INTERACTION OF OBLIQUE BLAST WAVE

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The interaction of oblique shock configuration with an infinite yawed thin wedge has been considered. It has been established that Mach reflection occurs when uniform relative outflow from the reflected shock is supersonic and that diffraction occurs when the uniform relative outflow is sonic or subsonic.

Chester[1] has considered the interaction of a normal shock wave with the leading edge of a yawed wedge which is an extension of Lighthill's[2] theory for the diffraction of normal blast. The problem of diffraction of oblique shock wave past a small bend has been solved by Srivastava[3,4] on the basis of techniques developed by Lighthill. Srivastava's results[3] have now been extended to the case of yawed wedge following Chester's[1] method. Two regions are important for the consideration of this problem (*i*) region between the incident and reflected shock and (*ii*) region behind the reflected shock. It has been established that the region between the incident and reflected shock remains undisturbed for any angle of yaw for regular reflection of oblique shock wave. The region behind the reflected-diffracted shock wave has been considered here and it has been proved that Mach reflection occurs when uniform relative outflow from the reflected shock is supersonic. It is also found that diffraction occurs if the uniform relative outflow is sonic or subsonic.

NOTATIONS

$0, 1, 2$ = These suffixes stand respectively for (*i*) region ahead of the incident shock (*ii*) region between the incident and reflected shock and (*iii*) region behind the reflected shock.

q = fluid velocity

a = sound velocity

p = pressure

U = velocity of the shock line (line of intersection of incident and reflected shock planes)

S = entropy

α = semi-angle of the Mach cone

α_0 = angle of incidence

α_2 = angle of reflection

α_s = sonic angle

β = angle of yaw

ρ = density of the medium

μ = angle between the axis of Mach cone and shock line

γ = adiabatic index for air $= 1 \cdot 4$.

FORMULATION AND SOLUTION OF THE PROBLEM

When an oblique shock configuration (Fig. 1) meets the leading edge of an infinite yawed wedge such that the medium ahead of the incident shock is at rest ($q_o = 0$), the flow variables inside the incident and reflected shocks are given by

$$\left.\begin{aligned} q_1 &= \frac{5}{6}\,\overline{U}\left(1 - \frac{a_o^2}{\overline{U}^2}\right) \\ p_1 &= \frac{5}{6}\,\rho_o\left(\overline{U}^2 - \frac{a_o^2}{7}\right) \\ \rho_1 &= 6\rho_o \Big/ \left(1 + \frac{5a_o^2}{\overline{U}^2}\right) \\ \overline{U} &= U \sin\alpha_o \end{aligned}\right\} \quad (1)$$

as the region between the incident and reflected shocks remains undisturbed after the interaction of the oblique shock configuration with the wedge.

Behind the reflected shock there will be a region of uniform flow which is unaffected by the presence of the wedge. Fluid velocity, pressure and density in this region are given by

$$\left.\begin{aligned} \overline{q_2} &= \overline{q_1} + \frac{5}{6}\,(U^* - \overline{q_1})\left\{1 - \frac{a_1^2}{(U^* - \overline{q_1})^2}\right\} \\ p_2 &= \frac{5}{6}\,\rho_1\left\{(U^* - \overline{q_1})^2 - \frac{a_1^2}{7}\right\} \\ \rho_2 &= 6\rho_1 \Big/ \left\{1 + \frac{5a_1^2}{(U^* - \overline{q_1})^2}\right\} \\ \overline{q_1} &= -\,q_1 \cos(\alpha_o + \alpha_2) \\ U^* &= U \sin\alpha_2 \\ \overline{q_2} &= q_2 \sin\alpha_2 \end{aligned}\right\} \quad (2)$$

As the shock line is moving with velocity U the point of intersection of the leading edge and the shock line moves with velocity $U/\sin\beta$ along the leading edge of the wedge. The shock is brought to rest by imposing a velocity $U / \sin\beta$ along the leading edge in the direction opposite to the direction of motion of the shock front (Fig. 2).

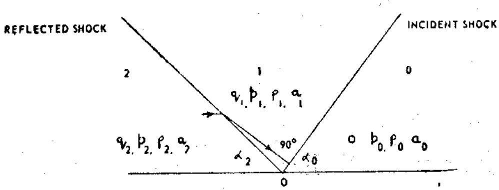

Fig. 1—Configuration for oblique shock reflection.

