A NOTE ON DIFFRACTION OF OBLIQUE SHOCK WAVE

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The region between incident and reflected shocks remains undisturbed for all values of γ, γ being the ratio of specific heats, after the shock configuration has crossed the small bend.

In an earliar paper[1] it was established that the region between the incident and reflected shocks remains undisturbed for all incident shock strengths after the shock configuration has crossed the corner. This result was established for $\gamma = 1.4$, γ being the ratio of specific heats. In this note the same result has been proved for general value of γ.

Following the notation of Fig. 1, the Rankine–Hugoniot equation across the incident shock gives

$$q_1 = \frac{2\bar{u}}{(\gamma+1)}\left(1 - \frac{a_o^2}{\bar{u}^2}\right) \tag{1}$$

$$\rho_1 = \frac{\rho_o(\gamma+1)}{(\gamma-1) + \frac{2a_o^2}{\bar{u}^2}} \tag{2}$$

$$p_1 = \frac{\rho_o}{(\gamma+1} \left[2\,\bar{u}^2 - a_o^2\,\frac{(\gamma-1)}{\gamma}\right] \tag{3}$$

where $\bar{u} = u \sin\alpha_o$ and $a_o = \sqrt{\dfrac{\gamma p_o}{\rho_o}}$

By using the argument of Srivastava & Ballabh[1] it is proved that the region between the incident and reflected shock will or will not be disturbed according as

$$\alpha_o > \text{or} < \alpha_o^*$$

where $$\cot^2\alpha_o^* = \frac{(\gamma+1)}{2}\cdot\frac{(\eta-1)}{\eta^2}, \quad \eta = \frac{\rho_1}{\rho_o} \tag{4}$$

Now it will be shown that α_o^* is greater than the extreme value of the angle of incidence. The extreme angle equation is given by

$$x^2(1+\eta^2x^2)^2 = (1+\eta x^2)(\eta-1)\{(\gamma+1)(\eta-1)+2\}\{(\gamma-1)(1+\eta x^2)+2\} \tag{5}$$

where $x = \cot\alpha_{ext}$.

Solving (4) and (5) we get the points of intersection as

$$\eta = \frac{(\gamma-1)}{(\gamma+1)} < 1$$

$$\eta = \frac{(2\gamma+3)}{(2\gamma+6)} < 1$$

$$\eta = \infty$$

$$\eta = 1$$

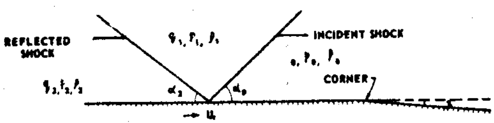

Fig. 1—Diffraction of oblique shock waves.

The first three values are inadmissible as $1 \leqslant \eta \leqslant \frac{(\gamma+1)}{(\gamma-1)}$. At $\eta = 1$ we find that $\left(\frac{dt}{d\eta}\right)_{\eta=1}$ for the curve (5) is $2(\gamma+1)$, where $t = \cot^2 \alpha_{ext}$ and $\left(\frac{dt}{d\eta}\right)_{\eta=1}$ for the curve (4) is $\frac{(\gamma+1)}{2}$. It follows therefore that the curve (α_{ext} versus η) always lie below the curve (α_o^* versus η). This proves the result.

Now when an oblique shock configuration passes a small bend, Mach reflection or diffraction occurs depending upon the relative outflow behind the reflected shock before the shock configuration has crossed the corner. When the relative outflow is supersonic, Mach reflection occurs and when it is subsonic or sonic diffraction occurs[2]. This result is independent of γ and hence is true for all γ. The result that is proved in this note therefore becomes quite useful for the consideration of diffraction and Mach reflection effects for all values of γ after the shock configuration has crossed the corner.

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REFERENCES

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