FREE CONVECTION PLOW OF OLDROYD LIQUID PAST A HOT VERTICAL POROUS PLATE

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The problem of flow of an elaatico-viscous liquid past a hot vertical porous flat plate has been considered by taking into account the effect of free convection when a body force 'g' acts in a direction opposite to that in which flow takes place. Numerical calculations have been made to study the effect of free convection and elasticity of the fluid on the fluid velocity. The fluid velocity and the skin-friction both increase with the Grashof number. The skin-friction is not affected by the elastic elements. The fluid velocity increases with the relaxation time but decreases with the increase in retardation time.

In recent gears, the study of free or natural convection phenomenon has gained considerable importance in view of its application in the field of aeronautics, atomic power,. chemical engineering and electronics. Although a number of studies on natural convection flow and heat transfer of Newtonian fluids have been reported in literature. Little work seems. to have been done in case the fluid is non-Newtonian. The aim of the present investigation is to study the problem of flow of Oldroyd elastico-viscous liquid past a hot vertical porous flat plate by taking into account tbe effect of free convection when a body force 'g' per unit mass is acting in the negative X-direction parallel to the plate. Recently, the author[1] has studied a similar problem with Rivlin-Ericksen[2] elastico-viscous liquid.

RHEOLOGICAL EQUATION OF STATE

The constitutive equations for the Oldroyd[3] B liquid is given by

$$(1 + \lambda_1 D/D_t)\, p'^{ik} = 2\eta_0 (1 + \lambda_1 D/Dt)\, e^{ik}, \tag{1}$$

$$p_{ik} = - p g_{ik} + p'_{ik} \tag{2}$$

where p_{ik} is the stress tensor, λ_1, λ_2 are the times of relaxation and retardation respectively, e^{ik} is the rate of strain tensor, p is an isotropic tensor, η_0 is the limiting viscosity at small rates of shear and g_{ik} is the metric tensor of a fixed coordinate system x^i. D/Dt denotes convected differentiation of a tensor quantity in relation to the material in motion and for a contravariant tensor b^{ik} is

$$(D/Dt) b^{ik} \equiv \partial b^{ik}/\partial t + v^m \partial b^{ik}/\partial x^m - b^{im} \partial v^k/\partial x^m - b^{mk} \partial v^i/\partial x^m \tag{3}$$

where v^m is the velocity vector. The material defined by equation (1) is essentially a liquid, and the physical model is such at $A > A$. As λ_1 and λ_2 tend to equality the material tends to become exactlv a Newtonian liquid of constant viscosity η_0. In mobile liquids λ_1 and λ_2 are small fractions of a second.

FORMULATION AND INTEGRATION OF EQUATIONS

Taking the X-axis along the plate and Y-axis normal to it, the equation of continuitv is

$$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0 \tag{4}$$

The momentum equations are

$$\rho\{u\partial u/\partial x+v\,\partial u/\partial y\}=-\partial p/\partial x-\rho g+\partial p'^{xx}/\partial x+\partial p'^{xy}/\partial y, \quad (5)$$

$$\rho\{u\partial v/\partial x+v\,\partial v/\partial y\}=-\partial p/\partial y+\partial p'^{xy}/\partial x+\partial p'^{yy}/\partial y+\partial(\mu_\circ I)/\partial y, \quad (6)$$

where $\mu_\circ$ is the coeffisient of bulk viscosity and

$$I=v_{i,i}$$

Since the plate is infinitely long all physical quantities, except pressure p, depend on y only. Following a previous work by the author[4], the equations of motion and continuity reduce to

$$\rho v du/dy=-\partial p/\partial x-\rho g+dp'^{xy}/dy \quad (7)$$

$$\rho v dv/dy=-\partial p/\partial y+d(\mu_\circ I)/dy+dp'^{yy}/dy \quad (8)$$

and

$$d(\rho v)/dy=0 \quad (9)$$

In accordance with the usual practice in free-convection, the density is treated to be a variable only in forming the buoyant force $\rho\beta g\theta$, otherwise it is a constant. β is the co fficient of thermal expansion and $\theta=T-T_\infty$, T and T_∞ are temperatures at any point and at far away from the plate respectively. Considering this fact, equation (9) gives

$$v=\text{constant}=v_\circ \quad (10)$$

and equations (7) and (8) reduce to

$$\rho v_\circ du/dy=\rho\beta g\theta+dp'^{xy}/dy \quad (11)$$

$$0=-\partial p/\partial y+dp'^{yy}/dy \quad (12)$$

The constitutive equation (I) now gives

$$p'xx+\lambda_1\left[v_\circ\ dp'^{xx}/dy-2\ du/dy\ p'^{xy}\right]=-2\eta_\circ\lambda_2(du/dy)^2, \quad (13)$$

$$p'xy+\lambda_1\left[v_\circ\,dp'^{xy}/dy-du/dy\ .\ p'^{yy}\right]=\eta_\circ\left[du/dy+\lambda_2 v_\circ\,du^2/dy^2\right] \quad (14)$$

$$p'^{yy}+\lambda_1 v_\circ\,dp'^{yy}/dy=0 \quad (15)$$

Equation (15) shows that $p'_{yy}=0$, is a particular solution of this equation. Hence putting $p'^{yy}=0$ in (14) we get

$$p'^{xy}+\lambda_1 v_\circ\ dp'^{xy}/dy=\eta_\circ\left[du/dy+\lambda_2 v_\circ\ d^2u/dy^2\right] \quad (16)$$

and

$$\partial p/\partial y=0 \quad (17)$$

Elimination of p'^{xy} between (11) and (16) gives

$$\eta_\circ\lambda_2 v_\circ\,d^3u/dy^3+(\eta_\circ-\lambda_1\rho v_\circ^2)\,d^2u/dy^2-\rho v_\circ\,du/dy+\lambda_1 v_\circ\,\rho g\beta d\theta/dy+\rho\beta g\theta=0 \quad (18)$$

