get

$$\frac{1}{\beta} = 1 + \frac{\dfrac{\frac{2jwC_1}{R_1}}{\frac{2}{R_1}+2jwC_1} + \dfrac{\frac{2jwC_1}{R_1}}{2jwC_1+\frac{2}{R_1}} + \dfrac{\frac{2jwC_2}{R_2}}{\frac{2}{R_2}+2jwC_2} + \dfrac{\frac{2jwC_2}{R_2}}{\frac{2}{R_2}+2jwC_2}}{\dfrac{\frac{1}{R_1^2}}{\frac{2}{R_1}+2jwC_1} + \dfrac{-w^2C_1^2}{2jwC_1+\frac{2}{R_1}} + \dfrac{\frac{1}{R_2^2}}{\frac{2}{R_2}+2jwC_2} + \dfrac{-w^2C_2^2}{2jwC_2+\frac{2}{R_2}}}$$

$$\beta = \frac{\left[\frac{1}{R_1^2R_2} - \frac{w^2C_1^2}{R_2} + \frac{1}{R_1R_2^2} - \frac{w^2C_2^2}{R_1}\right] + jw\left[\frac{C_2}{R_1^2} - w^2C_1^2C_2 + \frac{C_1}{R_2^2} - w^2C_2^2C_1\right]}{\left[\frac{1}{R_1^2R_2} - \frac{w^2C_1^2}{R_2} - \frac{4w^2C_1C_2}{R_1} + \frac{1}{R_1R_2^2} - \frac{w^2C_2^2}{R_1} - \frac{4w^2C_1C_2}{R_2}\right] + jw\left[\frac{C_2}{R_1^2} - w^2C_1^2C_2 + \frac{4C_1}{R_1R_2} + \frac{C_1}{R_2^2} - w^2C_1C_2^2 + \frac{4C_2}{R_1R_2}\right]} \tag{27}$$

Taking the real and the imaginary terms in the numerator and equating each to zero the resonant conditions can be obtained as long as the denominator is not zero.

Taking the imaginary term, we get

$$w^2C_1C_2\,(C_1 + C_2) = \frac{C_2}{R_1^2} + \frac{C_1}{R_2^2}$$

$$w^2 = \frac{1}{C_1C_2R_1R_2}\;\frac{\left[C_2\,\frac{R_2}{R_1} + C_1\,\frac{R_1}{R_2}\right]}{(C_1 + C_2)}$$

$$w = \sqrt{w_{01}\,w_{02}\;\frac{\left[\frac{1}{R_1w_{02}} + \frac{1}{R_2w_{01}}\right]}{(C_1 + C_2)}} \tag{28}$$

when $\quad w_{01} = \dfrac{1}{C_1R_1}$ and $w_{02} = \dfrac{1}{C_2R_2}$

The value of 'w' given by the imaginary term equated to zero corresponds to the phase minimum.

Taking the real term, we get

$$w^2\left(\frac{C_1^2}{R_2} + \frac{C_2^2}{R_1}\right) = \frac{1}{R_1^2R_2} + \frac{1}{R_1R_2^2}$$

$$w^2\left(\frac{C_1^2R_1 + C_2^2R_2}{R_1\,R_2}\right) = \frac{R_1+R_2}{R_1^2\,R_2^2}$$

$$w^2 = \frac{(R_1 + R_2)}{R_1 R_2 \left[\frac{C_1}{w_{01}} + \frac{C_2}{w_{02}} \right]}$$

$$= w_{01} w_{02} \left[\frac{(R_1 + R_2)}{R_1 R_2 (C_1 w_{02} + C_2 w_{01})} \right]$$

Therefore $w = \sqrt{w_{01} w_{02} \left[\frac{(R_1 + R_2)}{R_1 R_2 (C_1 w_{02} + C_2 w_{01})} \right]}$ (29)

Case 6 : The triple-T network shown in Fig. 7 is formed by adding to the first twin-T, the resistance section of the second twin-T.

$$\frac{1}{\beta} = 1 + \frac{\dfrac{\frac{2 jwC_1}{R_1}}{\frac{1}{R_1} + jwC_1} + \dfrac{\frac{j wC_2}{R_2}}{\frac{1}{R_2} + j wC_2}}{\dfrac{\frac{1}{R_1^2} - w^2 C_1^2}{\frac{2}{R_1} + 2 jwC_1} + \dfrac{\frac{1}{R_2^2}}{\frac{2}{R_2} + 2 jwC_2}}$$

and

$$\beta = \frac{\left(\frac{1}{R_1^2 R_2} + \frac{1}{R_1 R_2^2} - \frac{w^2 C_1^2}{R_2} \right) + jw \left(\frac{C_2}{R_1^2} + \frac{C_1}{R_2^2} - w^2 C_1^2 C_2 \right)}{\left[\frac{1}{R_1^2 R_2} + \frac{1}{R_1 R_2^2} - \frac{w^2 C_1^2}{R_2} - \frac{4 w^2 C_1 C_2}{R_1} - \frac{2 w^2 C_1 C_2}{R_2} \right] + jw \left[\frac{C_1}{R_2} \left(\frac{4}{R_1} + \frac{1}{R_2} \right) + \frac{C_2}{R_1} \left(\frac{2}{R_2} + \frac{1}{R_1} \right) - w^2 C_1^2 C_2 \right]} \quad (30)$$

