PROPAGATION OF SOUND IN SEDIMENTS

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The theories of sound propagation in sediments are more or less extensions of those applicable to atmospheric acoustics and to emulsions and suspensions. The theoretical and experimental work of various investigators on the velocity and absorption of sound in sediments has been reviewed. The sound velocity in many high porous fine grained sediments has been found to be less than that in water. As the mineral concentration increases beyond 23 per cent (77 per cent porosity) the system is no longer a suspension and assumes a measureable rigidity and, therefore, sound velocity increases with further increase in concentration of particles. The absorption in sediments is mainly due to viscous losses when the size of the mineral particles smaller than the wave length of sound and scattering losses also occur when the size is comparable to wave length. Some important unsolved problems have also been outlined.

The study of propagation of sound in aqueous suspensions has aroused wide interest because of its applications in oceanography and navigation for determining the thickness of soft mud-layer in the sea-bottom. The waters of open oceans can be treated as dilute suspensions of diatoms and other small organisms. The natural sediment is a mixture of water, mineral grains, colloidal particles and ions. Mineral grain composition is unique in each sediment sample. The particles are of different sizes and shapes; many are in contact with each other while many more are in suspension. Since the sediments are non-isotropic materials whose physical properties vary in depth and lateral dimension, the laws of elasticity cannot be applied to sediments. On account of this reason the elastic constants of sediments have not been determined statically and valid inter-relationship between them have not been established. The aim of this paper is to review the field of work and outline some of the major problems still unsolved.

Velocity in sediments

Wood[1] noted that in a dispersion of one or more fluids in one another, the bulk adiabatic compressibility equals the sum of the compressibilities of individual components multiplied by their proportional volumes in the total volume. A corresponding relation holds for bulk density. A simple expression for the velocity of sound C_{ws} can be written as follows :

$$C_{ws}^2 = \frac{1}{[\Sigma f_{si} \ \beta_{si}] \ [\Sigma f_{si} \ \rho_{si}]} \qquad (1)$$

where f_{si} are the volume fractions, β_{si} the compressibilities, and ρ_{si} the densities of i components. Herzfeld[2] derived an expression for the adiabatic compressibility of solids in suspension in terms of the velocity of sound, density of solid and liquid, and the compressibility of the liquid. This was based on extensive theoretical considerations of the effect of scattering by numerous small rigid spherical particles in the sound field. It was suggested that the compressibility of solid substances could be determined by suspending the solid in the form of particles in a liquid and measuring the velocity of sound in suspension. Randall[3] applied the results for this purpose but extremely small or even negative compressibilities were found.

Urick[4] applied equation (1) to the case of suspensions or emulsions under the assumption that suspended particles are infinitesimally small compared with the wave length of sound, hence scattering effects could be neglected. If ρ and K represent the density and compressibility, β the volume percentage of particles, V the velocity and subscripts 0, 1 and 2 refer to the suspension, suspending liquid and suspended particles respectively then

$$V_1^2 = \frac{1}{\rho_1 K_1}$$

and

$$V_\circ = \frac{1}{(\rho_\circ K_\circ)^{\frac{1}{2}}} = \left[\frac{1}{\{\rho_2 \beta + \rho_1 (1-\beta) \quad \{K_2 \beta + K_1 (1-\beta)\}}\right]$$

$${}^2 = \left(\frac{V_1}{V_\circ}\right)^2 = (1 + a\beta)(1 + b\beta)$$

where

$$a = \left(\frac{\rho_2 - \rho_1}{\rho_1}\right) \text{ and } b = \left(\frac{K_2 - K_1}{K_1}\right)$$

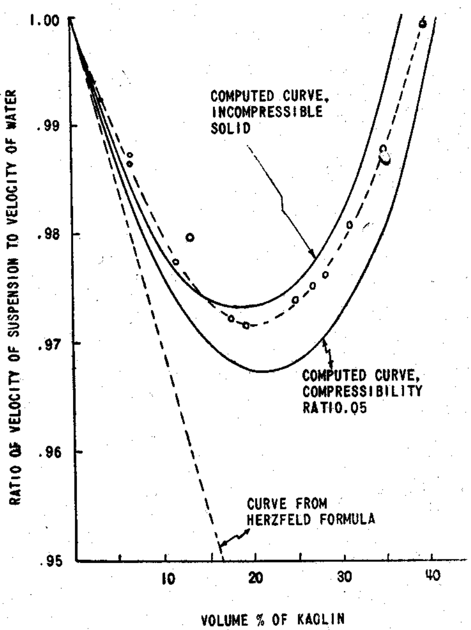

FIG. 1—Sound velocity ratio *vs* concentration, kaolin suspensions.

Thus the velocity ratio γ is a parabolic function of the concentration β and will, therefore, have a maximum or a minimum at some particular concentration β_m.

