# STUDY OF SYMMETRICAL MULTIPLE-T NULL RC NETWORKS IN PARALLEL

A. SUNDARABABU

Air Force Technical College, Bangalore

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**This paper deals with the study of similar T-networks paralleled successively. The variation of not only the frequency but also the 'Q' value of such combinations with an increase in the number of capacitance and resistance arms have been studied. General formulae for the frequency and the 'Q' of a symmetrical RC multiple-T are also given.**

The study of RC twin-T networks have been made by several workers each giving a method of his own. Bowers[1] pointed out that various spot frequencies can be obtained by varying the series and shunt arms of a single twin-T, but such a twin-T used in the feedback circuit of an oscillator alters the regeneration with a considerable shift in phase and a consequent variation in amplitude. A symmetrical RC multiple-T used in the feedback circuit of an oscillator does not produce an alternation in the regeneration because the phase and the amplitude minima occur at the same frequency. The behaviour of RC multiple-T networks[2] has been studied in detail by taking into consideration two twin-Ts of slightly differing frequencies and combining them in different combinations.

## RESONANT FREQUENCY OF PARALLEL SYMMETRICAL RC MULTIPLE-T'S

Similar resistance and capacitance arms of a twin-T are taken and paralleled together as shown in Fig. 1. If $n_r$ is the number of resistance arms and $n_c$, the number of capacitance arms in the combination, then

$$\beta = \frac{(n_r/R^2) - n_c\, w^2 c^2}{(n_r/R^2) - n_c\, w^2 c^2 + (n_r + n_c)\ 2jwc/R} \tag{1}$$

$\beta$ is zero at resonance and hence $\frac{n_r}{R^2} - n_c\, w^2 c^2$ is equal to zero

$$\therefore w = \frac{1}{CR}\sqrt{n_r/n_c}$$

For a twin-T, $$w_D = \frac{1}{CR}$$

Hence $w_M$ for a symmetrical multiple-T is given by $$w_M = w_D\sqrt{\frac{n_r}{n_c}} \tag{2}$$

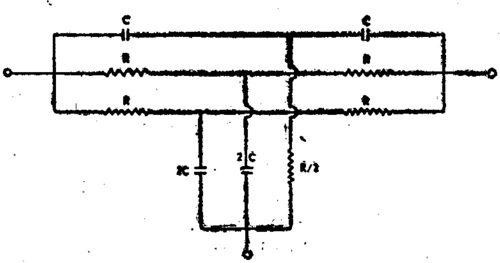


FIG. 1—Parallel Symmetrical Multiple Ts

The factor $\sqrt{\frac{n_r}{n_c}}$ is the multiplying factor for the frequency of any combination. The variation of $w_M$ with $n_r$ and $n_c$ has been studied theoretically and practically and it is found from the graphs that keeping $n_r$ constant and increasing the value of $n_c$, $\sqrt{\frac{n_r}{n_c}}$

decreases and the curves given in Fig. 2 crowd together as the value of $n_r$ is increased. When $n_c$ is kept constant and $n_r$ is increased, $\sqrt{\frac{n_r}{n_c}}$ increases and these curves, also shown in Fig. 2, come closer together for higher values of $n_c$. At the beginning of these curves, they appear to converge and at the end they appear to diverge, the divergence decreasing with the increasing values of $n_c$. From these curves, it is possible to calculate the resonant frequency of any combination of similar capacitance and resistance arms in parallel if the resistance and capacitance values are known.

## EXPERIMENTAL VERIFICATION

Ten sets of resistance and capacitance arms have been made carefully, selecting the same value components. The resonant frequency of such a twin-T combination selected is equal to 12·75 Kc/s. The resonant frequencies of several combinations, keeping the number of capacitance arms constant at 1, 2, 3......upto 10 and increasing the number of resistance arms from 1 to 10 are measured with the help of a cathode follower circuit having the output of the T-combination connected to its input and the output of the cathode follower to a bridge rectifier circuit. An audio signal generator output is given to the T-input and the resonant frequency corresponding to the minimum deflection in the bridge meter in noted. Knowing this frequency and the values of resistances and capacitances, $\sqrt{\frac{n_r}{n_c}}$ is calculated and plotted against $n_c$ or $n_r$, keeping one constant and varying the other.

