PAY-OFF IN A TACTICAL AIR GAME

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A problem in tactical air-game with an additional role for the ground support forces has been formulated. A mix of different types of aircrafts for each of the three roles *e.g.*, counter-air, air-defence and ground support operations has further been considered. A simple approach has been outlined to calculate the pay-off function and it is shown that with the help of the theory of games, the optimal strategies for both the sides can be obtained.

The problem of optimal deployment of tactical air force in various theatre air tasks can be analysed as a multi-move game between the two sides. A similar problem has been considered by Berkowitz and Dresher[1] and the usual tasks assigned are as follows:—

Counter-air—These operations are against the enemy theatre air base complex and organisation in order to destroy his aircraft, personnel and facilities.

Air-defence—These represent air-defence operations against the enemy's counter-air operations.

Ground Support—The targets for ground support operations are concentrations of enemy troops or fortified positions. This also includes interdiction, reconnaissance and air lift.

In the present work, the ground support forces have been given an additional role of attacking the air-defence forces of the opposite side in order to reduce them before they actually attack the counter-air forces. It is obvious that this additional role assigned by a party to its ground support forces will increase the number of its counter- air forces that survive the attack of the air-defence forces of the other party and hence greater damage is done to the other party.

Besides, the problem has been generalised by considering a mix of different types of aircrafts, for each role, with different kill capabilities depending upon their flying altitude and weapon characteristics.

It is necessary to point out at this stage that distinction between one "type" of aircraft from another is made on the basis of "Operating Expenditure". Likewise, in choosing various alternative combinations of different aircrafts, the total Operating Expenditure, and not the total "number" of aircrafts has been kept constant. This is very realistic since in an actual situation, it is the total "Operating Expenditure" and not the total "number" of aircrafts that ultimately decides the choice between the various combinations of different types of aircraft.

It may also be made clear that the "Operating Expenditure" does not mean purely rupee and paise, it is actually in a deepersense. We may, for example, interpret "Operating Expenditure" as 15, 10 and 5 units of petrol or some other essential commodity the availability of which in time may be priceless to the commander.

MODIFIED DEFINITION OF THE PAY-OFF FUNCTION

The pay-off function as defined by Berkowitz and Dresher is in terms of the difference of distance advanced by both the sides which is a function of ground support forces of each side. In defining this, they have taken the strength of the ground support forces at the beginning of the strike but it should be borne in mind that it will continuously go on diminishing and at the end of the strike, it is likely to differ much from that in the beginning. In view of this, the pay-off has also been modified by taking the mean of the ground support forces at the beginning and end of the strike as their strength throughout the compaign. Though this is also an approximation but more accurate than the previous one.

THE PROBLEM

Let the total strength of planes with two sides A and B be S_1 and S_2 and let the number of types of planes be m and n respectively. We further suppose that the way of distributing the total aircraft strength (m types) among the three types of roles be as under—

$$\text{Counter-air}: \sum_{i=1}^{m} N_{j1}\,;\ \text{Air-Defence}: \sum_{i=1}^{m} N_{j2}\,;\ \text{and Ground support}: \sum_{i=1}^{m} N_{j3}\,.$$

Similarly for the other side we have

$$\text{Counter-air}: \sum_{j=1}^{n} N_{j1}\,;\ \text{Air-defence}: \sum_{j=1}^{n} N_{j2}\ \text{and Ground support}: \sum_{j=1}^{n} N_{j3}\,.$$

In addition, the following information is given.—

K'_{ij} —the number of kills achieved by a single i-type aircraft of the counter-air force when sent against the j-type aircraft of the ground support force of the other party.

K''_{ij} —the number of kills achieved by a single i-type aircraft of the air-defence force when sent against the j-type aircraft of the counter-air force of the other party.

K'''_{ij} —the number of kills achieved by a single i-type aircraft of the ground support when sent against the j-type aircraft of the air-defence force of the other party.

Q'_{ji} —the number of kills achieved by a single j-type aircraft of the counter-air force when sent against i-type aircraft of the ground support force of the other party.

Q''_{ji} —the number of kills achieved by a single j-type aircraft of the air-defence force when sent against the i-type aircraft of the counter-air force of the other party.

Q'''_{ji} —the number of kills achieved by a single j-type aircraft of the ground support force when sent against the i-type aircraft of the air-defence force of the other party.

