USE OF A SHEATH IN CONCEALING UNDERGROUND EXPLOSIONS

K. Viswanathan

Defence Science Laboratory, Delhi

(*Received* 10 *January* 1966)

A theoretical formula is obtained for the decoupling of the seismic signals from underground explosions due to the introduction of a sheath of a stronger material on the cavity walls. The elastic-elastic decoupling clearly enhances the value of the elastic-nonelastic decoupling factor already known. The theory has applications in the concealing of underground nuclear and chemical explosions.

The idea of concealing underground explosions was proposed recently by Latter *et al*[1]. Usually the tamped (coupled) explosion causes the surrounding medium to flow, crack and behave nonlinearly due to the large pressure released by the explosion. Such non-elastic behaviour causes increased coupling because the medium can undergo large displacements when it can flow as a liquid. If, however, the explosion is conducted in a cavity large enough to eliminate any non-elastic behaviour it is found that the associated seismic signals are considerably reduced. This can be explained thus : when a disturbance propagates through the earth, the high-frequency components of the waves are absorbed by the earth and only the low-frequencies propagate to large distances[2]. The amplitude of the low-frequency seismic waves at large distances is directly proportional to the product of the critical stress and the cube of the corresponding critical radius. For the tamped case, where non-elastic behaviour occurs, the critical pressure is attained at a larger radius than in the perfectly elastic case and hence the radiation of seismic signals at any particular distance is larger.

Using this theory, Latter *et al*[1]. showed that the wave amplitude at any distance from a tamped nuclear shot in tuff is 300 times the wave amplitude for the same shot from a cavity in salt. In order to test this theory the U. S. Atomic Energy Commission sponsored the Project Cowboy and several experimental tests were conducted with high explosives in salt, the results of which were reported by Murphy[3] and Herbst *et al*[4]. All these experiments confirm the decoupling predicted though their values differ on account of their dependence on the various parameters of the experiment.

The investigations so far have answered only one question : how far can we decouple the seismic waves from an underground explosion by choosing a proper cavity (with the optimum radius) in a given medium? An interesting conclusion of Latter *et al*[1]. in this connection is that once the hole exceeds some critical size it does not pay to make it any bigger.

In this paper larger decoupling has been achieved by introducing a sheath of stronger material around the cavity. The formula derived indicates clearly that further decoupling is possible. It has been found that this decoupling will be more if the material of the sheath is stronger and if the ratio of its inner and outer radii is smaller.

THEORY

Consider an explosion of yield W in a spherical cavity of radius a in a medium consisting of a sheath M_1, of internal radius a welded at its external radius equal to b to an otherwise infinite medium M_2. Both M_1 and M_2 are assumed to be homogeneous, isotropic and perfectly elastic. Also a is assumed sufficiently large so that there is no non-elastic behaviour in the media considered. It is assumed as usual that the energy W is suddenly distributed uniformly over the volume of the cavity giving rise to a step-function pressure on the wall given by

$$p = p(a) = 3(\gamma - 1)\ W/4\ \pi\ a^3 \tag{1}$$

where γ is the ratio of specific heats of the gas in the cavity.

Spherical symmetry being assumed the only non-vanishing displacements in M_1 and M_2 are radial and are solutions of the equations

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2} \right) u_j = \frac{1}{\alpha_j^2}\ \frac{\partial^2\ u_j}{\partial\ t^2}\ ;\ (j = 1, 2) \tag{2}$$

where $\alpha_j = \left(\frac{\lambda_j + 2\ \mu_j}{\rho_j} \right)^{\frac{1}{2}}$ are the respective speeds of sound and other symbols have their usual meanings.

Defining the Fourier transform $\hat{u}_j\ (\omega)$ of u_j by the representation

$$u_j = \int_{-\infty}^{\infty} \hat{u}_j\ (\omega)\ e^{i w t}\ d\omega \tag{3}$$

equations (2) become

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2} + k_j^2 \right) \hat{u}_j\ (\omega) = 0 \tag{4}$$

where $$k_j = \omega/\alpha_j$$

The proper solutions of (4) in M_1 and M_2, representing outgoing waves in M_2 are

$$\left.\begin{aligned} \hat{u}_1\ (\omega) &= A\ \frac{\partial}{\partial r}\left(\frac{e^{i k_1 r}}{r} \right) + B\ \frac{\partial}{\partial r}\left(\frac{e^{-i k_1 r}}{r} \right), \\ \hat{u}_2\ (\omega) &= C\ \frac{\partial}{\partial r}\left(\frac{e^{-i k_2 r}}{r} \right) \end{aligned}\right\} \tag{5}$$

where A, B and C are to be determined from the conditions on $r = a$ end $r = b$. These conditions, in the usual notation, are

