STABILITY CRITERIA OF PROJECTILE DURING FLIGHT IN VARYING DENSITY ATMOSPHERE

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Stability of the projectile in a varying density atmosphere is investigated under general set up of aerodynamic forces and torques. For circular trajectories, necessary and sufficient stability criteria are developed. Relationship between stability and optimality is outlined. The results have been illustrated by numerical examples.

NOMENCLATURE

M = mass of projectile

$\dot{S}$ = projectile velocity

$\ddot{S}$ = projectile acceleration

θ = angle which the direction of motion of the projectile makes with abscissa

ϕ = angle between abscissa and longitudinal axis of the projectile

δ = $\phi - \theta$ = angle of yaw

ρ = atmospheric density

d = cross-sectional diameter of the projectile

H = altitude

β = altitude factor

K_D, K_L, K_M, K_S, K_H = aerodynamic coefficients

λ = frequency parameter

μ = gravitational constant times mass of the Earth

R = radius of the Earth

K = radius of gyration of projectile about transverse axis through its centre of gravity

Subscripts :

1 = indicates reference values

0 = indicates small increments

Stability problems are of paramount importance and have been attracting many investigators in various fields of engineering and technology. During operation of a projectile, certain forces (disturbing forces) may occur which are not fully considered in the mathematical analysis of the system in action. For the successful operation of any projectile, the system should remain stable under these disturbing forces and it is, therefore, necessary to consider stability.

Considering only lift and drag, Morth & Speyer[1] have investigated the deviation of the vehicle from the planar-glide path by taking the ideal case of no yaw. However,

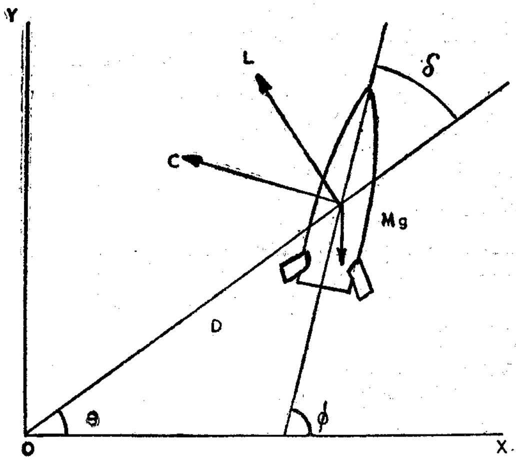

Fig. 1

Nielsen & Synge[2] have shown that the inevitable existence of yaw loads to a complex set of aerodynamic forces and torques which act on the projectile (e.g. rocket) and affect its stability considerably.

This paper discusses the stability of the projectile under the complete set of Nielsen & Synge aerodynamic forces and torques. The stability criteria for projectile motion in general and for circular trajectory in particular have been derived taking into consideration variation of atmospheric density and gravitational force with altitude.

EQUATIONS OF MOTION

Taking the local horizontal in the direction of motion as positive direction of abscissa and upward vertical as positive direction of ordinate (Fig. 1), the equations of motion of the projectile can be written[2] as

$$M \ddot{S} = - \frac{M\mu \sin \theta}{(R+H)^2} - C \sin \delta - D$$

$$M \dot{S} \dot{\theta} = - \frac{M\mu \cos \theta}{(R+H)^2} + C \cos \delta + L$$

$$MK^2 \ddot{\phi} = - K_M \rho d^3 \dot{S}^2 \sin (\phi - \theta) - K_H \rho d^4 \dot{S} \dot{\phi}$$

where

$D = K_D \rho d^2 \dot{S}^2$ = drag acting through the centre of gravity of the projectile. It opposes the direction of motion.

$L = K_L \rho d^2 \dot{S}^2 \sin \delta$ = lift acting through the centre of gravity. It acts in the direction perpendicular to the direction of motion of the projectile.

$C = K_S \rho d^3 \dot{S} \dot{\phi}$ = cross spin force or pitching force. This acts through the centre of gravity and is perpendicular to the longitudinal axis of the projectile in the direction of ϕ increasing when $\dot{\phi}$ is positive.

$K_M \rho d^3 \dot{S}^2 \sin \delta$ = restoring moment tending to decrease the numerical value of yaw.