MATHEMATICAL EXPRESSION FOR THE PAY-OFF FUNCTION

Loss in B's air-defence force is

$$\sum_{j=1}^{n} \sum_{j=1}^{m} N_{i3} K'''_{ij} \tag{1}$$

Therefore B's remaining air-defence is

$$\sum_{=1}^{n} N_{j2} - \sum_{j=1}^{n} \sum_{i=1}^{m} N_{i3}\ K'''_{ij} \tag{2}$$

Number of planes of counter-air force of A killed by the air-defence of B given by (2) can be written as :

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(N_{j2}-\sum_{i=1}^{m} N_{i3}\ K'''_{ij}\right)Q''_{ji}\right] \tag{3}$$

Therefore A's remaining counter-air force is

$$\sum_{i=1}^{m}N_{i1}-\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(N_{j2}-\sum_{i=1}^{m}N_{i3}\ K'''_{ij}\right)Q''_{ji}\right] \tag{4}$$

This gives the reduction in the ground support forces of B as

$$\sum_{j=1}^{n}\sum_{i=1}^{m}\left[\left\{N_{i1}-\sum_{j=1}^{n}\left(N_{j2}\ Q''_{ji}-Q''_{ji}\sum_{i=1}^{m}N_{i3}K'''_{ij}\right)\right\}K'_{ij}\right] \tag{5}$$

So the remaining ground forces of B after the strike is

$$\sum_{j=1}^{n}N_{j3}-\sum_{j=1}^{n}\sum_{i=1}^{m}\left[\left\{N_{i1}-\sum_{j=1}^{n}\left(N_{j2}\ Q''_{ji}-Q''_{ji}\sum_{i=1}^{m}N_{i3}K'''_{ij}\right)\right\}K'_{ij}\right]$$

$$=\sum_{j=1}^{n}\left[N_{j3}-\sum_{i=1}^{m}\left\{N_{i1}K'_{ij}-K'_{ij}\sum_{j=1}^{n}\left(N_{j2}Q''_{j}-Q''_{ji}\sum_{i=1}^{m}N_{i3}K'''_{ij}\right)\right\}\right] \tag{6}$$

Thus the mean of the ground support forces at the beginning and end of the strike for B is given by

$$P_2=\sum_{j=1}^{n}\left[N_{j3}-1/2\sum_{i=1}^{m}\left\{N_{i1}K'_{ij}-K'_{ij}\sum_{j=1}^{n}\left(N_{j2}Q''_{ji}-Q''_{ji}\sum_{i=1}^{m}N_{i3}K'''_{ij}\right)\right\}\right] \tag{7}$$

Similarly the value of this mean for A is given by

$$P_1=\sum_{i=1}^{m}\left[N_{i3}-1/2\sum_{j=1}^{n}\left\{N_{j1}Q'_{ji}-Q'_{ji}\sum_{i=1}^{m}\left(N_{i2}K''_{ij}-K''_{ij}\sum_{j=1}^{n}N_{j3}Q'''_{ji}\right)\right\}\right] \tag{8}$$

Thus, according to the modified definition of the pay-off function, the pay-off P from B to A can be written as

$$P=P_1-P_2$$

$$=\sum_{i=1}^{m}\left[N_{i3}-1/2\sum_{j=1}^{n}\left\{N_{j1}Q'_{ji}-Q'_{ji}\sum_{i=1}^{m}\left(N_{i2}K''_{ij}-K''_{ij}\sum_{j=1}^{n}N_{j3}Q'''_{ji}\right)\right\}\right]$$

$$-\sum_{j=1}^{n}\left[N_{j3}-1/2\sum_{i=1}^{m}\left\{N_{i1}K'_{ij}-K'_{ij}\sum_{j=1}^{n}\left(N_{j2}Q''_{ji}-Q''_{ji}\sum_{i=1}^{m}N_{i3}K'''_{ij}\right)\right\}\right] \tag{9}$$

The value of the pay-off can be determined with the help of (9) knowing all the parameters involved.

OPTIMAL STRATEGIES AND VALUE OF THE GAME IN A PARTICULAR CASE

Let us suppose that there are three types of aircraft with A and let their operating expenditure be 15, 10, 5 units respectively. Assuming the total operational budget to be 100 units and that at least one aircraft of each type has to be included for every role, the possible number of ways of allocating aircrafts for the three roles would be as given in Table 1.

TABLE 1

Combination	Roles		
	Counter-air	Air-defence	Ground Support
1st	1+2+1	1+1+1	1+1+1
2nd	1+1+3	1+1+1	1+1+1
3rd	1+1+1	1+2+1	1+1+1
4th	1+1+1	1+1+3	1+1+1
5th	1+1+1	1+1+1	1+2+1
6th	1+1+1	1+1+1	1+1+3
7th	1+1+2	1+1+1	1+1+1
8th	1+1+2	1+1+1	1+1+2
9th	1+1+1	1+1+2	1+1+2

Similarly let there be two types of aircrafts with B and let their operating expenditure be 20 and 10 units respectively. If the total operational budget is 100 units and at least one aircraft of each type has to be included for every role then the possible number of ways of allocating would be as given in Table 2.

TABLE 2

Roles	Combination		
	1st	2nd	3rd
Counter-air	1+2	1+1	1+1
Air-defence	1+1	1+2	1+1
Ground support	1+1	1+1	1+2

Thus knowing K'_{ij}, K''_{ij}, K'''_{ij}, Q'_{ji}, Q''_{ji}, and Q'''_{ji}, we can formulate the pay-off matrix which, with the help of theory of games[2], will easily give optimal strategies for both the sides and the value of the game. The solution may come out in terms of pure strategies or mixed strategies for both the sides or a combination of pure and mixed strategies depending upon the values of the various parameters involved.

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