CONCENTRIC ARRAYS OF RING SHAPED APERTURES AS NEUTRAL DENSITY FILTERS

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Neutral density filters with flat transmission curves find extensive use in optical instruments, but really neutral filters *i.e.*, with flat transmission are rather difficult to fabricate. A new method of making neutral filter has been discussed here from the oretical as well as practical aspect.

Neutral density filters are normally used in optical instruments to reduce the amount of visible light entering the instrument, so that it can be aimed at or near bright sources, such as sun or search-light.

Neutral density filters in the real sense *i.e.*, filters with flat transmission curves are very difficult to obtain. Jacob's suggested the use of a fogged photographic plate or wire mesh as a neutral filter. According to him a wire mesh is most excellent provided that care is taken to control the light scattered by the wires. While following this lead, we observed that it was not easy to fabricate and use such a mesh with success *i.e.*, without affecting the image in anyway. It was hence considered that a concentric array of ring shaped aperture may be successfully used to reduce the intensity in the image.

PRACTICAL DETAILS

In order to evolve an easy and successful method for the production of a concentric array of ring-shaped aperture, the following procedure was adopted.

A master pattern was made in which black concentric rings were drawn at the interval of 0·04 inches starting from a diameter of 0·08 inches and increasing to a maximum of 20 inches. The whole pattern was reduced twenty times by photographic method. In the twenty times reduced pattern, bright and dark rings were separated from each other at an interval of 0·002 inches. This pattern when observed at a distance of 10 inches from the eye, the angle subtended by any two consecutive rings is slightly less than a minute. As eye may fail to resolve such a ring pattern, it was considered most suitable for introduction in an optical system. In order to protect the pattern, it was sandwitched between two cover plates, with the help of optical cement.

The pattern when examined for its transmission showed an integrated transmission of 45 per cent in the red region, 46 per cent in blue region and 44 per cent in the green region. Thus, the transmission was more or less flat and without peak. This filter, when used with a binocular on its eye cups, did not cause any strain to the eye. The definition was also not affected in any way.

In order to vary the percentage transmission, it was considered that different demagnification could be tried. A few more filters were prepared from the master with demagnification varying from 15—25. It was observed that the pattern with a demagnification ranging from 18—22 can be employed as neutral filter beyond which it tends to have lensatic affect.

THEORETICAL CONSIDERATIONS

Following the usual law of intensity *i.e.*, it is directly proportional to the square of the amplitude, an analysis has been attempted for the intensity transmitted by concentric arrays of ring-shaped apertures.

Firstly, consider inside one of the transparent annuli a thin circular strip of light and the amplitudes for this strip with given extreme differences of phase at a point close to the axis of the aperture. Secondly, to integrate the result over the entire concentric array of ring-shaped aperture, we take into account two factors

(*i*) The amplitude for any annular array at an off axis point is obtained by adding the amplitudes contributed thereby the individual transparent annuli.

(*ii*) For any free annulus, we arrive at the required amplitude by subtracting from the amplitude given by the externally bounding circle that which would come from the internally bounding circle.

Now consider the elementary strip KPM inside any one of the transparent annuli and let F_o be a piont on the axis of aperture. Take OF_o as the axis of X, OM as the axis of Y and OK as axis of Z. Let P be a point on the circle such that angle $KOP = \theta$. Then the co-ordinates of P are $(0 - r, \sin\theta, r\cos\theta)$. Then the disturbance due to an element at P will arrive at a point F_1 distance Y_1 parallel to the Y-axis from F_o, with a difference of phase $PF_1 - KF_1$ from that at K (Fig. 1).

Since the co-ordinates of F_1 are $(f, Y_1, 0)$

$$PF_1 = \{f^2 + (Y_1 + r\sin\theta)^2 + r\cos^2\theta\}^{\frac{1}{2}}$$
$$= \sqrt{f^2+r^2}\left\{1 + \frac{rY_1\sin\theta}{f^2+r^2}\right\}.$$

Neglecting terms in Y_1^2 etc. as being of second order

$$KF_1 = \{f^2 + Y^2_1 + r^2\}^{\frac{1}{2}} = \sqrt{f^2+r^2} \text{ approx.}$$

$$\therefore\ PF_1 - KF_1 = \frac{rY_1\sin\theta}{\sqrt{f^2 + r^2}} = W\sin\theta$$