$K_H \rho d^4 \dot{S} \dot{\phi}$ = damping moment tending to decrease the numerical value of ϕ. Substituting the values of C, D and L in the above equations we have

$$M\ddot{S} = -\frac{M\mu \sin\theta}{(R+H)^2} - K_S\ \rho\, d^3 \dot{S}\,\dot{\phi} \sin(\phi-\theta) - K_D\ \rho\, d^2 \dot{S}^2$$

$$M\ \dot{S}\dot{\theta} = -\frac{M\mu \cos\theta}{(R+H)^2} + K_S\ \rho\ d^3 \dot{S}\,\dot{\phi} \cos(\phi-\theta) + K_L\ \rho\ d^2 \dot{S}^2 \sin(\phi-\theta) \qquad (1)$$

$$M K^2 \ddot{\phi} = -K_M\ \rho\ d^3 \dot{S}^2 \sin(\phi-\theta) - K_H\ \rho\ d^4 \dot{S}\,\dot{\phi}$$

where K_D, K_L, K_S, K_M, and K_H are aerodynamic coefficients. Also altitude-velocity relationship is given by

$$\dot{S}\sin\theta - \dot{H} = 0 \qquad (2)$$

Variation of atmospheric density with altitude can be assumed as

$$\rho = \rho_1\ e^{-\beta H} \qquad (3)$$

Now differentiating (1) where S_o, θ_o, ϕ_o and ρ_o are corresponding small increments and converting ρ_o into H_o by the relation $\rho_o = -\beta\rho H_o$, the perturbation equations are expressed as

$$M\ddot{S}_o + \dot{S}_o\left[2K_D\,\rho\, d^2\dot{S} + K_S\, d^3\rho\,\dot{\phi}\sin(\phi-\theta)\right] + \theta_o\left[\frac{M\mu\cos\theta}{(R+H)^2} - K_S\ \rho\, d^3\dot{S}\ \dot{\phi}\cos(\phi-\theta)\right]$$

$$+ \dot{\phi}_o\left[K_S\ \rho\ d^3\ \dot{S}\sin(\phi-\theta)\right] + \phi_o\left[K_S\ \rho\ d^3\ \dot{S}\,\dot{\phi}\cos(\phi-\theta)\right]$$

$$-\beta\ \rho\ \ H_o\left[K_D d^2\dot{S}^2 + K_S\ d^3\dot{S}\,\dot{\phi}\sin(\phi-\theta) + \frac{2\ M\ \mu\sin\theta}{\beta\rho(R+H)^3}\right] = 0 \qquad (4)$$

$$\dot{S}_o\ \left[M\ \dot{\theta} - 2\ K_L\ \ \rho\ d^2\dot{S}\sin(\phi-\theta) - K_S\ \rho\ d^3\ \dot{\phi}\cos(\phi-\theta)\right] + \dot{\theta}_o\ \left[M\ \dot{S}\right]$$

$$+\ \theta_o\left[-\frac{M\mu\sin\theta}{(R+H)^2} + K_L\,\rho\, d^2\dot{S}^2\cos(\phi-\theta) - K_S\ \rho\ d^3\ \dot{S}\,\dot{\phi}\sin(\phi-\theta)\right]$$

$$+\ \dot{\phi}_o\left[-K_S\,\rho\, d^3\,\dot{S}\cos(\phi-\theta)\right] + \phi_o\left[-K_L\,\rho\, d^2\dot{S}^2\cos(\phi-\theta) + K_S\,\rho\, d^3\,\dot{S}\,\dot{\phi}\right.$$

$$\left.\sin(\phi-\theta)\right] + \beta\rho H_o\left[K_L\ d^2\dot{S}^2\sin(\phi-\theta) + K_S\ d^3\,\dot{S}\,\dot{\phi}\cos(\phi-\theta) - \frac{2M\mu\ \cos\theta}{\beta\rho(R+H)^3}\right] = 0 \qquad (5)$$

$$\dot{S}_o\ \left[2\,K_M\,\rho\, d^3\,\dot{S}\sin(\phi-\theta) + K_H\,\rho\, d^4\,\dot{\phi}\right] + \theta_o\ \left[-K_M\rho\ d^3\ \dot{S}^2\cos(\phi-\theta)\right]$$

$$+\ \ddot{\phi}_o\ \left[M\ K^2\right] + \dot{\phi}_o\ \left[K_H\,\rho\ d^4\ \dot{S}\right] + \phi_o\ \left[K_M\rho\ d^3\ \dot{S}^2\cos(\phi-\theta)\right]$$

$$-\ \beta\ \rho\ H_o\ \left[K_M d^3\ \dot{S}^2\sin(\phi-\theta) + K_H d^4\ \dot{S}\ \dot{\phi}\right] = 0 \qquad (6)$$

and from (2)

$$\dot{S}_o\ [\sin\theta] + \theta_o\ [\ \dot{S}\cos\theta\ ] - \dot{H}_o = 0 \qquad (7)$$

