ON THE INTERACTION OF OBLIQUE SHOCK WAVE WITH A YAWED WEDGE

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(*Received* 27 *April* 1966)

When an oblique shock configuration encounters an infinite yawed wedge, it has been found that the region between the incident and the reflected shock, called intermediate region, remains undisturbed for any angle of yaw.

The phenomenon of diffraction of an oblique shock configuration meeting an infinite unyawed wedge has been studied by Srivastava & Ballabh[1]. It has been shown that the intermediate region, the region between the incident and reflected shocks remains undisturbed after crossing the corner. Later on, a brief review of this work has been done by Srivastava[2].

By the method of Chester[3], we have studied the interaction of an oblique shock with an infinite yawed wedge. It has been established that the intermediate region remains undisturbed after crossing the edge for any angle of yaw.

BASIC EQUATIONS AND MATHEMATICAL FORMULATION

Let the velocity of the incident shock be $U \sin \alpha_o$, where α_o is the angle of incidence and let the wedge be yawed through an angle β (Fig. 1). Then the point of intersection of the shock-wall line (line formed by the incident and reflected shock plane on the wall) and the leading edge travels with a velocity $U/\sin\beta$ along the leading edge.

Since the region of disturbance is always adjacent to the wall, it can be assumed that far away from the wall the intermediate region remains undistubed. In the region of uniform flow let the fluid velocity, pressure, density and sonic speed be denoted by q_1, p_1, ρ_1 and a_1 respectively and ahead of the incident shock let the corresponding quantities be O, p_o, ρ_o and a_o. The shock transition relations across the incident shock give

$$q_1 = \frac{5}{6}\,\bar{U}\left(1 - \frac{a_o^2}{\bar{U}^2}\right),\ p_1 = \frac{5}{6}\,\rho_o\left(\bar{U}^2 - \frac{1}{7}\,a_o^2\right),\ \rho_1 = 6\,\rho_o \Big/ \left(1 + 5\,a_o^2/\bar{U}^2\right) \quad (1)$$

where $\bar{U} = U \sin \alpha_o$ and γ (adiabatic index) $= 1{\cdot}4$.

Let a velocity $- U/\sin\beta$ along the leading edge be superimposed on the whole field so that the oblique shock configuration is reduced to rest. The uniform fluid velocity in the intermediate region for stationary shock configuration is then given by $\vec{V_1}$

where
$$V_1^2 = \frac{U^2}{\sin^2\beta} + q_1^2 - 2\,U\,q_1 \sin\alpha_o \quad (2)$$

Let us take the angle α_o^* for which

$$U^2 + 2\,q_1^2 - 2\,U\,q_1 \sin\alpha_o^* = a_1^2 \quad (3)$$

After some transformations (3) gives

$$\cot^2\alpha_o^* = \frac{6}{5}\,\frac{\eta - 1}{\eta^2}, \quad \eta \text{ being } \frac{\rho_1}{\rho_o} \quad (4)$$

The relation between η and the extreme angle of incidence, α_e (Bleakney & Taub[4]) is given by

$$x^2 (1 + \eta^2 x^2)^2 = (1 + \eta x^2) (\eta - 1) \{ 2.4 (\eta - 1) + 2 \} \{0.4 (1 + \eta x^2) + 2 \} \quad (5)$$

where $x = \cot \alpha_e$

It has been shown by Srivastava & Ballabh[1] that the curves given by (4) and (5) has only one real point of intersection at $\eta = 1$ for $1 \leqslant \eta \leqslant 6$ and

$$\frac{d}{d\eta} (\cot^2 \alpha_o^*)_{\eta = 1} < \frac{d}{d\eta} (x^2)_{\eta = 1}$$

This gives $\alpha_o^* > \alpha_e$

It means $$U^2 + q_1^2 - 2\, U q_1 \sin \alpha_o > a_1^2 \quad (6)$$

α_o being any angle of incidence for which regular reflection is possible.