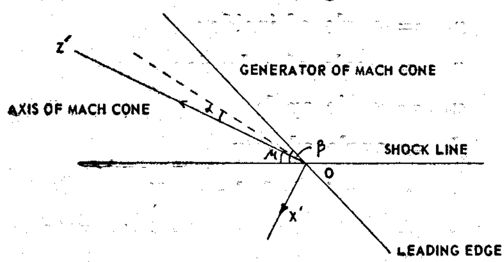

Fig. 2—Configuration in the x'—z' plane.

The velocity in the region of uniform flow behind the reflected shock is given by V_2, where

$$\left.\begin{aligned} V_2^2 &= \frac{U^2}{\sin^2 \beta} + q_2^2 - 2\,U\,q_2 \\ \text{or} \quad \frac{V_2^2}{a_2^2} &= \frac{U^2 \cot^2 \beta}{a_2^2} + \left(\frac{U - q_2}{a_2}\right)^2 \end{aligned}\right\} \tag{3}$$

Following Bleakney & Taub[5] it is noted that when the angle of incidence is less than the sonic angle, $\left(\frac{U - q_2}{a_2}\right)$ is always greater than unity and so from (3), V_2 is supersonic for all values of β. When $\frac{U - q_2}{a_2} = 1$, β has to be less than $\pi/2$ for V_2 to be supersonic. When $\frac{U - q_2}{a_2} < 1$ (*i.e.* when uniform relative flow from the reflected shock is subsonic) V_2 will be supersonic if

$$\sin^2 \beta < \frac{U^2}{a_2^2 + 2q_2\,U - q_2^2}$$

which gives the restriction on β. For given angle of incidence and incident shock strength, angle of reflection could be obtained from Bleakney and Taub's equations.[5] When α_2 is known, q_2 and a_2 can be found out from (1) and (2) and hence the restriction on β is found out.

The perturbations behind the reflected shock introduced by the presence of the wedge are confined to the region bounded by the reflected shock, the wedge and the Mach cone having its vertex as the point of intersection of shock line and leading edge. The axis of the Mach cone is in the direction of V_2 and makes an angle μ with the shock line such that

$$\tan \mu = \frac{U - q_2}{U \cot \beta} = \frac{a_2\,k}{U \cot \beta} \tag{4}$$

where $$k = \frac{U - q_2}{a_2}$$

The semi-angle of the Mach cone α is given by the relation

$$\sin \alpha = \frac{a_2 \sin \mu}{U - q_2} = \frac{\sin \mu}{k} \tag{5}$$

Let (x', y', z') be a coordinate system with origin at O, z'-axis along the axis of the cone; x'-axis perpendicular to Oz' lying in the plane of z' and the leading edge and y' perpendicular to the plane of x' and z'. If the flow variables in the perturbed region are given by V'_2, p'_2, ρ'_2, S'_2 the equations of conservation of mass, momentum and entropy are

$$\left.\begin{aligned} V'_2 \cdot \nabla\,\rho'_2 &= -\rho'_2\,\nabla \cdot V'_2 \\ (V'_2 \cdot \nabla)\,V'_2 &= -\frac{1}{\rho'_2}\,\nabla\,p'_2 \\ V'_2 \cdot \nabla\,S'_2 &= 0 \end{aligned}\right\} \tag{6}$$

Now if $V'_2 = (u_2, v_2, V_2 + \omega_2)$, p'_2, ρ'_2 differ by small amount from their values in the uniform region behind the reflected shock, equations (6) after linearisation and the use of Chester's transformations[1]

$$\left.\begin{aligned} x = \frac{x'}{z' \tan \alpha}, \quad y = \frac{y'}{z' \tan \alpha}, \quad p = \frac{p'_2 - p_2}{a_2 \rho_2 q_2} \\ \rho = \frac{a_2}{\rho_2 q_2}\left(\rho'_2 - \rho_2\right), \quad u = \frac{u_2}{q_2 \cos \alpha}, \quad v = \frac{v_2}{q_2 \cos \alpha}, \quad \omega = \frac{-\omega_2}{q_2 \sin \alpha} \end{aligned}\right\} \quad (7)$$

become

$$\left.\begin{aligned} x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} &= \frac{\partial p}{\partial x} \\ x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} &= \frac{\partial p}{\partial y} \\ \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}\right) p, \rho, \omega &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \end{aligned}\right\} \quad (8)$$

From (8), $\left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + 1 \right) \left(x \frac{\partial p}{\partial x} + y \frac{\partial p}{\partial y} \right) = \nabla^2 p$ (9)

This equation is hyperbolic if $x^2 + y^2 > 1$ and elliptic if $x^2 + y^2 < 1$. Thus $x^2 + y^2 = 1$ separates the regions of uniform and non-uniform flow. The disturbances are bounded by the unit circle, the reflected shock and the wedge.

