A NOTE ON PARALLEL HYDROMAGNETIC SHOCKS

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It has been shown here thht the transition from State 1 to State 3 can be effected by a parallel hydromagnetic shock of the same strength as the ordinary fluid dynamic parallel shock.

Pant and Misra[1] have obtained the result—if a switeh-on shock take the gas from state 1 to state 2 and from there a parallel switch-off shock takes the gas to state 3, then the transition from state 1 to state 3 is the same as produced by an ordinary fluid dynamic parallel shock of a specified strength. It has been shown here that it is not necessary for the two shocks to be switch-on and switch-off[2]—they can be any two parallel hydromagnetic shocks and the transition from state 1 to state 3 can be effected by a parallel hydromagnetic shock of the same strength as the ordinary fluid dynamic parallel shock.

Let $T_\alpha\,(\alpha = 1, 2)$ be the two parallel shocks and p_r , ρ_r ; u_r^i, u_{rn}, H_{rn} and H_{rt} $(r = 1,2,3)$ be the pressure, density, velocity components of a particle of the fluid, normal fluid velocity, normal and tangential components of the magnetic field in the states 1, 2 and 3. The strengths S_α of the shocks T_α are defined as

$$S_\alpha \overset{\text{def}}{=} \frac{\rho_{\alpha+1} - \rho_\alpha}{\rho_\alpha} \tag{1}$$

The Rankine–Hugoniot jump conditions when the fluid passes a hydromagnetic shock are given by[3]

$$\left[\, u^i{}_{\alpha+1} \right] = \frac{S_\alpha\, u_{\alpha n}\, H_{\alpha t}\, H_{\alpha n}}{4\pi\rho_\alpha\, u^2_{\alpha n} - (1+S_\alpha)\, H^2_{\alpha n}}\, l^i_\alpha - \frac{S_\alpha}{1+S_\alpha}\, u_{\alpha n}\, n^i{}_\alpha \tag{2}$$

$$\left[H^i{}_{\alpha+1} \right] = \frac{4\,\pi\, S_\alpha\, \rho_\alpha\, u^2_{\alpha n}\, H_{\alpha t}}{4\pi\rho_\alpha\, u^2_{\alpha n} - (1 + S_\alpha)\, H^2_{\alpha n}}\, l^i{}_\alpha \tag{3}$$

$$\left[p^*{}_{\alpha+1} \right] = \frac{S_\alpha}{1+S_\alpha}\, \rho_\alpha\, u^2_{\alpha n} \quad \text{where } p^* = p + \frac{H^2}{8\pi} \tag{4}$$

and the summation convention is not observed with respect to the repeated indices. Also l^i_α and n^i_α are the components of the unit tangent and normal to the shock surfaces T_α.

THEOREM

If a hydromagnetic shock takes the gas from state 1 to state 2 and from there another parallel hydromagnetic shock takes the gas to state 3, then the transition from state 1 to state 3 can be effected by a parallel hydromagnetic shock of strength δ given by

$$\delta = S_1 + S_2 + S_1 S_2 \tag{5}$$

where S_1 and S_2 are the strengths of the two parallel shocks.

Proof: With the above notations, we have

$$p_2{}^* - p_1{}^* = \frac{S_1}{1+S_1}\, \rho_1\, u^2_{1n} \tag{6}$$

$$p_3{}^* - p_2{}^* = \frac{S_2}{1+S_2}\, \rho_2\, u^2_{2n} \tag{7}$$

Adding (6) and (7) we get

$$p_3{}^* - p_1{}^* = \frac{S_1}{1+S_1}\, \rho_1\, u^2_{1n} + \frac{S_2}{1+S_2}\, \rho_2\, u^2_{2n} \tag{8}$$

Substituting $u_{2n} = \dfrac{u_{1n}}{1+S_1}$ and putting $\dfrac{\rho_2}{\rho_1} = 1 + S_1$ in (8) we have

$$p_3^* - p_1^* = \rho_1 u^2_{1n} \left\{ \frac{S_1}{1+S_1} + \frac{S_2}{(1+S_1)(1+S_2)} \right\} \tag{9}$$

or

$$p_3^* - p_1^* = \rho_1 u^2_{1n} \left\{ \frac{S_1 + S_2 + S_1 S_2}{1 + (S_1 + S_2 + S_1 S_2)} \right\} \tag{10}$$

or
$$p_3^* - p_1^* = \rho_1 u^2_{1n} \frac{\delta}{1+\delta} \tag{11}$$

If the transition from state 1 to state 3 is to be effected by a hydromagnetic shock of strength δ, it is obvious that (11) has to be satisfied. Again we have for states 1 and 2

$$u_2^i - u_1^i = \frac{S_1 u_{1n} H_{1t} H_{1n} l_1^i}{4\pi \rho_1 u^2_{1n} - (1+S_1) H^2_{1n}} - \frac{S_1}{1+S_1} u_{1n} n_1^i \tag{12}$$

And for states 2 and 3

$$u_3^i - u_2^i = \frac{S_2 u_{2n} H_{2t} H_{2n} l_1^i}{4\pi \rho_2 u^2_{2n} - (1+S_2) H^2_{2n}} - \frac{S_2}{1+S_2} u_{2n} n_1^i \tag{13}$$

Adding (12) and (13) we get

$$u_3^i - u_1^i = \left\{ \frac{S_1 u_{1n} H_{1t} H_{1n}}{4\pi \rho_1 u^2_{1n} - (1+S_1) H^2_{1n}} + \frac{S_2 u_{2n} H_{2t} H_{2n}}{4\pi \rho_2 u^2_{2n} - (1+S_2) H^2_{2n}} \right\} l_1^i - \left\{ \frac{S_1}{1+S_1} u_{1n} + \frac{S_2}{1+S_2} u_{2n} \right\} n_1^i \tag{14}$$

.. e also have

$$H_2^i - H_1^i = \frac{4\pi S_1 \rho_1 u^2_{1n} H_{1t}}{4\pi \rho_1 u^2_{1n} - (1+S_1) H^2_{1n}} l_1^i \tag{15}$$

Multiplying both sides of (15) by l_1^i we obtain

$$H_{2t} - H_{1t} = \frac{4\pi S_1 \rho_1 u^2_{1n} H_{1t}}{4\pi \rho_1 u^2_{1n} - (1+S_1) H^2_{1n}} \tag{16}$$

Substituting $\dfrac{u_{1n}}{1+S_1}$ for u_{2n} and $\rho_1 (1+S_1)$ for ρ_2 in (14) we see that as a consequence of (16), (14) becomes

$$u_3^i - u_1^i = \frac{(S_1 + S_2 + S_1 S_2) u_{1n} H_{1t} H_{1n}}{4\pi \rho_1 u^2_{1n} - (1+S_1+S_2+S_1 S_2) H^2_{1n}} l_1^i - \frac{S_1 + S_2 + S_1 S_2}{1 + S_1 + S_2 + S_1 S_2} u_{1n} n_1^i \tag{17}$$

or

$$u_3^i - u_1^i = \frac{\delta u_{1n} H_{1t} H_{1n}}{4\pi \rho_1 u^2_{1n} - (1+\delta) H^2_{1n}} l_1^i - \frac{\delta}{1+\delta} u_{1n} n_1^i \tag{18}$$

which proves the therem.

REFERENCES

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2. BLEVISS, Z. O., *Heat Trans. Fluid Mech. Inst.* (1959)
3. KANWAL, R.P., *Proc. Roy Soc.*, **257** (1960), 263.