The results were verified by measuring the velocity with the ultrasonic interferometer at 1 Mc/s. The velocity is plotted as the ratio of velocity in suspension to that of the liquid in which kaolin particles are suspended (Fig. 1). The velocity at first falls as kaolin is added to the liquid, reaches a minimum at 20 per cent kaolin by volume and then rises. Also plotted in Fig. 1 is a curve computed from Herzfeld's formula. The applicability of the additive formula was further illustrated by some emulsions of xylene-in-water and water-in-xylene. This was exemplified by data on fresh horse blood which is a suspension of blood corpuscles in plasma.

Wood[1] and Urick[4] have assumed that both density and compressibility of the mixture are the additive of corresponding quantities for the two materials but no justification has been given for this. Also the derivation requires in addition to a density mixture law, an assumption concerning the compressibility of the mixture in terms of the constituent compressibilities. Chambre[5] derived expressions for the velocity of a plane compressional wave in a gross mixture (porous or suspension medium) which obeys a simple density mixture law and found that his results agreed with the results of Wood and others [1,4]. He showed further that the assumption concerning the compressibility of the mixture is a direct consequence of the density mixture law and hence is not required.

Urick and Ament[6] suggested an improvement to the homogeneous assumption theory (1) by developing a first order theory which, in allowing for first order scattering effects, gives a complex propagation constant whose real part yields the velocity and the imaginary part gives the absorption co-efficient. Velocity measurements at megacycles on a series of bromoform-in-water and mercury-in-water emulsions are in agreement with the theory at low and moderate concentrations upto 25—30%. Ament[7] carried the work further by determining effective dynamic densities for gross mixtures in the form of either a porous solid or a suspension.

Iida[8,9] and Takahashi & Sato[10] showed that compressional velocity should increase in proportion to the sixth root of the depth of burial of a sediment. The same result was obtained by Gassmann[11] who dealt with spheres in hexagonal close packing. Sato[12] gave expressions for the compressional wave propagation in media with small holes and in media containing small obstacles as an extension of the theory of Mackenzie[13] for a solid containing spherical holes.

A simple theory with macroscopic approach has been outlined by Morse[14]. The theory was first suggested by Rayleigh[15] and is more or less an extension of that outlined by Kosten and Zwikker[16]. It is similar to the porous medium theory developed by Morse and Bolt[17], Scott[18] and Beranek[19]. Ferrero and Sacerdote[20] found experimentally that the sound velocity remained constant in the frequency range 0·5 to 1·5 Kc/s for three lead shot samples (particle radii of 0·185, 0·128 and 0·098 cm) air being the fluid in each case. No attempt was made by Ferrero to compare his results with theory.

The theory was extended by White and Sengbush[21]. Using as a model an aggregate of randomly stacked spherical particles of four different sizes, Brandt[22] developed theoretical expressions to explain the influence of pressure, porosity and liquid saturation on sound speed in a porous granular medium. Kerner[23] determined theoretically the gross elastic moduli of a composite medium in terms of the properties of its components. Gutenberg *et. al.*[24], summarized the values for the velocity of longitudinal elastic waves from seismic data. Biot gave a general treatment of elastic wave propagation in porous solids at low frequencies (where Poiseuille flow holds)[25], and also at high frequencies (where Poiseuille flow does not hold)[26].

A sediment is a somewhat rigid structure of solid grains with fluid filled channels and therefore, compressional waves are transmitted by the fluid and shear and compressional waves are also transmitted by the frame. The compressional velocity $C_{\omega s}$ may be related to the bulk properties of the sediment by the well known equation,

$$\overset{2}{C_{\omega s}} = \frac{K_{\omega s} + \frac{4}{3}\mu_{\omega s}}{\rho_{\omega s}} \tag{2}$$

where $\mu_{\omega s}$ is the rigidity modulus of the aggregate, $\rho_{\omega s}$ the bulk density and

$$\rho_{\omega s} = f_\omega \ \rho_\omega + \Sigma f_{si} \ \rho_{si} \tag{3}$$

and $K_{\omega s}$ is the bulk modulus of the aggregrate.

Laughton [27, 28] investigated the justifiction of using the relation

$$\frac{1}{K_{\omega s}} = f_\omega \ \beta_\omega + \Sigma \left(\frac{f_{si}}{K_{si}}\right) \tag{4}$$

Further a structure bulk modulus Kc was introduced by him so that for a two component system the aggregate modulus is

$$K_{\omega s} = \frac{K_s \ K_\omega \left(1 + \frac{f_\omega}{f_s}\right)}{K_\omega + \left(\frac{f_\omega}{f_s}\right) K_s} + K_c \tag{5}$$

He studied the effects of hydrostatic pressure and compacting pressure on compressional velocity in natural and artificial sediments.