## EVALUATION OF Q

Considering the RC symmetrical parallel-T combination as a series tuned circuit, its $Q$ can be found out. $\beta$ for the multiple-T considered is of the form $(P/P+jq)$ which is of the form $e^{r\theta} . \cot\theta = -\frac{P}{q}$. But for a series tuned circuit $\cot\theta = Q\left(r - \frac{1}{r}\right)$

where $r = \frac{w}{w_M}$

$$\therefore\ Q\left(r - \frac{1}{r}\right) = -\frac{n_r - n_c\ w^2c^2R^2}{2\,wcR\,(n_r + n_c)} \qquad (3)$$

Substituting the value $CR = \frac{1}{w_M}\sqrt{\frac{n_r}{n_c}}$,

we get $$Q = \frac{\sqrt{n_r\ n_c}}{2\,(n_r + n_c)} \qquad (4)$$

It is found from the formula that the $Q$ factor is entirely dependent on only the number $n_r$ of resistance arms and $n_c$ of the capacitance arms. It does not depend on the values of the components. Substituting $n_r = n_c = 1$ for a twin-T, $Q$ is equal to 0·25 which is

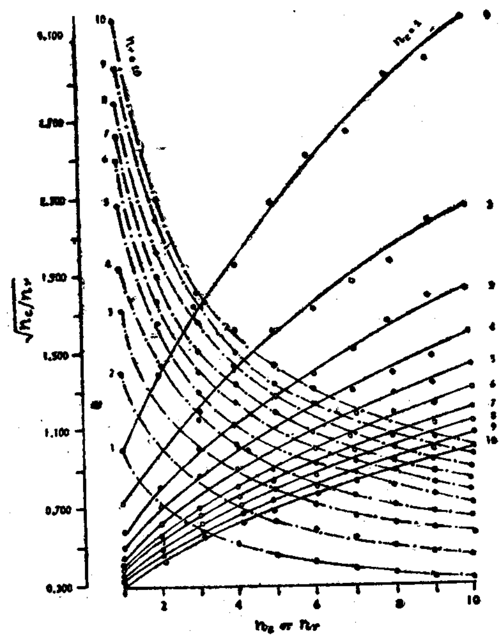

FIG. 2—Variation of $\sqrt{n_r/n_c}$ with $n_c$ or $n_r$.

the value obtained by other methods. It is found that by keeping $n_c$ constant and varying $n_r$ or *vice versa,* the $Q$ value of the combination increases to a maximum value equal to 0·25 when $n_r = n_c$ and decreases for a further increase in the value of $n_c$ or $n_r$.

## EXPERIMENTAL VERIFICATION

The experimental values of $Q$ are found out by knowing the resonant frequency of the symmetrical multiple-T as well as the resonant frequency of the twin-T and calculating the values from the formulae $Q = \dfrac{1}{2\left(\dfrac{w_M}{w_D} + \dfrac{w_D}{w_M}\right)}$

Fig. 3 gives the variation of $Q$ with $n_r$ or $n_c$ obtained practically. It is found from the curves, that as the value of $n_c$ or $n_r$ increases, the curves crowd together not only at the leading portions but also at their trailing portions though they spread out at the middle portion.

## TWO RESONANT FREQUENCIES FOR THE SAME Q

If $W_{M\,1}$ and $W_{M\,2}$ are the resonant angular frequencies of two similar symmetrical multiple-T Rc networks, then $w_{M1} = w_D \sqrt{\dfrac{n_{r\,1}}{n_{c\,1}}}$ and $w_{M2} = w_D \sqrt{\dfrac{n_{r\,2}}{n_{c\,2}}}$ where $n_{r\,1}$, $n_{c\,1}$ and $n_{r_2}$, $n_{c_2}$ are the numbers of resistance and capacitance arms in the first and second multiple-Ts respectively. The value of $Q$ for the first network is given by $Q_1 = \sqrt{\dfrac{n_{r_1}\; n_{c_1}}{2\,(n_{r\,1} + n_{c\,1}}}$

and for the second network by $Q_2 = \sqrt{\dfrac{n_{r_2}\; n_{c_2}}{2\,(n_{r_2} + n_{c_2})}}$. Since the $Q$ function is symmetric with respect to $n_r$ and $n_c$ , $Q_1 = Q_2$ when (*i*) $n_{r_1} = n_{r_2}$ and $n_{c_1} = n_{c_2}$ and (*ii*) $n_{r_1} = n_{c_2}$. and $n_c = n_{r\,2}$

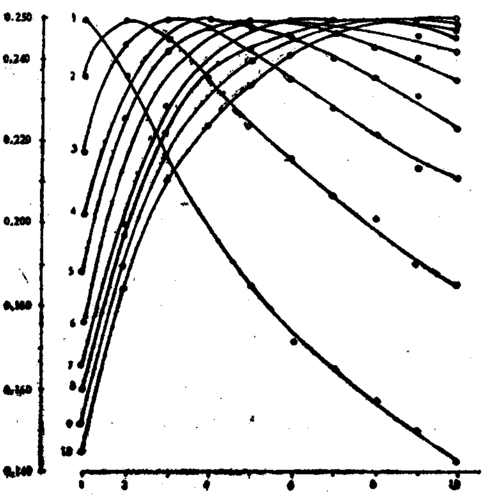

FIG. 3—Variation of $Q$ with $n_c$ or $n_r$.