Now,

$$\frac{U^2}{\sin^2 \beta} + q_1^2 - 2\, U q_1 \sin \alpha_o = U^2 \cot^2 \beta + U^2 + q_1^2 - 2\, U q_1 \sin \alpha_o$$

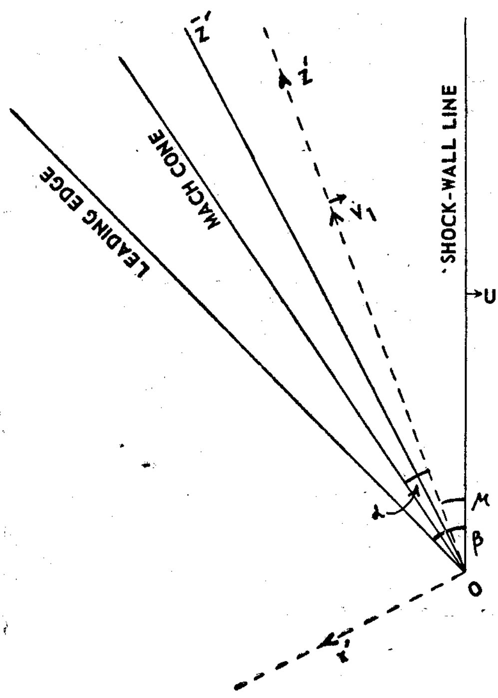

Fig. 1—Configuration for the oblique shock—yawed wedge interaction(i) solid lines n the plane of the wall and (ii) dashed lines in the $(x'—z')$ plane

Hence $V_1^2 > a_1^2$

for any value of β, the angle of yaw. Therefore it can be treated as a problem of steady flow within the intermediate region which is, in many respects, similar to the well-known cone field problem of Busemann[5].

Let us take rectangular cartesian coordinates (x', y', z') in the field with the origin at the point O, z'-axis in the direction of $\vec{V_1}$ and x'-axis in the plane containing the leading edge and the z'-axis (Fig. 1).

Let $\vec{V'_1}$, p_1', ρ_1', a_1' and S_1' be the velocity pressure, density, sonic speed and entropy respectively in the assumed region of perturbed flow within the intermediate region. The equations of conservation of mass, momentum and entropy are then

$$\left.\begin{aligned} \vec{V}_1 \cdot \nabla \rho_1' &= - \rho_1' \nabla \cdot \vec{V}_1' \\ (\vec{V}_1' \cdot \nabla)\vec{V}_1' &= - \frac{1}{\rho_1'} \nabla p_1' \\ \vec{V}_1' \cdot \nabla S_1' &= 0 \end{aligned}\right\} \quad (7)$$

where $\vec{V}_1' = (u_1', v_1', V_1 + \omega_1')$; u_1' v_1' and ω_1' being perturbations in velocities along x', y' and z'-axes respectively.

For supersonic flow it is assumed that the perturbations are small and (7) is linearised by using the transformations:

$$\left.\begin{aligned} x &= \frac{x'}{z' \tan\alpha}, \; y = \frac{y'}{z' \tan\alpha}, \; p = \frac{p_1' - p_1}{a_1 \rho_1 q_1} \\ \rho &= \frac{a_1(\rho_1' - \rho_1)}{\rho_1 q_1}, \; u = \frac{u_1'}{q_1 \cos\alpha}, \; v = \frac{v_1'}{q_1 \cos\alpha}, \; \omega = \frac{-\omega_1'}{q_1 \sin\alpha} \end{aligned}\right\} \quad (8)$$

giving a second order partial differential equation

$$\nabla^2 p = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) \quad (9)$$

The choice of the axes is such that the transformations are the same as those of Chester[3].

The characteristics of (9) are all tangents to the circle $x^2 + y^2 = 1$; no other line except an arc of the unit circle itself can be part of a boundary between a region where some solution of (9) is constant and one where it is not. It is reasonable to assume that the region of non-uniform flow is enclosed by this circle. The unit circle is equivalent to the Mach cone $x'^2 + y'^2 = z'^2 \tan^2\alpha$ the axis of which is along the z'-axis, its semi-angle being α.