The most useful equation for predicting sound speed in sediment is

$$\overset{2}{C_{\omega s}} = \frac{K_{\omega s} + \frac{4}{3}\mu_{\omega s}}{\Sigma f_{si} \ \rho_{si} + f_\omega \rho_\omega} \tag{6}$$

where $K_{\omega s}$ is determined as in eq. (4).

Assuming that the sediment volume fraction of sand grains is proportional to the rigidity modulus, eq. (6) can be written in the form

$$\overset{2}{C_{\omega s}} = \frac{K_{\omega s} + b\,(1-\eta)\,\psi}{\Sigma f_{si} \ \rho_{si} + f_\omega \rho_\omega} \tag{7}$$

where b is a constant, $(1-\eta)$ the particle concentration, ψ is the diameter of grains coarser than 0·0625 mm and η the porosity.

Eq. (7) agrees with experimental results better than eq. (1), when a value of 0·8 is used for b, appropriate mineral densities and compressibilities are used and ψ is given a value of 1·0 at η = 0·4, 0·3 and 0·525 and zero at η = 0·8.

Sutton *et. al.* [29] studied 37 samples and their analysis of data shows that the best equation relating sound speed to the more important independent variables is eq. (7). Nafe and Drake [30] studied the variation of porosity, density, and velocity of compressional and shear waves with depth in shallow and deep water marine sediments. Shumway [31] reported the variation of sound velocity with temperature in water saturated sediments.

Emery *et. al.*[32] studied the sediments of sea water in the vicinity of San Diego. Hamilton, George Shumway, Menard and Shipek[33] studied 35 stations in the same area at water depths to 90 ft. Sound velocity measurements were made *in situ* at 100 Kc/s by pulsing between two barium-titanate transducers inserted into the sediment by a diver and connected by water tight cables with driving and receiving units located above water. Using diver taken samples the velocity and attenuation were measured in the laboratory between 23 and 41 Kc/s by the resonance method whose simple theory was given by Toulis [34]; the equipment used and techniques involved have been described by Shumway[35]. The nomenclature followed by these authors[33] is that recommended by Shepard[36] so far as the presence of relative amounts of sand, silt and clay are concerned, but a category of sediment is further divided according to particle diameters. The sediments studied were taken from the upper six inches with water-sediment surface defined rather than being "soupy".

The loss in decibels per reflection for normal incidence was computed for each station using Rayleigh equation presented by Mackenzie[37] in the following form :

$$\text{Loss in } db/\text{reflection} = 20 \log_{10} \left[\frac{[\,pq - \{1 - (p^2 - 1)\cot^2 \phi\}^{\frac{1}{2}}\,]}{[\,pq + \{1 - (p^2 - 1)\cot^2 \phi\}^{\frac{1}{2}}\,]} \right]$$

where p=velocity in sediment/velocity in water, q =density of sediment/density of water and ϕ=grazing angle.

The values of the velocity of sound, reflection loss in *db* for normal incidence and other acoustic data from *in situ* probe measurements at 100 Kc/s have been tabulated by these authors[33]. A separate table gives sound velocity, attenuation coefficient α in *db/ft* at various frequencies and other physical data for individual samples by the resonance-chamber-methad. Sound velocities by the resonance method ranged from about 4900 ft/sec. to about 5900 ft/sec. A few velocities lower than those in sea water were also obtained, Mifsud[38], Ericson[39] and Paterson [40] have also reported acoustical measurements in natural and artificial sediments, but *in situ* very few determinations were made.

Hamilton[41] noted that in high porosity sediments of the sea floor off San Diego, at several stations, the velocity of sound in the sediments was 2—3% less than the velocity of sound in the water just above the bottom. Taylor[42] studied the elastic properties of the sediments. Sutton *et. al*[29]. measured sound velocity and certain other physical properties in samples from 26 cores taken in the Atlantic Ocean from water depths between 840 and 4780 m. Only a few of their samples had median diameters in the sand range and all of them were oozes with calcium carbonate content higher than 85%.

Shumway[43,44] reported sound velocity and absorption data for 111 unconsolidated marine sediments ranging from shallow water sands to deep sea clays. This suite includes most of the common marine sediment types and the data provide the general range of values for sea floor sediments, Samples were collected by divers from water depths less than about 45 m at 26 new shallow water stations in San Diego Bay and the adjacent continental shelf. A group of samples were taken by core, dredge and snapper from continental shelf off Pigeon Point California, whose geologic setting is given by Moore and Sumway[45] and also from continental border land area off San Diego where a varied topography provides a variety of sediments ranging from sands to clays. Deep sea sediments were obtained from cores collected in the Eastern Pacific Ocean.