Putting in equations (4) to (7)

$$S_o = S_1 e^{\lambda t}, \ \theta_o = \theta_1 e^{\lambda t}, \ \phi_o = \phi_1 e^{\lambda t} \text{ and } H_o = H_1 e^{\lambda t}$$

The frequency equation in the determinant form, after dropping out λ common from the first column, can be written as

$$\begin{vmatrix} A_{11}\lambda + A_{12} & B_{11} & C_{11}\lambda + C_{12} & D_{11} \\ A_{21} & B_{21}\lambda + B_{22} & C_{21}\lambda + C_{22} & D_{21} \\ A_{31} & B_{31} & C_{31}\lambda^2 + C_{32}\lambda + C_{33} & D_{31} \\ A_{41} & B_{41} & 0 & -\lambda \end{vmatrix} = 0 \quad (8)$$

where

$$A_{11} = M, \quad B_{11} = \left[\frac{M\mu \cos\theta}{(R+H)^2} - K_S \ \rho \ d^3 \ \dot{S} \dot{\phi} \cos(\phi - \theta) \right]$$

$$A_{12} = [\, 2\, K_D \rho\, d^2 \dot{S} + K_S \rho\, d^3 \dot{\phi} \sin(\phi - \theta)], \quad B_{21} = MS$$

$$A_{21} = [\, M \dot{\theta} - 2K_L \rho \ d^2 \dot{S} \sin(\phi - \theta) \ - \ K_S \rho \ d^3 \dot{\phi} \ \cos(\phi - \theta)]$$

$$A_{31} = [2\, K_M \ \rho \ d^3 \dot{S} \sin(\phi - \theta) + K_H \ \rho\, d^4 \dot{\phi}]$$

$$B_{22} = \left[-\frac{M\mu \sin\theta}{(R+H)^2} + K_L \rho\, d^2 \dot{S}^2 \cos(\phi - \theta) - K_S \rho\, d^3 \dot{S} \dot{\phi} \sin(\phi - \theta) \right]$$

$$A_{41} = \sin\theta, \ B_{31} = [-K_M \ \rho \ d^3 \dot{S}^2 \cos(\phi - \theta)]$$

$$B_{41} = \dot{S} \cos\theta \quad (9)$$

$$C_{11} = [K_S \ \rho \ d^3 \dot{S} \sin(\phi - \theta)], D_{11} = -\beta\rho \left[K_D\, d^2 \dot{S}^2 + K_S \ d^3 \dot{S} \dot{\phi} \sin(\phi - \theta) + \frac{2\, M\mu \sin\theta}{\beta\rho\, (R+H)^3} \right]$$

$$C_{12} = [K_S \ \rho \ d^3 \dot{S} \dot{\phi} \cos(\phi - \theta)]$$

$$D_{21} = \beta\rho \left[K_L \ d^2 \dot{S}^2 \sin(\phi - \theta) + K_S \ d^3 \dot{S} \dot{\phi} \ \cos(\phi - \theta) - \frac{2\, M\, \mu \cos\theta}{\beta\, \rho\, (R+H)^3} \right]$$

$$C_{21} = [-K_S \ \rho \ d^3 \dot{S} \cos(\phi - \theta)], \quad D_{31} = -\beta\rho\, [K_M\, d^3 \dot{S}^2 \sin(\phi - \theta) + K_H\, d^4 \dot{S} \dot{\phi}]$$

$$C_{22} = [K_S \ \rho \ d^3 \dot{S} \dot{\phi} \sin(\phi - \theta) - K_L \ \rho \ d^2 \dot{S}^2 \cos(\phi - \theta)], \quad C_{31} = MK^2$$

$$C_{32} = K_H \rho\, d^4 \dot{S}, \quad C_{33} = K_M \ \rho \ d^3 \ \dot{S}^2 \cos(\phi - \theta)$$