$$\left.\begin{aligned} (\sigma_r)_1 &= -p\ H\ (t) \quad , & r &= a \\ (\sigma_r)_1 &= (\sigma_r)_2 \quad , & r &= b \\ u_1 &= u_2 \quad , & r &= b \end{aligned}\right\} \tag{6}$$

and

where $$H\ (t) = \begin{cases} 0\ , & t < 0 \\ 1\ , & t > 0 \end{cases}$$

The Fourier transforms of (6) are

$$\left.\begin{aligned} (\hat{\sigma}_r)_1 &= p\ i\ /\ 2\ \pi\ \omega \quad , & r &= a \\ (\hat{\sigma}_r)_1 &= (\hat{\sigma}_r)_2 \quad , & r &= b \\ \hat{u}_1\ (\omega) &= \hat{u}_2\ (\omega) \quad , & r &= b \end{aligned}\right\} \tag{7}$$

We obtain $\overset{\Delta}{\sigma}_r$ from the formula

$$\overset{\Delta}{\sigma}_r = \frac{\lambda}{r^2}\frac{\partial}{\partial r}(r^2\,\hat{u}) + 2\,\mu\,\frac{\partial \hat{u}}{\partial r} \tag{8}$$

Therefore from (5), we get

$$\left.\begin{aligned}\left(\overset{\Delta}{\sigma}_r\right)_1 &= \frac{4\,\mu_1}{\alpha_1^2\, r}\Big[A\, e^{i k_1 r}\left(\frac{\alpha_1^2}{r^2} - \frac{i\,\omega\,\alpha_1}{r} - \frac{\lambda_1 + 2\,\mu_1}{4\,\mu_1}\,\omega^2\right) \\ &\quad + B\, e^{-i k_1 r}\left(\frac{\alpha_1^2}{r^2} + \frac{i\,\omega\alpha_1}{r} - \frac{\lambda_1 + 2\,\mu_1}{4\,\mu_1}\,\omega^2\right)\Big] \\ \left(\overset{\Delta}{\sigma}_r\right)_2 &= \frac{4\,\mu_2}{\alpha_2^2\, r}\, C\, e^{-i k_2 r}\left(\frac{\alpha_2^2}{r^2} + \frac{i\,\omega\,\alpha_2}{r} - \frac{\lambda_2 + 2\,\mu_2}{4\,\mu_2}\,\omega^2\right)\end{aligned}\right\} \tag{9}$$

Applying (7) in (5) and (9) we get

$$A\, f_1\,(a) + B\, g_1\,(a) = p\ a\ i\ \alpha_1^2/8\,\pi\,\mu_1\,\omega \tag{10}$$

$$A\, f_1\,(b) + B\, g_1\,(b) = \mu_2\,\alpha_1^2\, g_2\,(b)\, C\,/\,\mu_1\,\alpha_2^2 \tag{11}$$

and

$$A\,\frac{\partial}{\partial b}\left(\frac{e^{i k_1 b}}{b}\right) + B\,\frac{\partial}{\partial b}\left(\frac{e^{-i k_1 b}}{b}\right) = C\,\frac{\partial}{\partial b}\left(\frac{e^{-i k_2 b}}{b}\right) \tag{12}$$

where

$$\left.\begin{aligned} f_j\,(x) &= e^{i k_j x}\left(\frac{\alpha_j^2}{x^2} - \frac{i\,\omega\,\alpha_j}{x} - \frac{\lambda_j + 2\,\mu_j}{4\,\mu_j}\,\omega^2\right) \\ g_j\,(x) &= e^{-i k_j x}\left(\frac{\alpha_j^{\,2}}{x^2} + \frac{i\,\omega\,\alpha_j}{x} - \frac{\lambda_j + 2\,\mu_j}{4\,\mu_j}\,\omega^2\right)\end{aligned}\right\} \tag{13}$$

On solving (10), (11) and (12), we obtain the following value for C:

$$C = \frac{p\,a\,\alpha_1^2}{8\,\pi\, i\,\omega\,\triangle}\left\{f_1\,(b)\,\frac{\partial}{\partial\, b}\left(\frac{e^{-i k_1 b}}{b}\right) - g_1\,(b)\,\frac{\partial}{\partial\, b}\left(\frac{e^{i k_1 b}}{b}\right)\right\} \tag{14}$$

where

$$\begin{aligned}\triangle = \Big[\,\mu_1\Big\{f_1\,(a)\, g_1\,(b) - f_1\,(b)\, g_1\,(a)\Big\}\frac{\partial}{\partial\, b}\left(\frac{e^{-i k_2 b}}{b}\right) + \frac{\mu_2\,\alpha_1^2}{\alpha_2^2}\, g_2\,(b) \\ \Big\{g_1\,(a)\,\frac{\partial}{\partial\, b}\left(\frac{e^{i k_1 b}}{b}\right) - f_1\,(a)\,\frac{\partial}{\partial\, b}\left(\frac{e^{-i k_1 b}}{b}\right)\Big\}\Big]\end{aligned} \tag{15}$$

