PHYSICAL PROPERTIES OF TEXTURES WITH LIMITING POINT GROUP SYMMETRIES

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The effect of symmetry on the physical properties of textures which possess conventional and magnetic limiting point group symmetries is discussed. Formulae to obtain the number of independent components of different magnetic property tensors in such materials and the results obtained in respect of pyromagnetism, magneto-electric polarizability and piezomagnetism are given.

TEXTURES

An isotropic body can be built from minute particles of a crystal provided care is taken to see that they are densely packed, homogenously distributed and randomly oriented. This, however, is not possible in some exceptional cases such as where the crystal chosen belongs to an enantiomorphous class. To say that the body is isotropic is equivalent to saying that the symmetry of the macroscopic texture is that of the full orthogonal group $R_{\infty i}$. Polycrystalline metals serve as examples of such isotropic bodies. When we assert that a polycrystalline body is isotropic, we have to adopt an appropriate interpretation of a symmetry operation as applicable to such bodies. We cannot demand, as we do in the case of single crystals, that a symmetry transformation should carry every atom to its equivalent position. It is sufficient if the density of packing, homogeneity and the type of randomness in respect of orientation are identical at all the equivalent positions-arising from a transformation. The transformation may then be taken as a symmetry transformation of the polycrystalline body. Such an interpretation of a symmetry operation leads us to recognize distinct classes of symmetry possible for such bodies. For example, if the crystal particles belong to a class which can exist in two enantiomorphous forms and if the particles are all of one such form, the polycrystalline body cannot possess the symmetry operations of reflection and inversion. It possesses the symmetry of the full rotation group R_{∞}.

For deriving other distinct classes of such macroscopic symmetry, let us start from particles of a crystal that belong to one of the C_n (n = 1, 2, 3, 4, 6) classes. If the unique axes of all the particles are aligned in the same direction and if there is complete randomness of orientation in the plane perpendicular to the unique axis, the body possesses the symmetry C_{∞}. This group is the limiting group of all C_n groups. In a similar way, it is possible to conceive of polycrystalline bodies which belong to the classes $C_{\infty v}$, $C_{\infty h}$, D_{∞} and $D_{\infty h}$ which are respectively the limits of C_{nv}, C_{nh} & S_{2n}, D_n and D_{nh} & D_{nd} groups. These five limiting groups together with R_{∞} and $R_{\infty i}$, which may be considered as the limiting groups of the cubic classes, are known as the 7 Curie groups.

The polycrystalline bodies described earlier may be regarded as particular examples of substances known as textures. A texture is, in general, defined as a composite body consisting of minute particles which are oriented in space in a definite way. Besides the polycrystalline bodies fibrous materials such as wood, layered media, liquid crystals consisting of molecules oriented in the same direction, electrets consisting of dipoles oriented

TABLE I

CURIE GROUPS AND THEIR MAGNETIC VARIANTS

No.	Schoenflies	Shubnikov	International*	Elements
1	C_∞	∞	∞	All rotations Cφ about a single axis.
2		$\underline{\infty}$	$\underline{\infty}$	All Cφ and complementary rotations $\underline{C\varphi}$ about a single axis.
3	$C_{\infty v}$	∞·m	∞m	All Cφ about a single axis and all mirrors m passing through that axis.
4		∞·$\underline{m}$	∞$\underline{m}$	All Cφ and all $\underline{m}$.
5		$\underline{\infty}$·m $\underline{\infty}$·$\underline{m}$	$\underline{\infty}$m $\underline{\infty}\underline{m}$	All Cφ, $\underline{C\varphi}$ and all m, $\underline{m}$
6	$C_{\infty h}$	∞: m	∞/m	All Cφ about a single axis and all rotation reflections $\tilde{C}\varphi$ about that axis.
7		∞: $\underline{m}$	∞/$\underline{m}$	All Cφ and all $\underline{\tilde{C}\varphi}$
8		$\underline{\infty}$: m $\underline{\infty}$: $\underline{m}$	$\underline{\infty}$/m $\underline{\infty}$/$\underline{m}$	All Cφ, $\underline{C\varphi}$ and all $\tilde{C}\varphi$, $\underline{\tilde{C}\varphi}$
9	D_∞	∞: 2	∞2	All Cφ about a single axis with C_2 axes in a plane perpendicular to it in all possible directions.
10		∞: $\underline{2}$	∞$\underline{2}$	All Cφ and $\underline{C_2}$
11		$\underline{\infty}$: 2 $\underline{\infty}$: $\underline{2}$	$\underline{\infty}$2 $\underline{\infty}\underline{2}$	All Cφ, $\underline{C\varphi}$ and C_2, $\underline{C_2}$
12	$D_{\infty h}$	m·∞:m	m∞/m	All the elements of the groups $C_{\infty v}$, $C_{\infty h}$ and D_∞.
13		m·∞:$\underline{m}$	m∞/$\underline{m}$	All Cφ, m, $\underline{\tilde{C}\varphi}$, $\underline{C_2}$.
14		$\underline{m}$·∞:$\underline{m}$	$\underline{m}$∞/m	All Cφ, $\underline{m}$, $\tilde{C}\varphi$, $\underline{C_2}$.
15		$\underline{m}$·∞:$\underline{m}$	$\underline{m}$∞/$\underline{m}$	All Cφ, $\underline{m}$, $\tilde{C}\varphi$, $\underline{C_2}$.
16		m·∞:m $\underline{m}$·∞:m m·∞:$\underline{m}$ $\underline{m}$·∞:$\underline{m}$	m∞/m $\underline{m}$∞/m m∞/$\underline{m}$ $\underline{m}$∞/$\underline{m}$	All the elements of group 12 and all their complements.
17	R_∞	∞/∞	∞∞	The full rotation group *i.e.* all possible rotations about all possible axes passing through a point.