The line of intersection of the incident and reflected shock is $x' \cot \mu = z'$ which, in the transformed (x, y) system, corresponds to $(\tan \mu / \tan \alpha, 0)$.

Now the condition $(\tan \mu / \tan \alpha) \gtrless 1$ with the help of (4) and (5) becomes

$$a_2^2 k^4 \gtrless \sin^2 \mu \, (a_2^2 k^2 + U^2 \cot^2 \beta)$$

or

$$a_2^2 k^4 \gtrless \frac{a_2^2 k^2 / U^2 \cot^2 \beta}{\left(1 + \dfrac{a_2^2 k^2}{U^2 \cot^2 \beta}\right)} (a_2^2 k^2 + U^2 \cot^2 \beta)$$

or

$$k \gtrless 1$$

or

$$\frac{U - q_2}{a_2} \gtrless 1$$

Therefore it is found that the uniform relative outflow from the reflected shock is supersonic, sonic or subsonic according as $(\tan \mu / \tan \alpha) \gtrless 1$.

When $(\tan\mu / \tan\alpha) > 1$, the point of intersection of the incident and reflected shock $\left(\dfrac{\tan\mu}{\tan\alpha}, 0\right)$ lies outside the unit circle which obviously means that the neighbourhood of the said point remains undisturbed which is not physically possible as this should be the region of maximum disturbance. Hence the shock line leaves the wedge and the reflection takes place in the medium itself.

The result can be established analytically as well. The equation of the reflected shock is $x' \cot\mu + y' \cot\alpha_2 \operatorname{cosec}\mu = z'$ for the shock line makes an angle μ with the z'-axis and the reflected shock makes an angle α_2 with the plane $y' = 0$.

This equation can be written as $(x + y \cot\alpha_2 \sec\mu) \cot\mu = \cot\alpha$ using the earlier transformations

$$x = \frac{x'}{z' \tan\alpha}, \qquad y = \frac{y'}{z' \tan\alpha}$$

Now using Busemann's transformations

$$\rho = \left\{1 - (1 - r^2)^{\frac{1}{2}}\right\} \Big/ r, \qquad x = r\cos\theta,\ y = r\sin\theta$$

equation of the reflected shock can finally be written as

$$\frac{2\rho}{1+\rho^2}\cos\theta = \frac{\tan\mu}{\tan\alpha} - \frac{2\rho}{1+\rho^2}\sin\theta\cot\alpha_2\sec\mu$$

and the wedge corresponds to $\theta = 0$.

The intersection of the reflected shock with the wedge is given by

$$\frac{2\rho}{1+\rho^2} = \frac{\tan\mu}{\tan\alpha}$$

which gives

$$\rho = \left[1 \pm \left\{1 - \left(\frac{\tan\mu}{\tan\alpha}\right)^2\right\}^{\frac{1}{2}}\right] \Big/ \left(\frac{\tan\mu}{\tan\alpha}\right)$$

Intersection of the shock with the wall is imaginary if $\left(\dfrac{\tan\mu}{\tan\alpha}\right) > 1$ which is the condition for the uniform relative outflow from the reflected shock to be supersonic. Therefore it is established that if the flow behind the reflected shock is supersonic the oblique shock configuration leaves the wedge and Mach reflection takes place.

When $(\tan\mu / \tan\alpha) \leqslant 1$, which is the case when the flow behind the reflected shock is subsonic or sonic, the reflected shock meets the wedge and diffraction takes place. In other words the shock line leaves the wall when angle of incidence is less than α_s and the shock meets the wedge when the angle of incidence is equal to or greater than the sonic angle.

Thus the problem of interaction of an oblique shock moving along the leading edge of a yawed wedge for all angles of incidence less than the sonic angle has been solved.

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