The equation of energy is

$$\rho c v_\circ\ d\theta/dy=Kd^2\theta/dy^2+p'^{xy}du/dy, \quad (19)$$

where c and K denote the specific beat and thermal conductivity of the fluid respectively. Neglecting visco-elastic energy dissipation term in (19), which is justified for slow motion as in the case of free convection flows, we get

$$\rho c v_o d\theta/dy = K d^2\theta/dy^2 \tag{20}$$

Equations (18) and (20) are to be solved under the boundary conditions.

$$\left.\begin{array}{lll} u=0, & \theta=\theta_o & \text{at } y=0 \\ u\to U, & \theta\to 0 & \text{as } y\to\infty \end{array}\right\} \tag{21}$$

Solution of equation (20) is physically possible only if v_o is negative *i.e.* there is Auid suction at the plate. Taking

$$v_o = -a, \ (a>0) \tag{22}$$

where 'a,' is a constant of the dimensions cf velocity, from **(20)** we get

$$\theta = \theta_o \exp. [-\rho a c\, y/K] \tag{23}$$

Elimination of θ between (18) and **(23)** gives

$$K_2\, d^3u/d\eta^3 - \left(1-k_1\right)d^2u/d\eta^2 - du/d\eta = UG\left(1+PK_1\right)\exp.\left(-P\eta\right), \tag{24}$$

where

$K_1 = \lambda_1\, \rho a^2/\eta_o$, Relaxation number
$K_2 = \lambda_2\, \rho a^2/\eta_o$, Retardation number
$\eta = \rho\, a\, y/\eta_o$, non-dimensional distance
$G = \beta g \eta_o\, \theta_o/\rho U a^2$, Grashof number
and $P = c\eta_o/k$, Prandtl number

The solution of (24) is

$$u = A + Be^{m_1\eta} + ce^{m_2\eta} - UG(1+PK_1)\, e^{-P\eta} \Big/ \left\{K_2 P^3 + (1+K_1)P^2 - P\right\} \tag{25}$$

where A, B, C are the constants of integration and

$$\left.\begin{array}{l} m_1 \\ m_2 \end{array}\right\} = \left[(1-K_1) \pm \left\{(1-K_1)^2 + 4K_2\right\}^{1/2}\right] \Big/ 2K_2, \tag{26}$$

which shows that for all values of K_1 and K_2 (both of them being small but positive)

$$m_1 > 0 \ ; \ m_2 < 0 \tag{27}$$

From the condition u $\to U$ as $\eta\to\infty$, we get $B=0$, otherwise 'u' will diverge to infinity and $A = U$.

Since u $=0$ at $\eta = 0$, we have from (25)

$$u/U = 1 - e^{m_2\eta} + G\left(1+PK_1\right)\left[e^{m_2\eta} - e^{-P\eta}\right] \Big/ \left\{K_2P^3 + (1-K_1)P^2 - P\right\} \tag{28}$$

If $G=0$ and $K_1 = K_2$, the solution (28) reduces to the familiar solution of Meredith and Griffith[5].

The skin-friction τ_o at the plate is

$$\tau_o = \rho a U(1 + G/P) \tag{29}$$

DISCUSSION

In Table 1, the calculated values of u/U are given for fixed values of K_1 and K_2 and various values for G and P. The fluid velocity increases with the Grashof number. This seems to be physically plausible for an increase in Grashof number implies an increase in buoyancy force leading to an increase in fluid velocity. From Table 1 we can also conclude that the fluid velocity decreases with the increase in Prandtl number.

Table 2 gives the calculated values of u/U sbowing the effects of elastic numbers on the fluid velocity. It can be seen that the fluid velocity increases with the relaxation time but it decreases with the increase in retardation time.

The expression (29) for skin-friction shows that it is independent of the elasticity of the liquid. It increases with the Grashof number but decreases with the increase in Prandtl number.

TABLE 1

EFFECT OF G AND P ON FLUID VELOCITY

$K_1=0\cdot8$ $K_2=0\cdot5$

η_o \ G	100	500	1000	P
0·4	73·68 33·14	366·88 164·18	733·38 327·98	0·5 1·0
0·8	101·42 43·70	505·02 216·02	1009·42 431·42	0·5 1·0
1·0	111·00 41·70	552·20 200·70	1103·70 400·70	0·5 1·0
2·0	101·08 27·73	501·76 135·01	1002·60 269·11	0·5 1·0

TABLE 2

EFFECT OF ELASTICITY ON THE FLUID VELOCITY

$P=1\cdot0$ $G=100$ $\eta=1\cdot0$

K_1 \ K_2	0·2	0·4	0·6	0·8
0·1	42·67	57·31	62·91	69·66
0·3		49·53	51·61	52·98
0·5			40·40	41·70
0·7				40·07

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REFERENCES

1. MISHRA, S.P., *Indian J. Pure & Appl. Phys.*, 3 (1965), 111.
2. RIVLIN R. S. & ERICKSEN, J.L., *J. Rat. Mech. Anal.*, 5 (1956), 179.
3. OLDROYD, J.G., *Proc. Royal Soc. London*, 200 A (1950), 523.
4. MISHRA, S.P., *Def. Sci. J.*, 15 (1965), 61.
5. MEREDITH, F. W. & GRIFFITH, A.A., "Modern Developments in Fluid Dynamics—Vol. 2" (Clarendon Press, Oxford.) 1938, p. 541.