*Symbols under this column have been obtained by suitably adapting the notation already in use for finite groups.

TABLE 1—*contd.*

No.	Schoenflies	Shubnikov	International	Elements
18		$\infty / \underline{\infty}$	$\infty\ \underline{\infty}$	All the elements of the group 17 and their complements.
19	$R_{\infty i}$	$\infty / \infty \cdot m$	$\infty\ \infty m$	The full orthogonal group *i.e.* all rotations about all possible axes passing through a point and their corresponding rotation-reflections.
20		$\infty / \infty \cdot \underline{m}$	$\infty\ \infty \underline{m}$	All the elements of group 19 except that the rotation-reflections are taken as complementary operations.
21		$\infty / \underline{\infty} \cdot m$ $\infty / \underline{\infty} \cdot \underline{m}$	$\infty\ \underline{\infty} m$ $\infty\ \underline{\infty}\,\underline{m}$	All the elements of group 19 and all their complements.

in the same direction and magnets with like orientation of electron spins have been cited by Shubnikev as examples of textures. Prom amongst the textures which possess a great variety of symmetry, those possessing the symmetry of the limiting groups are of particular interest. Physical properties of such textures are of some importance and particular mention may be made of the piezoelectric textures studied by Shubnikov *et al*[1]. in this context.

MAGNETIC TEXTURES

In the polycrystalline bodies, if a distinctive feature like the magnetic moment is associated with the crystal particles, it is possible that a spatial symmetry operation will bring the body into self-coincidence in all respects but may result in a reversal of the magnetic moments. Thus, as in the case of crystallographic point groups, we are led to define the reversal of magnetic moment and the combined operations of ordinary spatial transformations followed by reversal of magnetic moments as possible symmetry operations of such bodies. Such combined operations are called the complements of the corresponding conventional operations and they are designated by underscoring the symbol for the conventional operation. The reversal of magnetic moments by itself is denoted by $\mathscr{R}$ and it is the complement of the identity operation? The investigation of obtaining the possible point groups of such bodies, in the generalized sense, leads us to the derivation of the possible magnetic variants of the 7 Curie groups[3]. They are listed in Table 1.

The groups 2, 5, 8, 11, 16, 18 and 21 arc called grey groups; 1, 3, 6, 9, 12, 17 and 19 are called single coloured or colourless groups and the remaining seven, namely 4, 7, 10, 13, 14, 15 and 20 are called mixed or double coloured groups. The single coloured groups have already been shown to he the limiting groups of appropriate finite point groups. Each of the grey groups and the double coloured groups can also be shown to be the limiting groups of a series of finite magnetic point groups. The 21 groups may, therefore, be referred to as the limiting groups.

PHYSICAL PROPERTIES OF TEXTURES

We shall assume that any physical property of a texture possesses at least the symmetry of the texture. In other words, we assume thnt Neumann's principle is applicable to textures. The schemes of components of physical properties of textures represented by tensors of various ranks can be readily derived. We shall, however, mainly derive the numbers of independent components that physical properties can possess in textures belonging to these limiting groups. The derivation of the schemes of non-vanishing coefficients in these limiting groups is greatly facilitated by a knowledge of the number of independent coefficients. The numbers of independent components that tensors of various ranks can possess when subject to the symmetries of the 7 Curie groups, have been derived by Rahman[4] by extending a formula given earlier by Bhagavantam[5]. Bhagavantam and Pantulu[2] have extended these investigations to magnetic properties in respect of the 90 magnetic point groups. The method and formulae for obtaining the numbers of independent components of magnetic properties in textures possessing the 21 limiting point groups are given below.

For the 7 single coloured groups, the formulae already given by Rahman [4] have to be used. Magnetic properties are forbidden in bodies possessing the symmetry of grey groups and the number of coefficients is, therefore, zero in each one of these cases. Table 2 gives tho formulae for obtaining the numbers of independent components of magnetic property tensors in respect of the 7 double coloured or mixed limiting groups.