Equating the imaginary terms of the numerator to zero, we get

$$w^2 = \frac{1}{C_1 C_2 R_2^2} + \frac{1}{C_1^2 R_1^2}$$

and

$$w = w_{01} \left[1 + \left(\frac{w_{02}}{w_{01}} \right)^2 \frac{C_2}{C_1} \right]^{\frac{1}{2}} \text{ where } w_{01} = \frac{1}{C_1 R_1} \quad (31)$$

and

$$w_{02} = \frac{1}{C_2 R_2}$$

Equating the real terms of the numerator to zero, we get

$$w_2 = \frac{1}{C_1^2 R_1^2} + \frac{1}{C_1^2 R_1 R_2}$$

and

$$w = w_{01} \left[1 + \left(\frac{R_1}{R_2} \right) \right]^{\frac{1}{2}} \quad (32)$$

Taking the geometric mean, we get

$$w = w_{01} \left\{ \left[1 + \left(\frac{w_{02}}{w_{01}} \right)^2 \frac{C_2}{C_1} \right] \left[1 + \frac{R_1}{R_2} \right] \right\}^{\frac{1}{4}} \tag{33}$$

Case 7 : This triple-T is formed with the first twin-T and the capacitance section of the second twin-T (Fig. 8).

$$\frac{1}{\beta} = 1 + \frac{\dfrac{\frac{2\,jwC_1}{R_1}}{\frac{2}{R_1} + 2\,jwC_1} + \dfrac{\frac{2\,jwC_1}{R_1}}{2jwC_1 + \frac{2}{R_1}} + \dfrac{\frac{2\,jwC_2}{R_2}}{2jwC_2 + \frac{2}{R_2}}}{\dfrac{\frac{1}{R_1^2}}{\frac{2}{R_1} + 2jwC_1} + \dfrac{-w^2 C_1^2}{2\,jwC_1 + \frac{2}{R_2}} + \dfrac{-w^2 C_2^2}{2\,jwC_2 + \frac{2}{R_2}}}$$

$$\beta = \frac{\left(\frac{1}{R_1^2} - w^2 C_1^2 \right) \left(2\,jwC_2 + \frac{2}{R_2} \right) - w^2 C_2^2 \left(\frac{2}{R_1} + 2\,jwC_1 \right)}{\left(\frac{1}{R_1^2} - w^2 C_1^2 + \frac{4\,jw\,C_1}{R_1} \right) \left(2\,jw\,C_2 + \frac{2}{R_2} \right) + \left(\frac{2\,jwC_2}{R_2} - w^2\,C_2^2 \right) \left(\frac{2}{R_1} + 2jwC_1 \right)} \tag{34}$$

Equating the imaginary terms of the numerator to zero, we get

$$2\,jw\,C_2 \left(\frac{1}{R_1^2} - w^2\,C_1^2 \right) - 2\,jw^3\,C_1\,C_2^2 = 0$$

$$w^2\,(C_1^2 + C_1\,C_2) = \frac{1}{R_1^2}$$

$$w^2 = \frac{1}{R_1^2\,(C_1^2 + C_1\,C_2)} = \frac{1}{\frac{1}{w_{01}^2} \left[1 + \frac{C_2}{C_1} \right]}$$

$$\therefore\ w = \frac{w_{01}}{\left[1 + \frac{C_2}{C_1} \right]^{\frac{1}{2}}} \tag{35}$$

Equating the real terms to zero, we get

$$\frac{2}{R_1^2\,R_2} - \frac{2\,w^2\,C_1^2}{R_2} - \frac{2\,w^2\,C_2^2}{R_1} = 0$$

$$w^2 \left[\frac{C_1^2}{R_2} + \frac{C_2^2}{R_1} \right] = \frac{1}{R_1^2\,R_2}$$

$$w^2 \left[\frac{C_1^2\,R_1 + C_2^2\,R_2}{R_1\,R_2} \right] = \frac{1}{R_1^2\,R_2}$$

$$w^2 = \frac{1}{C_1^2 R_1^2 + C_2^2 R_2^2 \dfrac{R_1}{R_2}}$$

$$= \frac{1}{\dfrac{1}{w_{01}^2} + \dfrac{1}{w_{02}^2} \dfrac{R_1}{R_2}}$$

$$\therefore w = \frac{w_{01}}{\left[1 + \left(\dfrac{w_{01}}{w_{02}} \right)^2 \dfrac{R_1}{R_2} \right]^{\frac{1}{2}}} \tag{36}$$

Taking the geometric mean, the resonant frequency is given by

$$w = \frac{w_{01}}{\sqrt[4]{\left[1 + \left(\dfrac{w_{01}}{w_{02}} \right)^2 \dfrac{R_1}{R_2} \right] \left[1 + \dfrac{C_2}{C_1} \right]}} \tag{37}$$

