

KE-Rod Initial Velocity of Hollow Cylindrical Charge

Wang Shushan and Li Chuanzeng*

State Key Laboratory of Explosion Science and Technology, Beijing Institute of Technology, Beijing-100 081
*CNGC, Beijing North Hongqi Mechanical & Electronic Co Ltd., Beijing-101 149
E-mail: wss2003@bit.edu.cn

ABSTRACT

KE-rod warhead is a kind of forward interception warhead. To control the KE-rods to disperse uniformly, the hollow cylindrical charge is applied. Initial velocity is crucial to KE-rods distribution and the coordination between the fuze and the warhead. Therefore, based on the classical Gurney formula of cylindrical charge and tabulate interlayer charge, a mathematical model for calculating the KE-rod initial velocity of hollow cylindrical charge has been deduced based on certain assumptions, of which the basis theory is energy and momentum conservation. To validate this deduced equation, high-speed photography and metal-pass target experimental methods were applied simultaneously to test the initial velocity of designed KE-rod warhead. Testing results clearly indicate that the calculated results of the derived mathematical model coincides with the experimental results, and with the increase in hollow radius, the calculated results become much closer to the experimental results. But the calculated results of classical Gurney formula are far above the experimental results, and the relative error increases with increase in the hollow diameter. The derived mathematical model with satisfactory accuracy is applicable to calculate the KE-rod initial velocity of hollow cylindrical charge in engineering applications.

Keywords: Hollow cylindrical charge, initial velocity, KE-rod, Gurney formula, forward interception, mathematical model, KE-rod warhead, dual-beam laser fuze, forward interception warhead

1. INTRODUCTION

Forward interception is an important trait of anti-aircraft and anti-missile ammunitions. To improve the damage efficiency and interception probability, two key technologies: dual-beam laser fuze and forward interception warhead have been proposed¹. KE-rod warhead is a kind of forward interception warhead, and cylindrical charge with several different diameter hollows were applied to make the KE-rods obtain different velocities. In this way, the KE-rods could be controlled to disperse just like an 'unfolded umbrella'.

Because of the need for the coordination between the fuze and the warhead, the velocity of KE-rods must be matched with forward detection angles of dual-beam laser fuze. A mathematical model is used to calculate the KE-rod initial velocity of hollow cylindrical charge which becomes crucial to be calculated. Classical Gurney formula was broadly applied to calculate the initial velocity of solid cylindrical charge. However, it has many assumptions when it is calculated. So, there are certain deficiencies like: only reflecting the influence of explosive properties, mass ratio of charge and shell on the initial velocity of fragments, and ignoring the effect of other structural parameters and shell plasticity.

1.1 Historical Developments

With the development of detonation driving theory and application technology, many scholars revised the classical Gurney formula according to application requirements.

Hirsch² proposed a hard core model based on the fact that the acceleration of thin cylindrical shell is implemented by the explosive next to the shell. Flis³, *et al.* and Shao⁴, *et al.* proposed some models based on the detonation flow to describe the flying process of fragments. Vesty⁵, *et al.* also presented some empirical and semi-empirical formulae. Fan⁶, *et al.* presented a fitting formula by dimension analysis, which considers the materials intensity, but this formula is just validated by theory analysis and numerical simulation, and there are no experimental results.

Similarly, formulae were given by some researchers to calculate the initial velocity of different hollow cylindrical charge fragments. Richard⁷ proposed a formula based on the Gurney formula of an inert core charge, which simply revised the radial sparse wave effect, but there was no adequate theoretical basis. Li⁸ presented a formula to calculate the initial velocity of columnar charge, which supposed a load coefficient β . and substituted it into classical Gurney formula, and then revised it by experimental results. A columnar charge Gurney formula was also discussed by Jin⁹, *et al.* wherein only simple theoretical model on some assumptions was suggested, having no powerful experimental validation. Zhang¹⁰, *et al.* extended the application field of Jin's model⁹, and validated it by numerical simulation.

In this study, based on the classical Gurney formula of cylindrical charge and tabulate interlayer charge, a mathematical model has been derived on certain assumptions

for calculating the KE-rod initial velocity of hollow cylindrical charge, of which is based on the theory of energy and momentum conservation. To validate the applicability of this derived mathematical model, high-speed photography and metal-pass target experimental methods have been used to test the velocity of designed KE-rod warhead. The experimental results and the calculated results have also been comparatively analysed.