The displacements in M_2 can then be obtained from (5) in the form

$$\begin{aligned}\hat{u}_2\,(\omega) &= -\,C\left(\frac{i\,k_2}{r} + \frac{1}{r^2}\right) e^{-i k_2 r} \\ &= \frac{p\,a\,i\,\alpha_1^2}{8\,\pi\,\omega\,\triangle}\left\{f_1\,(b)\,\frac{\partial}{\partial\, b}\left(\frac{e^{-i k_1 b}}{b}\right) - g_1\,(b)\,\frac{\partial}{\partial\, b}\left(\frac{e^{i k_1 b}}{b}\right)\right\}\times \\ &\quad\left(\frac{i\,k_2}{r} + \frac{1}{r^2}\right) e^{-i k_2 r}\end{aligned} \tag{16}$$

Approximation for small frequencies

The waves of interest are the small frequency components as the high frequencies normally get absorbed by the earth. The important range for ω, for the seismic network considered by the Geneva Conference on test suspension, is reported to be from 0 to about 6 $\sec^{-1}$ corresponding to frequencies below 1 cps. For such low frequencies (16) can be simplified on the following lines:

$$\left\{ f_1(b) \frac{\partial}{\partial b}\left(\frac{e^{-ik_1 b}}{b} \right) - g_1(b) \frac{\partial}{\partial b}\left(\frac{e^{ik_1 b}}{b} \right) \right\}$$

$$= (\omega_1^2 - i\,\omega_1\,\omega - K_1\,\omega^2)\left(\frac{-i\,\omega}{\alpha_1\,b} - \frac{1}{b^2} \right) - (\omega_1^2 + i\,\omega_1\,\omega - K_1\,\omega^2)\left(\frac{i\,\omega}{\alpha_1\,b} - \frac{1}{b^2} \right)$$

$$= \frac{2\,i\,K_1\,\omega^3}{\alpha_1\,b}, \tag{17}$$

where $\omega_1 = \dfrac{\alpha_1}{b}$ and $K_1 = \dfrac{\lambda_1 + 2\,\mu_1}{4\,\mu_1}$

Also

$$\left\{ f_1(a)\,g_1(b) - f_1(b)\,g_1(a) \right\} \frac{\partial}{\partial b}\left(\frac{e^{-ik_2 b}}{b} \right)$$

$$= 2\,i \Big[\{ -\omega_o^2\,\omega_1^2 + [\,K_1(\omega_o^2 + \omega_1^2) - \omega_o\,\omega_1\,]\,\omega^2 - K_1^2\,\omega^4 \} \sin k_1(b-a)$$

$$+ (\omega_o\,\omega_1\,\omega + \omega^3 K_1)(\omega_o - \omega_1)\cos k_1(b-a) \Big] \frac{\partial}{\partial b}\left(\frac{e^{-ik_2 b}}{b} \right)$$

$$\simeq -\frac{2\,i\,\omega^3\,\alpha_1}{a^2\,b}\,(1 - a_o^3)\,(K_1 - \tfrac{1}{3})\,e^{-ik_2 b} \tag{18}$$

where $\omega_o = \alpha_1/a$, $a_o = a/b$

Similarly

$$\left\{ g_1(a) \frac{\partial}{\partial b}\left(\frac{e^{ik_1 b}}{b} \right) - f_1(a) \frac{\partial}{\partial b}\left(\frac{e^{-ik_1 b}}{b} \right) \right\} \frac{\alpha_1^2}{\alpha_2^2}\,g_2(b)$$

$$\simeq \frac{2\,i\,\omega^3\,\alpha_1}{a^2\,b}\Big[(1 - a_o^3)(K_1 - \tfrac{1}{3}) - K_1 \Big] e^{-ik_2 b} \tag{19}$$

Then from (15), (18) and (19) we get

$$\Delta = \frac{-2\,i\,\omega^3\,\alpha_1\,\mu_2}{a^2\,b}\Big[K_1 + (\mu_o - 1)(K_1 - \tfrac{1}{3})(1 - a_o^3) \Big] e^{-ik_2 b} \tag{20}$$

where $\mu_0 = \mu_1/\mu_2$

The amplitude of the distant field (for $r >> b$) can then be obtained from (16), (17) and (20) in the form

$$\left| \hat{u}_2(\omega) \right| = \frac{p\,a^3}{8\,\pi\,\mu_2\,\alpha_2\,r} \times \frac{K_1}{[\,K_1 + (\mu_o - 1)(K_1 - \frac{1}{3})(1 - a_o^3)\,]} \tag{21}$$