Case 8 : The resistance arm of the first twin-T is added to the second twin-T to form the triple-T network (Fig. 9).

$$\frac{1}{\beta} = 1 + \frac{\dfrac{\frac{2\,jwC_1}{R_1}}{\frac{2}{R_1} + 2\,jw\,C_1} + \dfrac{\frac{2\,jw\,C_2}{R_2}}{\frac{2}{R_2} + 2\,jw\,C_2} + \dfrac{\frac{2\,jw\,C_2}{R_2}}{\frac{2}{R_2} + 2\,jw\,C_2}}{\dfrac{\frac{1}{R_1^2}}{\frac{2}{R_1} + 2\,jw\,C_1} + \dfrac{\frac{1}{R_2^2}}{\frac{2}{R_2} + 2\,jw\,C_2} + \dfrac{-w^2 C_2^2}{\frac{2}{R_2} + 2\,jwC_2}}$$

$$\beta = \frac{\dfrac{1}{R_1^2}\left(\dfrac{2}{R_2} + 2\,jw\,C_2 \right) + \left(\dfrac{1}{R_2^2} - w_2 C_2^2 \right) \left(\dfrac{2}{R_1} + 2\,jwC_1 \right)}{\left(\dfrac{1}{R_1^2} + \dfrac{2jwC_1}{R_1} \right)\left(\dfrac{2}{R_2} + 2\,jw\,C_2 \right) + \left(\dfrac{1}{R_2^2} - w^2 C_2^2 + \dfrac{4\,jw\,C_2}{R_2} \right)\left(\dfrac{2}{R_1} + 2\,jwC_1 \right)} \tag{38}$$

Equating the imaginary terms of the numerator to zero, we get

$$\frac{2\,jw\,C_2}{R_1^2} + \frac{2\,jw\,C_1}{R_2^2} - 2\,jw^3 C_1 C_2^2 = 0$$

and

$$w = w_{01} \left[\frac{C_1}{C_2} + \left(\frac{w_{02}}{w_{01}} \right)^2 \right]^{\frac{1}{2}} \tag{39}$$

Equating the real terms of the numerator to zero, we get

$$\frac{2}{R_1^2 R_2} + \frac{2}{R_1 R_2^2} - \frac{2w^2 C_2^2}{R_1} = 0$$

$$w^2 = \frac{1}{C_2^2 R_1 R_2} + \frac{1}{C_2^2 R_2^2}$$

$$= w_{02}^2 \left[1 + \frac{R_2}{R_1} \right]$$

$$\therefore \; w = w_{02} \left[1 + \frac{R_2}{R_1} \right]^{\frac{1}{2}} \qquad (40)$$

Case 9 : The triple-T is formed by adding the capacitance section of the first twin-T to the second twin-T (Fig. 10).

$$\frac{1}{\beta} = 1 + \frac{\dfrac{\frac{2\,jwC_1}{R_1}}{2\,jwC_1 + \frac{2}{R_1}} + \dfrac{\frac{4\,jwC_2}{R_2}}{2\,jwC_2 + \frac{2}{R_2}}}{-\dfrac{w^2 C_1^2}{2\,jwC_1 + \frac{2}{R_1}} + \dfrac{\frac{1}{R_2^2} - w^2 C_2^2}{2\,jwC_2 + \frac{2}{R_2}}}$$

$$\beta = \frac{-\left(\frac{2}{R_2} + 2\,jwC_2\right) w^2 C_1^2 + \left(2\,jwC_1 + \frac{2}{R_1}\right)\left(\frac{1}{R_2^2} - w^2 C_2^2\right)}{\left(\frac{2\,jw\,C_1}{R_1} - w^2 C_1^2\right)\left(\frac{2}{R_2} + 2\,jwC_2\right) + \left(4\,jwC_2 + \frac{1}{R_2^2} - w^2 C_2^2\right)\left(2\,jwC_1 + \frac{2}{R_1}\right)} \qquad (41)$$

Equating the imaginary terms of the numerator to zero, we get

$$-2\,jw^3 C_1^2 C_2 + \frac{2\,jwC_1}{R_2^2} - 2\,jw^3 C_2^2 C_1 = 0$$

$$w^2 = \frac{1}{R_2^2 C_2 (C_1 + C_2)} = \frac{w_{02}^2}{\left(1 + \frac{C_1}{C_2}\right)}$$

$$w = \frac{w_{02}}{\left(1 + \frac{C_1}{C_2}\right)^{\frac{1}{2}}} \qquad (42)$$

Equating the real terms to zero, we get

$$-\frac{2w^2 C_1^2}{R_2} + \frac{2}{R_1 R_2^2} - \frac{2\,w^2 C_2^2}{R_1} = 0$$

$$w = \frac{w_{02}}{\left[1 + \left(\frac{w_{02}}{w_{01}}\right)^2 \frac{R_2}{R_1}\right]^{\frac{1}{2}}} \qquad (43)$$