TABLE 2

FORMULAE FOX NUMBER OF INDEPENDENT COMPONENTS

Group	Number of Components
$\infty . \underline{m}$	$\int_0^{2\pi} [\chi(\phi) - \psi(o)]\, d\phi \Big/ 2\int_0^{2\pi} d\phi$
$\infty : \underline{m}$	$\int_0^{2\pi} [\chi(\phi) - \psi(\phi)]\, d\phi \Big/ 2\int_0^{2\pi} d\phi$
$\infty : \underline{2}$	$\int_0^{2\pi} [\chi(\phi) - \chi(\pi)]\, d\phi \Big/ 2\int_0^{2\pi} d\phi$
$m.\infty : \underline{m}$	$\int_0^{2\pi} [\chi(\phi) - \psi(\phi) - \chi(\pi) + \psi(o)]\, d\phi \Big/ 4\int_0^{2\pi} d\phi$
$\underline{m}.\infty : m$	$\int_0^{2\pi} [\chi(\phi) + \psi(\phi) - \chi(\pi) - \psi(o)]\, d\phi \Big/ 4\int_0^{2\pi} d\phi$
$\underline{m}.\infty : \underline{m}$	$\int_0^{2\pi} [\chi(\phi) - \psi(\varphi) + \chi(\pi) - \psi(o)]\, d\phi \Big/ 4\int_0^{2\pi} d\varphi$
$\infty/\infty : \underline{m}$	$\int^{2\pi} [\chi(\varphi) - \psi(\pi-\phi)] \sin^2 \frac{\phi}{2}\, d\varphi \Big/ 2\int^{2\pi} \sin^2 \frac{\phi}{2}\, d\varphi$

$\chi(\phi)$ is the character of a pure rotation through ϕ and $\psi(\phi)$ is the character of a rotation-reflection through ϕ in the tensor representation. The special transformation properties of magnetic tensors under complementary operations have been taken account of in the formulae given in Table 2. For details, reference may be made to Bhagavantam and Pantulu[2].

The characters χ and ψ in the representations formed by tensors appropriate to (1) pyromagnetism, (2) magnetoelectric polarizability and (3) piezomagnetism are given below:

Property No.	$\chi\ (\varphi)$	$\psi\ (\varphi)$
(1)	$(1+2\cos\varphi)$	$(1-2\cos\varphi)$
(2)	$(1+2\cos\varphi)\times(1+2\cos\varphi)$	$(-1+2\cos\varphi)\times(1-2\cos\varphi)$
(3)	$(4\cos^2\varphi+2\cos\varphi)\times(1+2\cos\varphi)$	$(4\cos^2\varphi-2\cos\varphi)\times(1-2\cos\varphi)$

By substituting these expressions for χ and ψ in the formulae given in reference 4 and the formulae given in Table 2, we obtain the number of independent constants or these properties in respect of the single colour groups and mixed groups respectively. The results for all the groups are given in Table 3.

TABLE 3

NUMBER OF INDEPENDENT TENSOR COMPONENTS

Group No.	Property No. (1)	(2)	(3)
1	1	3	4
2	0	0	0
3	0	1	1
4	1	2	3
5	0	0	0
6	1	0	4
7	0	3	0
8	0	0	0
9	0	2	1
10	1	1	3
11	0	0	0
12	0	0	1
13	0	1	0
14	1	0	3
15	0	2	0
16	0	0	0
17	0	1	0
18	0	0	0
19	0	0	0
20	0	1	0
21	0	0	0

TABLE 4

CORRESPONDENCE BETWEEN THE LIMITING AND FINITE GROUES

Group No.	Property No. (2)	Property No. (3)	Group No.	Property No. (2)	Property No. (3)
∞	6	6	$m \cdot \infty : m$	..	6/mmm
$\underline{\infty} \cdot m$	6mm	6mm	$m \cdot \infty : \underline{m}$	$6/\underline{m}mm$	..
$\infty \cdot \underline{m}$	$6\underline{mm}$	$6\underline{mm}$	$\underline{m} \cdot \infty : m$	..	$6/m\underline{mm}$
$\infty : m$	..	6/m	$\underline{m} \cdot \infty : \underline{m}$	$6/\underline{mmm}$	..
$\infty : \underline{m}$	$6/\underline{m}$	..	$\infty/\underline{\infty}$	432	..
$\infty : 2$	622	622	$\infty/\infty \cdot m$	..	..
$\infty : \underline{2}$	$62\underline{2}$	$62\underline{2}$	$\infty/\infty \cdot \underline{m}$	$\underline{m}3\underline{m}$	..

By comparing the Table giving the number of independent coefficients in the 90 magnetic crystallographic groups[2] with Table 3, we find that the number of independent constants in a limiting group is the same as that in the crystallographic group of highest possible symmetry in the series of groups that tend to this limiting group. For properties (2) and (3), this turns out to be the hexagonal group. It follows that in the limiting group as well, the same scheme as in the corresponding hexagonal group will be obtained. Table 4 brings out this correspondence and enables the scheme for any limiting group being readily written out as it has already been given for the crystallographic groups[6, 7].

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