Standard methods were used for determining the grain size distribution[46], sieve for coarser sands, settling tube[47] for materials of intermediate size and pipette for fine materials. The velocity and absorption of sound were measured by the resonance technique[34,35] at frequencies between 20 Kc/s and 37 Kc/s and the values have been tabulated along with other physical data[43]. The values of sound velocity range from 1484 to 1785 m/sec. and the porosity range from 0·357 to 0·856. Fig. 2 shows the relationship of sound velocity to porosity at mean temperature 22·79°C. The velocity of sound in sea water of 35 % salinity at this temperature is 1529·3 m/sec. More than a third of the velocity values lie below this water value. The theoretical explanation for this phenomenon was apparently made by Urick[4] and field measurements by Frank Press and Maurice Ewing[48], Officer[49] and Hamilton[41]. The velocity at first falls as the concentration of particles increases, reaches a minimum at about 23 % concentration and then rises again.

These high porosity sediments are described acoustically by the velocity formula which applies to a suspension. As the mineral concentration reaches 23% (77% porosity) in a fine grained sediment the system is no longer a suspension and grain to grain contact causes the sediment to assume elastic properties which were previously either small or absent. Viscous drag is also no longer operative and the sediment assumes a measureable rigidity. Its Poisson's ratio is lowered from 0·5 for fluids and its compressibility is less than that of a fluid mixture. Due to these elastic properties, the velocity of sound increases with further increase in mineral concentration.

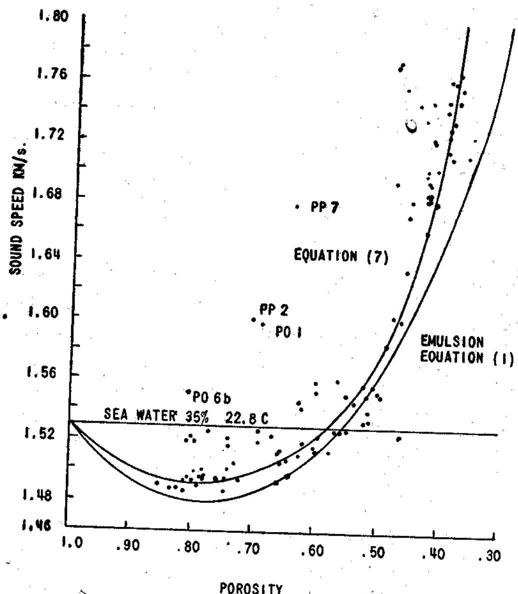

FIG. 2—Sound speed *vs* porosity for unconsolidated marine sediments.

For sediments in general it may be said that the smaller the median or mean diameter, the higher the porosity and, therefore, the lower the velocity of sound though the relationship is not linear.

Temperature and pressure changes occur in a sediment sample when it is brought from the sea floor to the laboratory. The effects of these factors have been discussed by Shumway[44]. The effect of frequency on sound velocity is practically negligible.

Sound absorption in sediments

Natural sediments are essentially suspensions of minerals in water having a rigidity depending upon the concentration of minerals. Theories of sound absorption for suspensions and granular aggregates have been, therefore, extended to these sediments by various investigators. The problem of absorption of sound by a suspension of one material in another has received greatest attention in connection with atmospheric acoustics as in the propagation of sound through fogs and dusts[50-52]. Rayleigh[15,53] was the first to calculate α of the amplitude in a beam of sound due to insertion therein of a single sphere small compared with the wave length.

Sewell[54] developed a theory for the extinction of sound in a viscous atmosphere by small spherical obstacles assuming the circumference of the obstacle to be small compared with the wave length of sound, the total volume occupied by the obstacle to be small compared with the volume of fluid and the quantity $a\left(\frac{\omega}{\mu}\right)^{\frac{1}{2}}$ to be large compared with unity. The expression finally developed is as follows :

$$2a = n\pi a^2 \left[3\sqrt{2}\ \frac{(\mu\omega)^{\frac{1}{2}}}{C} + \frac{6\mu}{Ca} + \frac{7}{9}\left(\frac{\omega a}{C}\right)^4 \right] \quad (8)$$

where 2α, the intensity absorption coefficient is defined by the equation $I = I_o e^{-2ax}$ and n is the number of obstacles per ml. a is the radius of the obstacle, μ is the kinematic viscosity of the medium (viscosity)/(density). C is the velocity of sound in the medium and $\omega = 2\pi\gamma$ where γ is the frequency.

The first term representing the loss due to turbulent friction in the immediate vicinity of an obstacle, contributes most significantly. The second term represents the loss due to stream flow friction, using Stoke's law for the slow motion of a sphere through a viscous medium, Brandt[55] obtained an expression for the loss due to stream flow friction. The third term represents the fraction of the incident energy which is scattered to a distance.

There were no experimental data to test the validity of Sewell's theory. However, in 1925, Altberg and Holtzmann[56] measured sound absorption in tobacco smoke but the results could not be compared with the theory as no analysis was made of the number and size of particles present in the smoke. Laidler and Richardson[50] measured the ultrasonic absorption in aerosols of magnesium oxide, stearic acid and lycoperdon spores and analysed all the three[57]. At the highest frequency (695 Kc/s) the experimental values of absorption were usually found to be much greater than the theoretical ones, lycoperdon spores showing closest agreement with the theory, presumably because they are more nearly spherical than smoke particles of stearic acid or magnesium oxide.