Under such conditions

$$w_{M_1}\; w_{M_2} = w_D{}^2 \sqrt{\frac{n_{r_1}\; n_{r_2}}{n \quad n_{r_2}}} \tag{5}$$

but $\dfrac{n_{r_1}\; n_{c_2}}{n_{c_1} \cdot n_{c_2}} = 1$

$$\therefore \quad w_D = \sqrt{w_{M_1}\; w_{M_2}} \tag{6}$$

Therefore, there are two frequencies for which the $Q$ value is the same and their geometric mean is equal to that of the resonant frequency of the twin-T with whose arms the multiple-Ts are formed.

The ratio of the two frequencies is given by

$$\frac{W_{M1}}{w_{M2}} = \sqrt{\frac{n_{r2}\; n_{c2}}{n_{r1}\; n_{c2}}} \tag{7}$$

When $$n_{r_1} = n_{c_2} = n_{r_2} \text{ and } n_{c_1} = n_{r_2} = n_c$$

then $$\frac{w_{M1}}{w_{M2}} = \frac{n_r}{n_c} \qquad (8)$$

The separation between the two frequencies is given by

$$w_{M_1} - w_{M_2} = w_D \left[ \sqrt{\frac{n_r}{n_c}} - \sqrt{\frac{n_c}{n_r}} \right] \qquad (9)$$

From equation (4) $$Q = \frac{1}{2\left[\sqrt{\frac{n_r}{n_c}} + \sqrt{\frac{n_c}{n_r}}\right]}$$

Substituting the values of $\sqrt{\frac{n_r}{n_c}}$ and $\sqrt{\frac{n_c}{n_r}}$ the value of $Q$ is given by

$$Q = \frac{1}{4\left[1 + \left(\frac{w_{M1} - w_{M2}}{2\, w_D}\right)^2\right]^{\frac{1}{2}}} \qquad (10)$$

Whether $w_{M1} > w_{M2}$ or $< w_{M2}$, $w_{M1} \sim w_{M2}$ is always positive.

From the above considerations it is seen that the same value of $Q$ can be obtained for low as well as high frequencies.

*Example*: When $n_r = 10$ and $n_c = 1$, the value of $Q$ is 0·144 and the corresponding multiplying factor $\sqrt{\frac{n_r}{n_c}}$ for frequency is $\sqrt{10}$, whereas when $n_r = 1$ and $n_c = 10$, the value of $Q$ is again 0·144 and the correspondig multiplying factor for frequency is $1 \ \sqrt{10}$ When the resonant frequency of the Twin-T is 12·75 Kc/s, the frequencies at which $Q$ has a value equal to 0·144 are 40·32 Kc/s and 4·03 Kc/s. The ratio of the two frequencies is equal to 10 and the separation between the two frequencies having the same $Q$ increases with the increase of the resonant frequency of the twin-T with whose arms the multiple-Ts are formed. The ratio of the larger frequency to the smaller frequency is given by $n_r / n_c$. Further, it can be seen that the geometric mean of the two frequencies with the same $Q$ is equal to the resonant frequency of the twin-T.

## CONCLUSION

The phase and the amplitude minimum in the case of a symmetrical RC multiple-T occur at only one frequency unlike in the case of a multiple-T formed from two twin-Ts of slightly differing frequencies. The resonant frequency of any combination is dependant on the ratio of the number of resistance to capacitance arms keeping the CR value constant. The $Q$ value of such a combination is entirely dependant on the values of $n_r$ and $n_c$ and not on the values of components as can be seen from the formula. There are two frequencies for a given value of $Q$ and their geometric mean is equal to the resonant frequency of the twin-T. The $Q$ has a maximum value of 0·25 when $n_r = n_c$. The symmetrical mupltiple-T networks can be used as tuning elements in very low frequency low power circuits.

## ACKNOWLEDGEMENT

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