2. MATHEMATICAL MODEL

Hollow cylindrical charge model is shown in Fig.1. The mathematical model is derived on the following basic assumptions:

- Instantaneous explosion of charge, and all released energy is completely converted to kinetic energy of explosion product and shell.
- Some explosion products expand to the hollow interior and other explosion products expand outwards to drive the shell.
- The velocity of explosion product has radial linear distribution.
- The explosion product expands evenly and the density is the same everywhere; ignoring the end sparse effect.

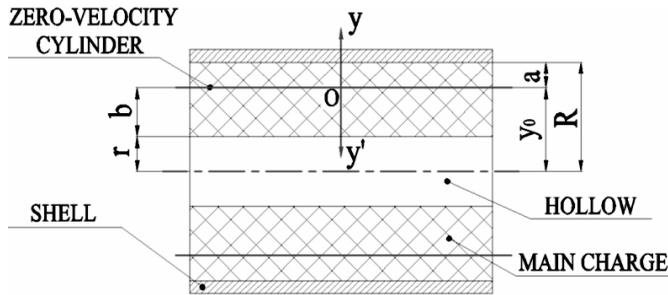


Figure 1. Sketch map of hollow cylindrical charge.

When detonating the main charge, explosion products expand inwards and outwards, there is a 'zero-velocity cylinder', from which the distance to outer surface of charge is a , and to the inner surface of charge is b .

Basing on the above assumptions, following equations have been derived:

$$v_a = \frac{a}{b} v_b \quad (1)$$

$$\begin{cases} v(y) = \frac{y}{a} v_a \\ v(y') = \frac{y'}{b} v_b \end{cases} \quad (2)$$

$$\rho(y) = \rho(y') = \frac{C}{\pi(R^2 - r^2)} \quad (3)$$

where, C is the charge mass per unit length; R is the radius of main charge; r is the radius of hollow cylindrical charge; v_a is the velocity of fragments; v_b is the explosion product velocity of the charge inner surface; $\rho(y)$ is the

density of (y - axis) explosion product; $\rho(y')$ is the density of (y' - axis) explosion product; $v(y)$ is the velocity of (y - axis) explosion product; $v(y')$ is the velocity of (y' - axis) explosion product.

According to the Momentum Conservation Law, the following equation is established:

$$\begin{aligned} Mv_a + \int_0^a v(y)\rho(y)2\pi(y+y_0)d_y \\ = \int_0^b v(y')\rho(y')2\pi(y_0-y')d_{y'} \end{aligned} \quad (4)$$

where, M is the shell mass per unit length; y_0 is the distance between 'zero-velocity cylinder' and the centre of the main charge.

Energy conservation equation is as follows:

$$\begin{aligned} CE = \frac{1}{2}Mv_a^2 + \frac{1}{2}\int_0^a v^2(y)\rho(y)2\pi(y+y_0)d_y \\ + \frac{1}{2}\int_0^b v^2(y')\rho(y')2\pi(y_0-y')d_{y'} \end{aligned} \quad (5)$$

where, E is the energy released per unit mass of charge.

Equations (1) to (3) when substituted into Eqn (4) will give:

$$y_0 = \frac{R(R+r) + \frac{2}{3}\beta(r^2 + Rr + R^2)}{(R+r)(1+\beta)} \quad (6)$$

where, β is load coefficient $\beta = C/M$.

Substituted the Eqns (1), (2), (3) and (6) into Eqn (5) and integrating, one gets:

$$v_a = \sqrt{2E} \cdot \sqrt{\frac{\beta}{1+\beta K}} \quad (7)$$

where,

$$\begin{aligned} K = \frac{2}{a^2(R+r)} \cdot \left[\frac{1}{4}(a^2 + b^2)(a-b) \right. \\ \left. + \frac{1}{3}(a^2 - ab + b^2)y_0 \right] \end{aligned} \quad (8)$$

3. EXPERIMENTAL METHODS

To validate the applicability of this derived mathematical model, using high-speed photography and metal-pass target experimental methods, the aim is to test the velocity simultaneously. The principles of two test methods are as follows:

- High-speed photography: This method can test the average velocity and velocity distribution of fragments. Each camera prepares two timers. One is used to record the starting time, and the other is used to record the ending time. There is an equipment to control the explosion of warhead and the startup of camera synchronously. When the flying fragments touch the target, spark is generated. At the same time, the bright spots appear in the film. According to the order of bright spots, the flying time of fragments can be assured,

and the average velocity can also be calculated.