Hartmann and Focke[58] measured the absorption of ultrasonics in both water and aqueous suspensions of lycopodium spores at seven different frequencies ranging from 990—2500 Kc/s in approximately equal steps. Although the absorption coefficient increases with increasing frequency, the agreement with theory can be said to be only qualitative becoming closer as the frequency increases. The difference in absorption coefficients for lycopodium suspensions and for water was compared with the classical theoretical expectation as derived by Sewell where the surface is supposed to be motionless, *i.e.* the particle is a rigid sphere into which no sound could penetrate, an effect negligible in aerosols but significant in hydrosols. Since this assumption is not sound, it seems to be certain that the boundaries of the surface are not strictly at rest. Also at higher frequencies, the spores would become relatively more rigid.

A model close to the actual conditions was analysed by Epstein[59]. Neither Sewell nor Epstein in their treatment took into account the thermal conductivity of the viscous fluid. Twelve years later, in 1953, Epstein and Richardson[60] completed their theory by taking into consideration both viscosity and thermal conductivity but the reduction of the general results to explicit formulae of attenuation was restricted to the case of liquid droplets suspended in gases with particular attention to water particles in air. In general the effect of viscosity and heat conduction are intermingled but the attenuation β can be divided into two parts :

$$\beta = \beta_\eta + \beta_\tau$$

where β_η may be called the coefficient of viscous attenuation and β_τ the coefficient due to thermal attenuation. Knudsen's measurements[61] afford experimental evidence for the validity of this theory. The paper of Tolman and Fine[62] is also worth study in this connection.

Lamb[63] modified and extended Sewell's theory to the case of rigid incompressible particles that are free to move in the sound field. He calculated the velocity potential of waves scattered from a mobile sphere and by integration over a surrounding volume, the energy loss.

Urick[64] cast Lamb's theory in a more convenient form and obtained for the attenuation due to a small single sphere free to move in (unit volume of) a viscous fluid, the formula

$$\alpha = \frac{2\pi}{9} k^4 a^6 + \frac{2\pi}{3} k \left(\sigma - 1\right)^2 a^3 \frac{S}{S^2 + (\sigma + \tau)^2} \qquad (9)$$

where a is the radius of the sphere, $k = \frac{2\pi}{\lambda}$, the wave length of the radiation,

$\sigma = \frac{\rho_1}{\rho_2}$ is the density of the particle relative to fluid

$$S = \frac{9}{4\beta a}\left(1 + \frac{1}{\beta a}\right), \qquad \tau = \left(1 + \frac{9}{4\beta a}\right)$$

β is the propagation constant of shear waves in the fluid, *i.e.* $\left(\frac{\omega}{2\gamma}\right)^{\frac{1}{2}}$, where ω is the pulsatance and γ the kinematic viscosity.

The first term in equation (9) represents a redistribution on scattering of sound energy due to the presence of the sphere and the second term on energy loss due to viscous effects at the surface of the sphere. At megacycles the second term predominates. Urick investigated the importance of this term experimentally and found the results in agreement with eq. (9) but under the conditions of his experiment the scattering effect was too small to measure. The frictional loss is due to the fact that particles in a sound field partake the motion of the fluid to an extent depending upon their mass the frequency of sound and the viscosity of fluid. Thus there exists a velocity difference between fluid and particles which for a viscous fluid results in a heat loss at the surface of each particle and therefore in a decrease in intensity of the sound field. It was found that the viscous drag absorption coefficient depends on the square of the ratio of the velocity difference between particles and the fluid to the velocity of the fluid, *i.e.* $\left(\frac{u_o}{v_o}\right)^2$.

Fig. 3 illustrates the theoretical variation of absorption with particle size at three frequencies 1, 5 and 15 Mc/s for a constant concentration (1 %) of particles of density 2·65 suspended in water. Extremely small particles tend to move to and fro along with the liquid in the sound field so that the ratio $\left(\frac{u_o}{v_o}\right)$ and hence the absorption are small. As the particle size increases the particles lag more and more behind the movement of the fluid but at the same time the total surface area of particles decreases with increasing size if the volume concentration remains constant. These opposing factors result in an absorption maximum. Fig. 3 also shows the apparent absorption caused by scattering. Urick measured the sound absorption in aqueous suspensions of irregular particles of kaolin and finely ground quartz sand by the pulse method similar to the one described by Teeter[65].