The perturbations introduced in the intermediate region are then enclosed by the incident and the reflected shock, the wedge and the Mach cone having its vertex at O (Fig. 1). The axis of the Mach cone makes an angle μ with the shock wall line such that

$$\tan^2\mu = \frac{U^2 + q_1^2 - 2Uq_1 \sin\alpha_0}{U^2 \cot^2\beta} \quad (10)$$

If the angle between the axis of the Mach cone and its projection on the surface of the wall is denoted by ϕ, we get

$$\tan^2\phi = \frac{q_1^2 \cos^2\alpha_0}{U^2 \operatorname{cosec}^2\beta + q_1^2 \sin^2\alpha_0 - 2Uq_1 \sin\alpha_0} \quad (11)$$

The semi-angle of the Mach cone is given by the relation

$$\sin\alpha = \frac{a_1}{V_1} \quad (12)$$

DISCUSSION

First we examine the condition in the region adjacent to the wall. In case this region is disturbed, the shock-wall line will lie inside the Mach cone.

This means that $\mu < \alpha$ (13)

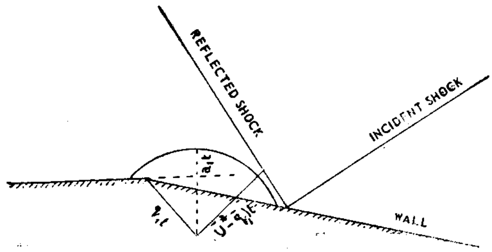

Fig. 2—Configuration in the x—y plane.

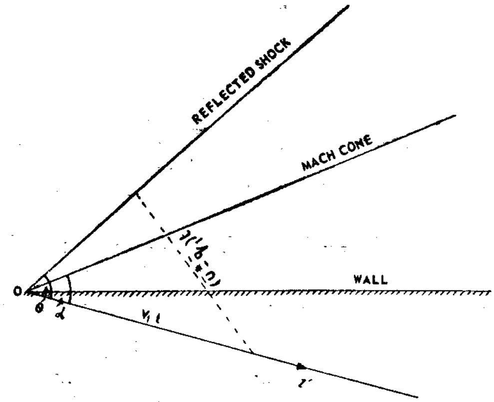

Fig. 3—Configuration in the plane perpendicular to the undiffracted reflected shock plane.

$$\text{or} \quad \frac{U^2 + q_1^2 - 2\,Uq_1 \sin\alpha_o}{U^2 \operatorname{cosec}^2\beta + q_1^2 - 2Uq_1 \sin\alpha_o} < \frac{a_1^2}{U^2 \operatorname{cosec}^2\beta + q_1^2 - 2\,Uq_1 \sin\alpha_o}$$

$$\text{or} \quad U^2 + q_1^2 - 2\,Uq_1 \sin\alpha_o < a_1^2 \tag{14}$$

But (14) contradicts the condition (6). This follows that the region in the neighbourhood of the wall remains undisturbed.

Secondly we add that the Mach cone cannot cross the reflected shock (Fig. 2 and 3). If the angle between the axis of the Mach cone and the plane of the reflected shock is denoted by θ, we obtain

$$\sin^2\theta = \frac{(\overset{*}{U} - \bar{q}_1)^2}{U^2 \operatorname{cosec}^2\beta + q_1^2 - 2\,Uq_1 \sin\alpha_o} \tag{15}$$

where $\bar{q}_1 = -\,q_1 \cos(\alpha_o + \alpha_2)$, $\overset{*}{\bar{U}} = U \sin\alpha_2$

and $\alpha_2 =$ the angle of reflection.

It can be verified that $\theta > \alpha$, since $(\overset{*}{U} - \bar{q}_1)$ the normal flow across the reflected shock in the intermediate region is greater than a_1. Therefore the intermediate region remains undisturbed when an oblique shock configuration encounters an infinite yawed wedge for any angle of yaw.

ACKNOWLEDGEMENTS

The authors are thankful to Dr. R. R. Aggarwal for his encouragement and the Director, Defence Science Laboratory, Delhi for his kind permission to publish this paper.

REFERENCES

1. SRIVASTAVA, R.S. & BALLABH, R., "Proc. of the First Congress on Theoretical and Applied Mechanics, India" (1955) 219.
2. SRIVASTAVA, R.S., "Diffraction of Oblique Shock Wave", (Aero Res. Coun. Nos. C.P. 612) 1962.
3. CHESTER, W., *Quart. J. Mech. Appl. Math.*, 6 (1953), 440.
BLEAKNEY, W. & TAUB, A.H., *Rev. Mod. Phys.*, 21 (1949), 584
BUSEMANN, A., *Luftfahrtfersch*, 20 (1943), 105