- **Metal-pass target method:** The target consists of A3 armour plate, colophony interlayer and a thin aluminum plate (Fig. 5). The colophony interlayer is used to insulate the A3 armour plate and the aluminum plate. Both A3 armour plate and the aluminum plate are connected with the signal generator. When the fragments penetrate through the target, A3 armour plate and the aluminum plate are switched on, and signals are generated. At the same time, the data collection instrument records the time. When the fragments penetrate through the thin aluminum plate, crack expands outwards, A3 armour plate and the aluminum plate are disconnected. In this way, the flying time of fragments is recorded. As a short flight distance of fragments, the velocity attenuation can be ignored, so, the testing velocity could be close to the real one.

3.1 Structure of Experimental Warhead

Experimental warhead (Figs 2 and 3) consists of main charge, shell, KE-rods, and end covers, etc.

The overall dimensions of the experimental warhead are $\Phi 80 \text{ mm} \times 190 \text{ mm}$; dimension of each KE-rod is $\Phi 6 \text{ mm} \times 50 \text{ mm}$; the dimensions of three hollows (from top to bottom) are: $\Phi 52 \text{ mm} \times 57.5 \text{ mm}$, $\Phi 42 \text{ mm} \times 55 \text{ mm}$, and $\Phi 14 \text{ mm} \times 57.5 \text{ mm}$.

KE-rod material is 45 steel, having hardness HB 265-275 and the tensile strength $> 600 \text{ MPa}$. All rods are glued on the shell surface with epoxy resin. Shell material is Bakelite (thickness 2 mm). Top and bottom cover material is LY12 aluminum. Main charge is cast composition B charge, whose density is 1.65 g/cm^3 . Warhead is detonated by electric detonator, which is placed in the center of the top cover.

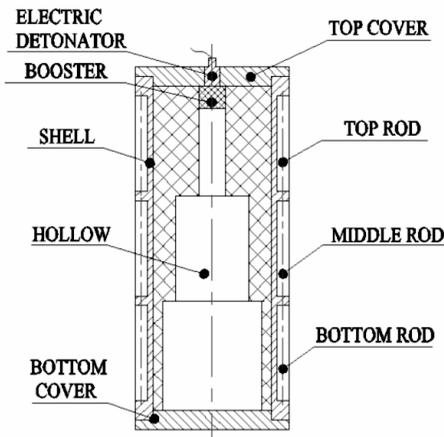


Figure 2. Structure of experimental warhead.

3.2 Testing System

According to the principles of two testing methods, three warheads were prepared to do static explosion test. The main testing instrument of high-speed photography was high-speed camera E-10USA, whose recording rate was 1000 frames/s; The main testing instrument of metal-



Figure 3. Experimental warhead.

pass target experimental method were the ray oscillograph SC16A, digital time relay JSNS, and low frequency signal generator XD7.

The layout diagrams and on-site photograph are shown in Figs 4 and 5; the dimensions of A3 armour plate used in high-speed photography were $300 \text{ mm} \times 150 \text{ mm} \times 10 \text{ mm}$, and the dimensions of composite target used in metal-pass target were $100 \text{ mm} \times 150 \text{ mm} \times 3 \text{ mm}$. At the time of conducting the experiment, warhead was placed in the centre of testing field, and the height at which it was kept was 500 mm.

