FURTHER STUDIES ON SHOCK WAVE IN A DUSTY MEDIUM

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Using the general form of the equation of state, the thermodynamic parameters have been studied across a shock wave in a dusty medium.

Carrier[1] studied shock wave in a dusty medium and since the solution of the problem of interest was described by a many parameter family of functions, no attempt was made to compile comprehensively the approximate number. Instead a few examples were worked out and the integration, which lead to the detailed results, were specified. Srivastava and Mishra[2] obtained the Rankine Hugoniots equations across the shock waves in a dusty medium and then using the perfect gas equations obtained the jump in entropy, enthalphy etc. The purpose of the present paper is to obtain the equation of state, the velocity of sound, the shock strength, the jump in entropy, enthalpy etc. using the general form of the gas equation.

FUNDAMENTAL EQUATIONS

Hirschfelder, Bird and Spotz[3] gave the general form of the gas equation as

$$p/\rho RT = 1 + b\rho + c\rho^2 + \ldots\ldots\ldots\ldots\ldots \text{ etc.} \tag{1}$$

where p, ρ, T denote pressure, density and absolute temperature, R is the universal gas constant and $b, c, \ldots\ldots$ are functions of temperature and are small quantities. We take the above equation in the form

$$p = \rho\,(1 + b\rho)\; RT \tag{2}$$

Following Srivastava and Mishra[2], the Rankine Hugoniots equations are

$$[U^i\,] = -\,\overset{*}{\delta}_{e/}\, u_{1N/}\, X^i \;/\; (1 + \overset{*}{\delta}_{e/}) \tag{3}$$

$$[\,p\,] = \overset{*}{\delta}_{e/}\; \overset{*}{\rho}_1 /\, U^2_{1n/}\; (1 + \overset{*}{\delta}_{e/}) \tag{4}$$

$$[\,\rho\,] = \overset{*}{\delta}_{e/}\; \overset{*}{\rho}_1 / \tag{5}$$

where U^i are the components of velicity; $\overset{*}{\rho} = \rho + mn$, where m is the mass transfer and n is the number of dust particles in unit space ; $\delta^*_{e/}$ is the density shock strength ; X^i the components of the unit normal directed away from the surface and [ ] denotes the difference of value on the two sides of the shock surface of the quantity enclosed. Thus, if f be any flow parameter then

$$[\,f\,] = f_{2/} - f_{1/}$$

Tho internal energy per unit mass E is given by

$$E = (m_g \ C_g \ T + m_d \ C_d \ T)/(m_g + m_d) \tag{6}$$

$$E = \chi p / (\gamma - 1) \rho^* (1 + b\rho^*) \tag{7}$$

where

$$\left.\begin{aligned} p &= \rho^* (1 + b\rho^*) R T \\ R &= C_p - C_g \end{aligned}\right\} \tag{8}$$

and χ is a dimensionless quantity defined as

$$\chi \overset{def}{=} (m_g + m_d \ C_d / C_g)/(m_g + m_d) \tag{9}$$

and m_g , C_g (m_d, C_d) are the mass transfer and specific heat of gas (dust) particles.

RESULTS

Theorem—The equation of state can be written as

$$p \div \left\{\rho^{* \, (\lambda + \gamma - 1) / \lambda} (1 + b\rho^*) \right\} = k \exp.\left[\left\{(S - S_o) / \lambda \, C_g \right\} + (\gamma - 1) \ (\rho^* - \rho_o^*) \, b/\lambda \right] \tag{10}$$

where

$$k = p_o / \rho^{*(\lambda + \gamma - 1)/\lambda} (1 + b \ \rho_o^*)$$

Proof : The first law of thermodynamics states that

$$dQ = C_m \, dT + pd/\rho^*) \tag{11}$$

dQ is the amount of heat added, C_m is the specific heat of the medium (the mixture of gas and dust particles) defined as

$$\begin{aligned} C_m &\overset{def}{=} (\rho C_g + mnC_d)/\rho^* \\ &= \lambda \, C_g \end{aligned}$$

where

$$A = (\rho + mnC_d / C_g) / \rho^*$$

(2) gives

$$R \, dT = \left\{dp / \rho^* (1 + b\rho_1^*\right\} + pd/\rho^* (1 + b \ \rho^*) \tag{12}$$

Eliminate T between (11) and (12) and using

$$\mathbf{R} = C_g \ (\gamma - 1)$$

$$dS = dQ / T$$

where S is entropy, we get

$$dS / \lambda \, C_g = d \left\{ \log\left(p / \rho^{* \, (\lambda + \gamma - 1) / \lambda} (1 + b \ \rho^*)\right)\right\} - d\left\{(\gamma - 1) \, b \ \rho^* / \lambda \right\}$$

which on integration, yields (10)

Theorem—The velocity of sound a i given by

$$a^2 = \left\{ p / \lambda \rho^* (1 + b \rho^*) \right\} \left\{ (\gamma - 1) b^2 \rho^{*2} + 2 (\lambda + \gamma - 1) b \rho^* + (\lambda + \gamma - 1) \right\} \quad (13)$$

$$a^2 = \left\{ (\lambda + \gamma - 1) / \lambda \right\} \left\{ p (1 + 2 b \rho^*) / \rho^* (1 + b \rho^*) \right\} \quad (14)$$

Proof : Taking logarithm of both the sides of (1) and using $a^2 = \delta \rho^*$ we get (13). Neglecting small order term $(\gamma - 1) b^2 \rho^{*2}$, me get (14).