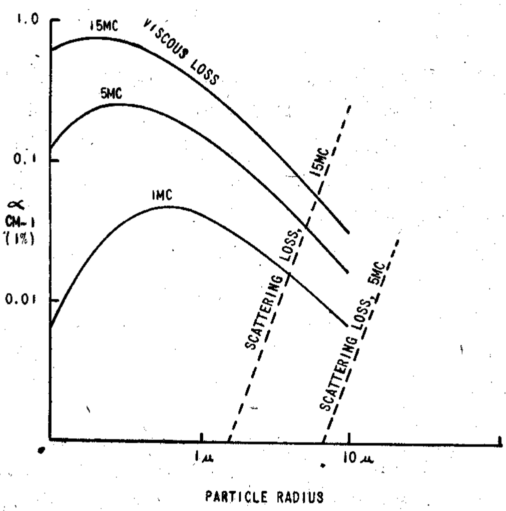

FIG. 3—Theoretical viscous and scattering loss *vs* particle size for frequencies 1, 5 and 15 MC/S, and for a volume concentration of 1%, density ratio 2·65.

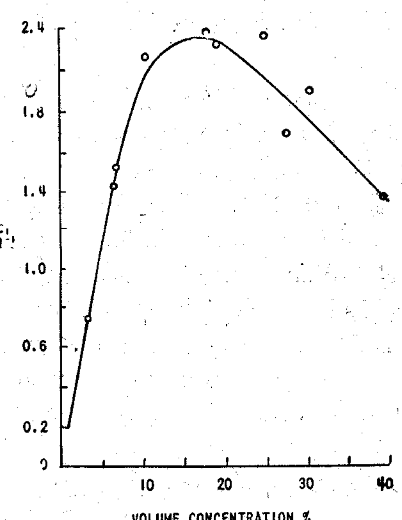

FIG. 4—1-mc absorption in kaolin suspensions for moderate concentrations.

The median diameter for kaolin particles was found to be 0·9 μ and for sand 2·2 μ. The absorption was found to increase linearly with volume concentration for kaolin and also for lycopodium spores of uniform diameter of 30 μ at 15 Mc/s. The linearity of absorption with concentration is in agreement with the theory and illustrates the additive nature of absorption process. It is seen from Fig. 4 representing the relation between kaolin concentration and absorption (1 Mc/s) that linearity holds good only at low concentrations where interaction between particles can be neglected.

Urick and Ament[6] measured absorption at megacycle frequencies on a series of bromoform-in-water and mercury-in-water emulsions and found the results to be in agreement with theory at low and moderate concentrations upto 25—30%. Absorption measurements were made also by Hueter, Morgan and Cohen[66] in homogenized and skimmed milk, by Carstensen, Kamli and Schwan[67] in blood plasma, by Robert Meister[68,69] in suspensions of plankton, by Rytov *et. al.*[70] and Vladimirskii and Galamin[71] in emulsions of mercury-in-water, by Allinson and Richardson[72] in emulsions of benzene in water and water in benzene and other emulsions. Other papers on sound absorption in emulsions are by Fry[73] Vladimirskii[74] and Ratinskaya[75].

Nyborg Rudnick and Schilling[76] measured sound attenuation at 10,18 and 26 Kc/s in "fine building sand" and "Hagerstown silt loam" but did not discuss the grain-size distribution for the samples used. However, the materials were ungraded samples of wide range particle size. Ferrero and Sacerdote[20] have reported data for attenuation in both sand and lead shot. Morse[14] derived expression for sound attenuation in sediments and found that the theory is in agreement with the experimental results of Ferrero[20] and Nyborg *et. al.*[76]

Hoyer *et. al.*[77] measured backward scattering coefficient, general scattering coefficient and reflection coefficients of water-filled sands as a function of grazing angle of the incident sound wave on water and water-filled sand plane interface at frequencies of 0·5 and 1·0 Mc/s. Nolle[78,79] computed transmission and reflection at plane boundary between fluid and fluid-filled sand. It was observed that the reflection loss approximately 10 *db* at normal incidence became very small at grazing angles smaller than the critical which is in agreement with ultrasonic experiments. Experimental studies were made of the propagation of longitudinal waves in several mixtures of water with quartz sands in the frequency range 400—1000 Kc/s the most probable particle size ranging from 0·01—0·07 cm and being smaller compared to wave length.

The more general case of scattering by particles comparable in radius with the wave length of sound does, however, give much greater weight to the scattering and it was this more general case which was investigated by Busby and Richardson[80]. To the extent that the attenuation produced by one sphere is unaffected by its neighbours, Busby and

Richardson adapt the formula of Lowan *et. al.*[81] to the present case and formulated an equation for the attenuation in a suspension in the form

$$\alpha = \frac{\lambda^2}{2\pi}\left[Sin^2 F_\circ\gamma + 3\left(\frac{\sigma - 1}{\sigma + \frac{1}{2}}\right)^2 Sin^2 F_1\gamma + \sum_{m=2}^{m=\infty} (2m+1)\, Sin^2 F_m \gamma \right]$$

$$+ \frac{2\pi k\,(\sigma - 1)^2 a^3 S}{3\,\{S^2 + (\sigma+\tau)^2\}} \qquad (10)$$

or

$$\frac{\alpha\lambda}{C} = \frac{3\pi}{\gamma^3}\left[Sin^2 F_\circ\gamma + 3\left(\frac{\sigma - 1}{\sigma + \frac{1}{2}}\right) Sin^2 F_1\gamma + \sum_{m=2}^{m=\infty} (2m+1)\, Sin^2 F_m \gamma \right]$$

$$+ \frac{\pi\,(\sigma - 1)^2 S}{S^2 + (\sigma + \tau)^2}$$

where C stands for volume concentration of spheres (volume of solid/total volume). This theoretical equation reduces to the Lamb-Urick equation [64] if a is made small compared with λ.