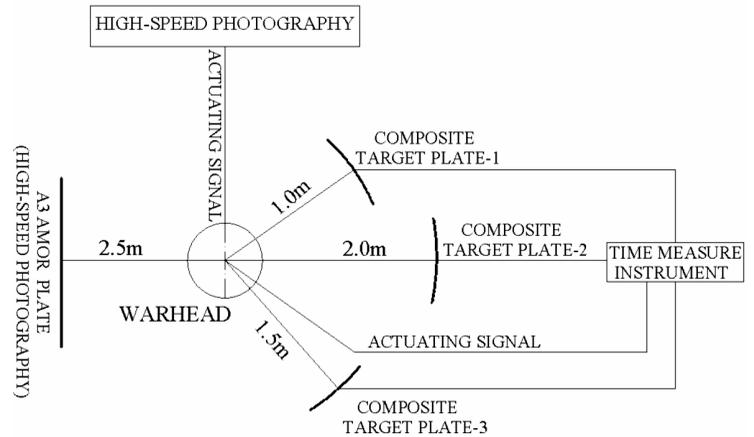


Figure 4. Layout diagrams of the experiment.

4. TEST RESULTS AND ANALYSIS

Using the above testing system to test the KE-rod initial velocity of hollow cylindrical charge, typical photographs of penetration results are shown in Figs 6 and 7.

The hollow cylindrical charge could avoid rods breaking up, so the perforation (Figs 6 and 7) was intact. Both figures also show the detonation-driving features of this warhead. The middle rods penetrate through the composite target vertically at 1.0 m, and turn 90° to penetrate target by cross-section at 1.5 m, then turn 90° again at 2.0 m. However, there is almost no turning of top and bottom



Figure 5. Experimental scene.

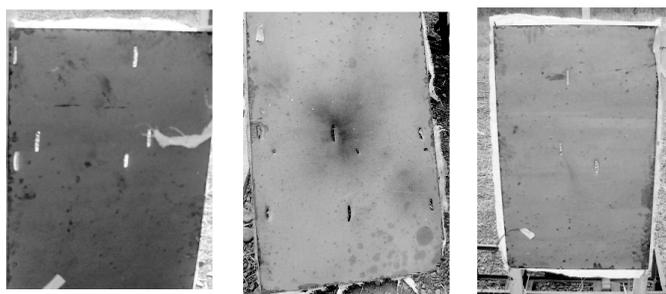


Figure 6. Test result photographs of the metal-pass target method.



Figure 7. Test result photographs of the high-speed photography.

rods from 1.0 m to 2.0 m. Study of the flying and penetration attitudes is useful to improve the coordination between the fuze and the warhead and evaluation of damage effectiveness.

The original record of the KE-rods flying time is shown in Fig. 8. According to the principle of metal-pass target method, the velocity can be calculated. Table 1 shows the test results of the initial velocity. It is obvious that there is velocity gradient among different section rods, and the velocity decreases with the increase in the hollow diameter.

Average testing velocity, calculated results of the derived model, and classical Gurney results have been compared in Table 2. It indicates clearly that the relative error between the calculated results of the derived model and the testing results is small, which can be controlled within -5 per cent. However, classical Gurney results are far above the testing results, the minimum relative error is 77 per cent; the maximum can reach

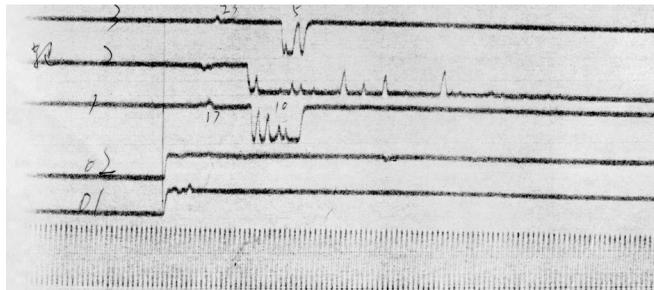


Figure 8. Original record of metal-pass target method.

Table 1. Test results of initial velocity

Warhead number	Initial velocity (m/s)		
	Top	Middle	Bottom
1	966.1	638.2	432.5
2	1104.7	674.3	473.1
3	989.8	638.7	448.3
Average velocity (m/s)	1020.2	650.4	451.3

158 per cent, and it was observed that the relative error increases with increase in the hollow diameter.

To show the relative error visually, three kinds of average results have been placed in one coordinate system (as shown in Fig. 9).