Now suppose that the same explosion is conducted inside the medium M_2 in the absence of M_1. Then if the radius R of the cavity in M_2 be again large enough to avoid non-elastic behaviours, the corresponding amplitude of the displacement for large distances from the centre of the cavity in this case would be[2]

$$\left| \hat{u}(\omega) \right| = \frac{p(R)\,R^3}{8\,\pi\,\mu_2\,\alpha_2\,r} \tag{22}$$

Since the yield is the same, we have by formula (1)

$$p(R) = \frac{3(\gamma - 1)W}{4\pi R^3}$$

$$\therefore p(a)\, a^3 = p(R)\, R^3 = \frac{3(\gamma - 1)W}{4\pi} \tag{23}$$

The elastic-elastic decoupling factor D in M_2 due to the introduction of the sheath material M_1 is then found to be

$$D = \left| \frac{\hat{u}(\omega)}{\hat{u}_2(\omega)} \right| = 1 + \frac{(\mu_\circ - 1)(K_1 - \frac{1}{3})(1 - a_\circ{}^3)}{K_1} \tag{24}$$

Clearly $D \geqslant 1$ since $a_\circ = \dfrac{a}{b} \leqslant 1$, $\mu_\circ = \dfrac{\mu_1}{\mu_2} \geqslant 1$ and

$$(K_1 - \tfrac{1}{3}) = \frac{3\lambda_1 + 2\mu_1}{12\mu_1} > 0.$$

DISCUSSION

It is interesting to note that a simple cubical relation (24) exists between the decoupling factor D and the ratio of the inner and interface radii for the elastic problem considered. The decoupling for given materials M_1 and M_2 depends only on the ratio $\dfrac{a}{b}$ and, therefore, given any value of $\dfrac{a}{b}$ it is always sufficient to take the critical value for a as there is no advantage in employing larger cavities. It is further observed that the critical value of the internal radius b of the medium M_2 in the present theory would necessarily be less than the critical radius in the absence of the sheath. This fact can be used to avoid the difficulties of obtaining big holes in the medium concerned.

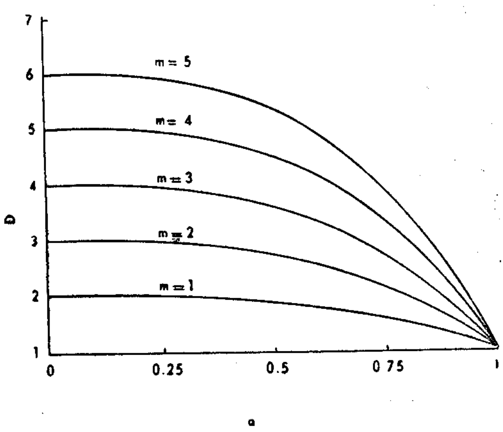

Fig. 1—Variation of the decoupling factor D with $a_\circ = \dfrac{a}{b}$ and $m = (K_1 - \frac{1}{3})(\mu_\circ - 1)/K_1$

Numerical values for the decoupling factor D can be obtained from (24) for various combinations of the materials M_1 and M_2 and for various values of $\dfrac{a}{b}$. Graphical representation of (24) for certain chosen cases are given in Fig. 1.

As a special example we may consider the Rainier explosion discussed by Latter *et al*[1]. The authors obtained an elastic-nonelastic decoupling factor of 50 for the explosion in tuff. If in our theory we take M_1 to be salt and M_2 to be the Rainier medium we have $K_1 = \frac{3}{4}$, $\mu_\circ = 5$ and the elastic-elastic decoupling factor $D = 1{\cdot}6$ for the case $\frac{a}{b} = \frac{9}{10}$. Thus the total decoupling factor for the actual Rainier shot, when the explosion takes place in tuff with a salt layer as sheath $\left(\text{with } \frac{a}{b} = \frac{9}{10}\right)$ will be $50 \times 1 \cdot 6 = 80$. In other words, for this particular case, we get as much as 60% increase as the elastic-elastic decoupling due to the introduction of a sheath. We can anticipate reasonably larger decoupling by choosing a stronger material for the sheath and by taking smaller values for the ratio $\frac{a}{b}$.

ACKNOWLEDGEMENTS

I wish to thank Dr. V. R. Thiruvenkatachar for his interest and criticism during the course of this work. My thanks are also due to the Director, Defence Science Laboratory, Delhi for his kind permission to publish this paper.

REFERENCES

1. LATTER, A.L., LELEVIER, R.E., MARTINELLI, E.A. & MCMILLAN, W.G., *J. Geophys. Res.* **66** (1961), 943.
2. LATTER, A.L., MARTINELLI, E.A. & TELLER, E., *Phys. Fluids*, **2** (1959), 280.
3. MURPHEY, B.F., *J. Geophys, Res.* **66** (1961), 947.
4. HERBST, R.F., WERTH, G.C. & SPRINGER, D.L., *ibid*, **66** (1961), 959.