When the radius of the sphere approaches the wave length, calculation of the scattering becomes difficult but Lowan *et. al.* have shown that the attenuation produced by such a rigid sphere on an impinging sound beam is given by

$$a = \frac{\lambda^2}{2\pi}\sum_{m=0}^{m=\infty} (2m+1)\, Sin^2 F_m \gamma$$

where F_m is a function of $\gamma \left(= \frac{2\pi a}{\lambda}\right)$ whose value may be expressed in Bessel functions. They found that for values of γ upto 10, the series in the above summation converged rapidly and they calculated and tabulated the cognate values of $F_m\,\gamma$.

Taking the above expression for α, the movement of the sphere in the liquid and viscous losses due to friction at the surface of the sphere, must be taken into consideration. Lamb's theory showed that the movement of the spheres affected only the first order term in the expansion which should be multiplied by the factor $\left(\frac{\sigma - 1}{\sigma + \frac{1}{2}}\right)$. Though Lamb applied the condition $a < \lambda$, his correction appears to be adequate for values of γ upto unity. Here the absorption due to scattering is expected to vary with radius.

Busby and Richardson[80] measured the attenuation of sound in suspensions of solid particles in water in the range of frequencies 1 to 10 Mc/s and of particle radius 8—100 μ and found that absorption was directly proportional to concentration and thus the interaction between the particles could be neglected. Some measurements were also made in natural sediments of mean diameters 450, 180 and 120 μ and also in an artificial sediment of glass spheres with mean diameter 120 μ and concurrence between artificial and natural sediment was found. It was observed that at high frequencies, Rayleigh scattering law ($\alpha \propto f^4$) seems to be obeyed by these sediments, but at low frequencies the absorption seems to be directly proportional to frequency. Attenuation was measured also at maximum packing fraction of 70% when the system no longer remains a suspension. At such large concentrations, there will be interaction between the particles and the effect of n particles on the sound wave will be n times the effect of one. There will be multiple scattering a factor difficult to introduce in any theory. Some of the multiscattered energy returns to the transducer and the viscous drag is also lessened.

Michel Auberger and John Rinehart[82] described a method for measuring attenuation in plastics and rocks and reported data for Plexiglas and granite in the frequency range 250—1000 Kc/s. Ide[83] measured the velocity in rock cylinders 5·08 cm in diameter and 25·4 cm long and also in glasses as a function of temperature. Knopoff and Mac Donald[84] measured the attenuation of small amplitude stress waves in solids. Birch[85,86] noticed that in dry crystalline rocks absorption is considerably higher than it is in isolated crystals of minerals involved.

Hamilton *et. al.*[33] observed that sea floor sediments of sand size gave attenuation between 3 and 10 *db/ft* while absorption in silt size sediments was lower than that for sand. The sandy coarse silt gave values of 6 to 11 *db/ft*. The lowest attenuations were found to be in finer grained and more porous materials.

Fig. 5 due to Shumway [44] exhibits an absorption maximum for sediment of intermediate porosity in the range 0·45 to 0·6 where the dominant sediment type is coarse silt. The highest measured absorption was 25·1 *db/m* and the lowest was 0·66 *db/m*. In general, low absorption values were found in those sediments which had high porosity, clays and fine silts. Most absorption values lie within the two broken lines arbitrarily drawn. Thus it may be said that at frequencies between 20 and 40 Kc/s most natural seafloor sediments probably have absorption values within the limits defined by these curves. An explanation for absorption maximum was given by Urick[64].

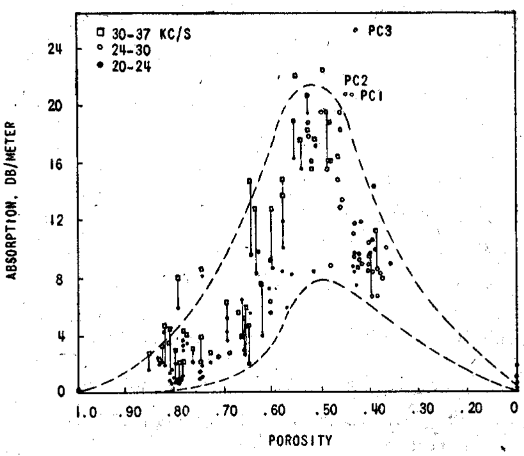

FIG. 5—Sound absorption *vs* porosity for unconsolidated marine sediments.