(8) can be reduced to

$$Q_o \lambda^5 + Q_1\lambda^4 + Q_2\lambda^3 + Q_3\lambda^2 + Q_4\lambda + Q_5 = 0 \quad (10)$$

where

$$
\begin{aligned}
Q_o &= A_{11}\, B_{21}\, C_{31} \\
Q_1 &= A_{12}\, B_{21}\, C_{31} + A_{11}\, B_{21}\, C_{32} + A_{11}\, B_{22}\, C_{31} \\
Q_2 &= [A_{11}\,(B_{22}\, C_{32} - B_{31}C_{21}) + A_{12}\,(B_{22}C_{31} + B_{21}C_{32}) + B_{21}(A_{11}C_{33} - A_{31}C_{11}) \\
&\quad + C_{31}(A_{11}B_{41}D_{21} + A_{41}B_{21}D_{11} - A_{21}B_{11})] \\
Q_3 &= [A_{11}(B_{22}C_{33} - B_{31}C_{22} - B_{41}C_{21}D_{31} + B_{41}C_{32}D_{21}) + A_{12}(B_{22}C_{32} - B_{31}C_{21} + B_{41}C_{31}D_{21}) \\
&\quad + B_{11}(A_{31}C_{21} - A_{21}C_{32} - A_{41}C_{31}D_{21}) + B_{21}(A_{12}C_{33} - A_{31}C_{12} + A_{41}C_{32}D_{11}) \\
&\quad + C_{11}(A_{21}B_{31} - A_{31}B_{22} - A_{41}B_{21}D_{31}) + D_{11}(A_{41}B_{22}C_{31} - A_{21}B_{41}C_{31})] \qquad (11) \\
Q_4 &= [A_{11}B_{41}(C_{33}D_{21} - C_{22}D_{31}) + A_{12}B_{41}(C_{32}D_{21} - C_{21}D_{31}) + A_{41}B_{11}\,(C_{21}D_{31} - C_{32}\, D_{21}) \\
&\quad + A_{41}B_{21}(C_{33}D_{11} - C_{12}D_{31}) + C_{11}D_{21}(A_{41}B_{31} - A_{31}B_{41}) + C_{11}D_{31}\,(A_{21}\, B_{41} \\
&\quad - A_{41}\, B_{22}) + D_{11}A_{41}(B_{22}C_{33} - B_{31}C_{21}) + D_{11}B_{41}(A_{31}C_{21} - A_{21}C_{32}) + A_{12}(B_{22}C_{33} \\
&\quad - C_{22}B_{31}) + B_{11}(A_{31}C_{22} - A_{21}C_{33}) + C_{12}(A_{21}B_{31} - A_{31}B_{22})], \\
Q_5 &= [A_{12}B_{41}(C_{33}D_{21} - C_{22}D_{31}) + A_{41}B_{11}(C_{22}D_{31} - C_{33}D_{21}) + C_{12}D_{21}(A_{41}B_{31} - A_{31}B_{41}) \\
&\quad + C_{12}D_{31}(A_{21}B_{41} - A_{41}B_{22}) + C_{22}D_{11}(A_{31}B_{41} - A_{41}B_{31}) + C_{33}D_{11}(A_{41}B_{22} - A_{21}B_{41})]
\end{aligned}
$$

For stability of the projectile all the roots of equation (10) should lie in the left half of λ – complex plane. A necessary condition for this would be

$$Q_r > 0 \qquad r = 0, 1, \ldots\ldots 5 \qquad (12)$$

The necessary and sufficient condition for the stability of the projectile will be by Routh–Hurwitz criterion $\quad D_r > 0 \qquad r = 1, \ldots\ldots, 5$

where

$$D_1 = Q_1,\; D_2 = \begin{vmatrix} Q_1 & Q_o \\ Q_3 & Q_2 \end{vmatrix} \qquad D_3 = \begin{vmatrix} Q_1 & Q_o & 0 \\ Q_3 & Q_2 & Q_1 \\ Q_5 & Q_4 & Q_3 \end{vmatrix}$$

$$D_4 = \begin{vmatrix} Q_1 & Q_0 & 0 & 0 \\ Q_3 & Q_2 & Q_1 & Q_0 \\ Q_5 & Q_4 & Q_3 & Q_2 \\ 0 & 0 & Q_5 & Q_4 \end{vmatrix} \quad \text{and} \quad D_5 = Q_5\, D_4 \qquad (13)$$

Evidently if

Q_1 and Q_5 are positive (14)

and $$D_r > 0, \qquad r = 2,3,4 \tag{15}$$
the projectile is stable.