Hence if the vibration reaching F_1 from K is expressed as $a\sin\dfrac{2\pi t}{T}$ where T is the period of light, it follows that the equation of the vibration reaching F_1 from P $= a\sin\left(\dfrac{2\pi t}{T} + W\sin\theta\right)$, where $W = \dfrac{rY_1}{\sqrt{f^2+r^2}} = \dfrac{rY_1}{f}$ (approx.) Similarly for point P^1 making angle $-\theta$ with the vertical OK, we have the vibration expressed as

$$A\sin\left(\frac{2\pi t}{T} - W\sin\theta\right)$$

These two vibrations will give a resultant

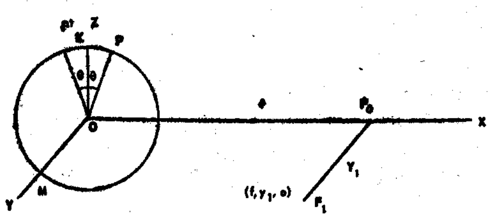

Fig. 1

vibration

$$= A \sin \frac{2\pi t}{T} \cos (W \sin\theta) = A \sin \frac{2\pi t}{T}$$

where A (the amplitude) $= a \cos (W \sin\theta)$

The amplitude (A_1) at Fl for the whole circular strip of light is obtained by integrating $a \cos (W \sin \theta)$ between the limits 0 and π, so that

$$A_1 = a \int_0^{\pi} \pi \cos (W \sin\theta)\, d\theta = 2a \int_0^{\pi/2} \cos (W \sin\theta)\, d\theta$$

$$= 2a\,\pi/2 \left[1 - \frac{W^4 \sin^2\theta}{2!} + \frac{W^4 \sin^4\theta}{4!} - \frac{W^6 \sin^6\theta}{6!} + \ldots\ldots\right] d\theta$$

$$= 2a\,\frac{\pi}{2} \left[1 - \frac{W^2}{2^2 (1!)^2} + \frac{W^4}{2^4 (2!)^2} - \frac{W^6}{2^6 (3!)^2} + \ldots\ldots\right] d\theta$$

$= a\pi J_\circ (W)$ where $J_\circ (W)$ is a Bessel function of zero order. Choosing 'a' so that A_1 is unity; $W = 0$ or in other words making central intensity unity as the standard, we have $a = \frac{1}{\pi}$ so that $A_1 = Jo\,(W)$.

Let the outermost nth circle of the concentric array of ring-shaped aperture be of unit radius and the other circles forming the array have radii $\frac{n-1}{n}, \frac{n-2}{n}, \frac{n-3}{n}, \ldots\ldots$ $\ldots\ldots\ldots \frac{2}{n}, \frac{1}{n}$ respectively.

If centre of the array is transparent and alternate annuli are opaque and transparent, then the resultant amplitude is given by

$$A_2 = 2\pi \left[\int_0^{\frac{1}{n}} J_\circ (W)\, rdr + \int_{\frac{2}{n}}^{\frac{3}{n}} J_\circ (W)\, rdr + \int_{\frac{n-2}{n}}^{\frac{n-1}{n}} J_\circ (W)\, rdr\right] \text{ when } n \text{ is even.}$$

As usually a neutral filter is employed in a parallel beam of light $W = \frac{ryl}{f}$ reduces to zero when f is infinity. Hence expression for the resultant amplitude reduces to the form

$$A_2 = 2\pi \left[\int_0^{\frac{1}{n}} rdr + \int_{\frac{2}{n}}^{\frac{3}{n}} rdr + \ldots\ldots\ldots\ldots\ldots + \int_{\frac{n-1}{n}}^{1} rdr\right]$$

when n is odd

$$= 2\pi \left[\int_{0}^{\frac{1}{n}} r dr + \int_{\frac{2}{n}}^{\frac{3}{n}} r dr + \ldots\ldots\ldots\ldots\ldots + \int_{\frac{n-2}{n}}^{\frac{n-1}{n}} r dr \right]$$

when n is even

$$A_2 = \frac{\pi}{n^2} \left[\left\{ 1^2 + 3^2 + 5^2 + \ldots\ldots + (n-2)^2 \right\} - \left\{ 2^2 + 4^2 + 6^2 + 8^2 + \ldots\ldots\ldots\ldots + (n-1)^2 \right\} \right] + \pi \quad \text{when } n \text{ is odd} \qquad (1)$$

$$= \frac{\pi}{n^2} \left[\left\{ 1^2 + 3^2 + 5^2 + \ldots + (n-1)^2 \right\} - \left\{ 2^2 + 4^2 + 6^2 + 8^2 + \ldots\ldots + \ldots\ldots\ldots\ldots + (n-2)^2 \right\} \right] \quad \text{when } n \text{ is even} \qquad (2)$$

Thus with given values of n, the resultant amplitude, and hence the intensity due to ring-shaped aperture employed as neutral filter can be determined with the help of equation (I) or (2) according as n is odd or even.

The fact that two consecutive rings cannot be resolved by the eye if the anlge subtended by these rings is slightly less than one minute at a least distance of distinct vision, has been made use of in making the suitable pattern.

CONCLUSION

If suitably made, the ring shaped aperture can be employed as neutral filter in the optical system.

We have two parameters which can be varied for making the neutral filters with different transmissions. One, the total number of circles n forming the pattern and the other demagnification of master pattern. Both the parameters have their own limitations. Drawing of large number of black concentric rings on the master sheet of given dimensions involve practical difficulty. As the rings become closer and closer, the work becomes more and more laborious. Demagnification has to be suitably chosen as not to deviate from angular subtense of one minute at the eye, between two consecutive black rings at a least distance of distinct vision. Hence a compromise is essential between the two parameters to achieve a satisfactory result.

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