Theom — The density shock strength $\delta^*_{\rho/}$ is given by

$$A\, \delta^{*}_{\rho/} + B\, \delta^{*}_{\rho/} + C = 0 \quad (15)$$

where

$$A = b\rho^*_{1/} \left\{ \left\{ (2\chi / \gamma - 1) + 1 \right\} + \left\{ (\lambda + \gamma - 1) / \lambda \right\} M^2_{1n} / (1 + 2 b \rho^*_{1/}) \right\} \quad (16)$$

$$B = \left\{ \frac{2\chi}{\gamma - 1} + 1 \right\} 2 b\rho^*_{1/} + \{ (\lambda + \gamma - 1)/\lambda \} M^2_{1n/} (6 b^2 \rho^2_{1/} + 4 b\rho^*_{1/} + \tfrac{1}{2}) \quad (17)$$

$$C = \left(\frac{2\chi}{\gamma - 1} + 1 \right) b\rho^*_{1/} + \{(\lambda + \gamma - 1)/\lambda\} M^2_{1n/} (2 b \rho^*_{1/} + 1)^2 \quad (18)$$

$$M_{1n/} = U_{1n/} / a_{1/}$$

as defined by Truesdell[4].

Proof : The energy equation is given in the form2

$$\tfrac{1}{2} (m_d + m_g) (U^2_{1/} - U^2_{2/}) - m_g C_g [T] - m_d C_d [T] = [p/\rho^*] (m_g + m_d)$$

which, by virtue of (6) and (7), is reduced to

$$(U^2_{1/} - U^2_{1/}) / 2 = \left\{ \frac{\chi}{(\gamma - 1)} \right\} [p/\rho^* (1 + b\rho^*)] + p/\rho^* \quad (19)$$

Equation (3) yields

$$U^i_{2/} = U^i_{1/} - \left(\frac{\delta \rho_/}{1 + \delta^* \rho_/} \right) U_{1n/} x^i$$

or

$$U_{2/}{}^2 = U_{1/}{}^2 - \{ \delta^*_{\rho/} (2 + \delta^*_{\rho/}) / (1 + \delta^*_{\rho/})^2 \} U_1{}^2{}_{n/} \quad (20)$$

using (14) in the form

$$p = a^2 \rho^* (1 + b\rho^*) / \{(\lambda + \gamma - 1)/\lambda\} (1 + 2 b\rho/^*)$$

and (2) in (19) and cancelling $\delta^*_{\rho/} / (1 + \delta^*_{\rho/})^2$ from both the sides, me get (15).

Theorem—The jump in enthalphy h is given by

$$[h] = \delta^* \rho_/ (1 + \delta^* \rho_/ /2) / (1 + \delta^* \rho_/)^2 \} U_1{}^2{}_{n/} \quad (21)$$

Proof : The enthalpy h is defined as

$$h = E + p/\rho^*$$

using (7) and (19) in the above equation, we get (21) *REMARKS* : It is striking to note that the jump in enthalpy as obtained in terms of the shock strength, is in the same form as was obtained by Srivastava and Mishra[2] using $p = \rho^* R T$. However the difference lies in the fact that $\delta^*_{\rho/}$ using here has got different value than the $\delta^*_{\rho/}$ used by them.

Theorem—The velocity of sound a is given by

$$a^2 = \left\{ p / \lambda \rho^* (1 + b \rho^*) \right\} \left\{ (\gamma - 1) b^2 \rho^{*2} + 2 (\lambda + \gamma - 1) b \rho^* + (\lambda + \gamma - 1) \right\} \quad (13)$$

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Proof : The energy equation is given in the form[2]

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which, by virtue of (6) and (7), is reduced to

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Theorem—The jump in entropy S is given by

$$[S] = \lambda \; C_g \; \log \left\{ \frac{+ \dfrac{\delta^*_{\rho/}}{1+\delta\rho_/} \cdot \dfrac{\rho^*{}_{1/} \; U^2{}_{n/}}{p_1}}{1 + \delta\rho_/ \; \dfrac{\lambda+\gamma-1}{\lambda} \left(1 + / \dfrac{b\rho_{1/}}{b p_{1/}} \; \delta\rho_/ \right)} \right\} -(r-1) \; Cg \; b\rho_{1/} \; \delta_{e/} \quad (22)$$

Proof : Writing (10) in the form

$$([S] / \lambda \, C_g) + (\gamma - 1) \quad [\quad = \left[\log \left\{ p_{,}' \rho^{* \, (\lambda + \gamma - 1)/\lambda} \, (1 + b \, \rho^*) \right\} \right]$$

and using (14) and (15), we get (22)

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