For any particular case, substituting the numerical values for various parameters in (9); Q_r $(r = 0, 1, \ldots\ldots, 5)$ can be evaluated from (11) and then from (14) and (15), the stability of the projectile can be tested.

CIRCULAR TRAJECTORY

We now transform the above results for circular trajectories ($\theta = 0$) with small yaw (ϕ small). After substitution from (9) in (11), putting $\theta = 0$ and neglecting terms of higer order of small quantities in ϕ and $\dot{\phi}$, we obtain

$$Q_5 = K_D \beta \rho^3 d^8 \dot{S}^5 \dot{\phi} (K_M K_S - K_L K_H) - 2 \Omega K_D K_M \rho^2 d^5 \dot{S}^4 \tag{16}$$

$$Q_4 = (\beta \dot{S}^2 + g)(K_M K_S - K_L K_H) M \rho^2 d^6 \dot{\phi} \dot{S}^2 - \Omega \rho d^3 \dot{S}^3 (M K_M + 2 K_D K_H \rho d^3) \tag{17}$$

$$Q_3 = M K^2 K_D K_S \beta \rho^2 d^5 \dot{S}^3 \dot{\phi} - (K_M K_S - K_L K_H)(2 M_g \phi + 2 K_D \rho d^2 \dot{S}^2 + M \beta \dot{S}^2 \dot{\phi}) \rho^2 d^6 \dot{S}^2$$
$$- \Omega M \rho d^2 \dot{S}^2 (K_H d^2 + 2 K_D K^2) \tag{18}$$

$$Q_2 = (\beta \dot{S}^2 + g)(K_L \dot{S} \phi + K_S d \dot{\phi}) M^2 K^2 \rho d^2 + M \rho d^2 \dot{S} (2 K_D K_L K^2 \rho d^2 \dot{S}^2$$
$$+ 2 K_D K_H \rho d^4 \dot{S}^2 + M K_M d \dot{S}^2 + M K^2 K_L g \phi) - M \rho^2 d^6 \dot{S}^3 (K_M K_S - K_L K_H)$$
$$- \Omega M^2 K^2 \dot{S} \tag{19}$$

$$Q_1 = M^2 \rho d^2 \dot{S}^2 (K^2 K_L + K_H d^2 + 2 K_D K^2) \tag{20}$$

$$Q_0 = M^3 K^2 \dot{S}$$

where

$$\Omega = \frac{2 M \mu}{(R + H)^3} \quad \text{and} \quad g = \frac{\mu}{(R + H)^2}$$

With above values of Q_i, (14) and (15) give criteria for the stability of the projectile in case of circular trajectories.

From (9) it follows that if one or more (not all) coefficients of (10) are negative, the projectile will become unstable. Q_0 is by its form always positive. For Q_5 and Q_4 to be positive, we have from (16) and (17) a necessary condition that $\dot{\phi}$ should be greater than the higher of the following two expressions

$$\frac{2 \Omega K_M}{\beta \rho d^3 \dot{S} (K_M K_S - K_L K_H)} \quad \text{or} \quad \frac{\Omega \dot{S} (M K_M + 2 K_D K_H \rho d^3)}{M \rho d^3 (\beta \dot{S}^2 + g)(K_M K_S - K_L K_H)}$$

This restriction on $\dot{\phi}$ if not satisfied, the projectile shall go unstable in case of circular trajectories.