Fluid exists in a frame work of grains and moves relative to this frame work under sonic agitation producing viscous losses which depend on total acoustically effective surface area of particles.

The acoustic surface area for a fluid-filled granular aggregate or a porous solid is defined as the area of a flat plate lying parallel to the direction of propagation which yields an energy loss due to viscosity, equivalent to viscous loss in the porous material.

The total surface areas of some deep sea sediments were measured by Kulp and Carn[87] and those of some clay soils were measured by Nelson & Hendricks [88] and Diamond & Kinter[89]. Their values ranged between 5 and 50 sq. m/gm of dry sediment. In comparison, the acoustic areas computed by Shumway[43,44] have values of about 0·05 sq. m/gm of dry sediment. He has also described the method of determining the acoustic surface area of a sediment. The values were also computed and tabulated for the samples studied. Detailed observation shows that the absorption α and acoustic surface area, A_s, may be related as

$$\alpha = M A_s$$

where M not only varies with porosity and grain size but it is also a more complicated factor.

Frequency dependence of absorption

The relation of absorption to frequency was examined by Shumway [44] for 65 samples and for the frequency range involved in the measurements

$$\alpha = K f^n$$

where K is constant.

The average value of n for the 65 samples was 1·79. Assuming energy loss through viscosity, the frequency dependence of absorption may be represented by the equation

$$\alpha = \frac{m + n f^{\frac{1}{2}}}{a + b f^{-\frac{1}{2}} + c f^{-1} + d f^{-3/2} + h f^{-2}}$$

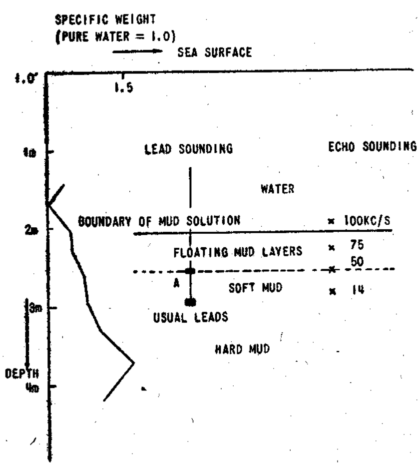

Fig. 6—Critical depth measurement by lead-sounding and echo-sounding methods.

where α is the absorption factor, f the frequency and a, b, c, d, h, m, and n, are constants.

Considering the views of Biot [25,26] and others the consequences would be that (*a*) for a given material the frequency dependence of absorption changes with frequency and (*b*) the frequency dependence of absorption for each fluid grain aggregate is unique.

Application of propagation studies to navigation

The existence of rigidity in a sediment has been acknowledged by a number of papers already mentioned but no attempt has been made to relate it to textural properties of the sediments. Thus the problem remains (*a*) How to relate the rigidity modulus (and frame incompressibility) to measurable sediment variables?

The deep layers of the ocean bottom consist of hard mud but going upwards the water content increases and this mixture of small particles and sometimes coagulate with organic substance, may be called "soft mud". The boundary of this mud is not well demarcated and particles float freely in the upper layer of sedimentation and the system resembles a solution. In such circumstances the problems are :

(*b*) What is the navigable depth in the case of a muddy bottom?

(*c*) How to determine depth by means of an appropriate and efficient method?

Though the investigation is still in progress, Owaki[90] has recently reported some preliminary results obtained experimentally at a port. Divising a special lead with a plate beneath it (*A* in Fig. 6) the critical depth was measured by lead sounding and also by echo-sounding using sound waves of frequencies 14, 50, 75 and 100 Kc/s. The higher frequencies gave low values and the depths obtained by lead sounding were much greater than any depth determined by echo-sounding. Assuming that the critical depth coincides with the demarcation between the soft and the compact mud, sounding by 50 Kc/s waves seems to give this depth. The probable errors in the value obtained by echo sounding are of the order of ± 5 cm; therefore, differences in measured depths are mainly due to frequency difference.

The left hand side of Fig. 6 shows the vertical distribution of the specific weight of mud ascertained by means of core sampling, and various depths obtained by lead and echo sounding. In order to obtain critical depth, it is necessary to choose a suitable frequency and intensity of transmitted wave. Also it is essential to find correlation between the density of mud and the depths at various frequencies and intensities.

Thus for the problem (*b*), one has to study what parameters (hardness, porosity, density etc.) will be best for the safety of navigation. For problem (*c*), one has to investigate the possibility of obtaining any level in the mud sediment that is specified by a certain value of some physical parameter like density.

Only then it would be possible to specify the weight, shape and dimension of the lead for such measurements and also to specify definite frequencies and intensities to be employed for echo sounding.

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