The additional three conditions which should be satisfied for the projectile to be stable are from equations (18) to (20)

$$MK^2K_D K_S \beta\rho d^3\dot{S}\dot{\phi} > [(K_M K_S - K_L K_H)(2Mg\phi + 2K_D \rho d^2\dot{S}^2 + M\beta\dot{S}^2\phi)\rho d^4 + \Omega M(K_H d^2 + 2K_D K^2)] \quad (21)$$

$$[(\beta\dot{S}^2 + g)(K_L \dot{S}\phi + K_S d\dot{\phi})MK^2\rho d^2 + \rho d^2\dot{S}(2K_D K_L K^2\rho d^2\dot{S}^2 + 2K_D K_H \rho d^4\dot{S}^2 + MK_M d \dot{S}^2 + MK^2K_L g\phi)] > \rho^2 d^6\dot{S}^3(K_M K_S - K_L K_H) + \Omega MK^2\dot{S} \quad (22)$$

and

$$K_L + 2K_D + \left(\frac{d}{K}\right)^2 K_H > 0 \quad (23)$$

The condition for border line instability is obtained by equating D_4 to zero.

STABILITY AND OPTIMALITY

During motion of a projectile certain inevitable perturbation forces come into play and due to these the projectile may begin to oscillate with increasing amplitude with time (oscillatory instability) in all the co-ordinates characterizing the motion of the projectile. For example the projectile may oscillate with increasing amplitude in its height, in angle which the longitudinal axis of the projectile makes with some reference line and in angle which the direction of motion of the centre of gravity of the projectile makes with the reference line. It may also happen that instead of oscillations the projectile deviates increasingly with time from the desired values of the co-ordinates only along one direction (positive or negative) of the co-ordinates (exponentially increasing instability). These cause discrepancies in the performance of the projectile which results in the failure of the desired mission (*e.g.* hitting a target, attainment of a desired velocity or required motion along a specified trajectory), the degree of failure depending upon the magnitude and duration of the oscillations and deviations. If the optimal performance of the projectile is to be that which is capable of attaining the desired mission within a specified permissible error, the stability criteria give the criteria for optimal performance of the projectile and may be called "Optimality criteria of projectile flight".

NUMERICAL ILLUSTRATION

Let us take the following parameters for a rocket :

d	$= 0{\cdot}4$ ft.	K_D	$= 0{\cdot}2$
K_L	$= 2$	K_M	$= 6$
K_S	$= 20$	K_H	$= 31$
M	$= 35$ lbs	$\dot{S}$	$= 10^3$ ft./sec
ϕ	$= 75°$	θ	$= 70°$
$\dot{\phi}$	$= 0{\cdot}1$ rad./sec	$\dot{\theta}$	$= 0{\cdot}01$ rad./sec
g	$= 32$ ft/sec^2	ρ	$= 0{\cdot}07$ lb. /ft^3.
MK^2	$= 20$ lb. ft^2.		

(24)

Value of β for Earth's atmosphere $= (\cdot 316 \times 10^{-4})\ ft^{-1}$. Taking the above value of g amd M we get

$$\Omega = (1\cdot 05 \times 10^{-4})\ \text{lb. sec.}^{-2}$$

Substituting the above values in equations (9) we get

$$\left.\begin{array}{lllllll} A_{11} & = & 35 & & B_{11} & = & 374\cdot 020 \\ A_{12} & = & 4\cdot 673 & & B_{21} & = & 35000 \\ A_{21} & = & (-3\cdot 569) & & B_{22} & = & 22211\cdot 375 \\ A_{31} & = & 4\cdot 803 & & B_{31} & = & (-27925\cdot 478) \\ A_{41} & = & 0\cdot 940 & & B_{41} & = & 342 \\ C_{11} & = & 8\cdot 148 & & D_{11} & = & (-\cdot 074) \\ C_{12} & = & 9\cdot 309 & & D_{21} & = & \cdot 045 \\ C_{21} & = & (-84\cdot 928) & & D_{31} & = & (-\cdot 078) \\ C_{22} & = & (-23270\cdot 417) & & C_{32} & = & 57\cdot 930 \\ C_{31} & = & 20 & & C_{33} & = & 27925\cdot 478 \end{array}\right\} \quad (25)$$

Fromvalues (25), equations (11) give

$$\begin{array}{lll} Q_0 & = & 2\cdot 45 \times 10^7 \\ Q_1 & = & 4\cdot 81 \times 10^7 \\ Q_2 & = & 3\cdot 42 \times 10^{10} \\ Q_3 & = & 3\cdot 53 \times 10^9 \\ Q_4 & = & -(2\cdot 17 \times 10^8) \\ Q_5 & = & 1\cdot 71 \times 10^6 \end{array}$$

Since Q_4 is negative hence by condition (12) the rocket is unstable.

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