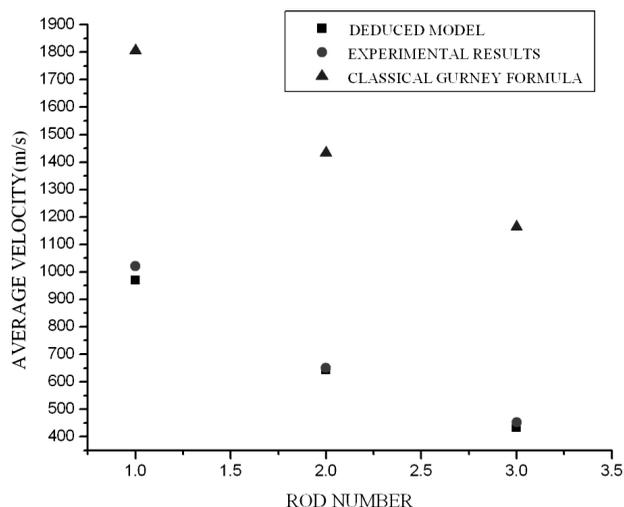


Figure 9. Comparative results in one coordinate system.

The comparative results show that the classical Gurney formula may be not applicable to calculate the KE-rod initial velocity of hollow cylindrical charge. However, the derived model can control relative error well, and also the

Table 2. Comparative results

KE Rod	Testing velocity (m/s)	Derived model		Classical Gurney	
		Velocity (m/s)	Relative error (percentage)	Velocity (m/s)	Relative error (percentage)
Top	1020.2	969.2	-5.0	1805.4	77.0
Middle	650.4	643.1	-1.1	1433.4	120.0
Bottom	451.3	432.9	-4.1	1164.3	158.0

calculated results coincide with the testing results. So, it is better to use this formula with satisfying accuracy to calculate the initial velocity in engineering applications.

The calculated results of the derived mathematical model are less than the testing results (Table 2 and Fig. 9). The reason is the explosion product of main charge expands to the inner of the hollows, where after colliding and mixing, it still has certain ability to drive the KE-rods. But the derived mathematical model does not take into account this effect. So, when using this derived model to calculate the initial velocity, the hollow radius r must meet the condition $r > 0$, and with the increase of cylindrical charge radius of the hollow, the calculated result is much closer to the experimental results.

The detonation driving of hollow cylindrical charge will decelerate the initial velocity of KE-rods, because part of the energy is dissipated by hollow sparse effect. But in this experiment, the sealing effect of end covers reduces the sparse effect to a certain extent, and the comparative results show that the derived mathematical model is still applicable without revising the end and the radial sparse effect.

5. CONCLUSIONS

A new kind of KE-rod warhead has been proposed, and its principle has been analysed. A mathematical model has been derived on certain assumptions for calculating the KE-rod's initial velocity of hollow cylindrical charge, of which the basis theory applicable is energy and momentum conservation. The initial velocity of designed KE-rod warhead has been tested using high-speed photography and metal-pass target experimental methods simultaneously. The test results indicate clearly that the calculated results of the derived mathematical model coincide with the experimental results, and the relative error can be controlled within 5 per cent. But the calculated results of the classical Gurney formula was far above the experimental results, and the relative error increased with hollow diameter. However, the deduced mathematical model ignores the driving effect of explosion product in the hollow; the calculated results of derived mathematical model are less than the testing results. So there is limitation in using. However, the calculated results are much closer to the experimental results with the increase of the radius of the hollow cylindrical charge. This derived model also does not take into account the end and the radial sparse effect; therefore, the calculated velocity is decreased. But the sealing effect of end covers reduce the sparse effect to a certain extent in the experiment. A comparison of the results also shows that the derived mathematical model is still applicable without revising the sparse effect. It is obvious that the derived mathematical model with satisfying accuracy may be applied to calculate the KE-rod initial velocity of hollow cylindrical charge.

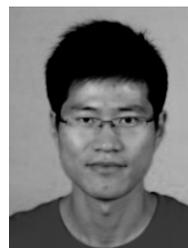
REFERENCES

1. Li, Chuanzeng; Han, Lei & Wang, Shushan. The double laser beam forward detection techniques for the anti-missile projectiles of the naval guns. *Proc. of SPIE*, 2009, **7515**, 75150Q. pp. 1-7.
2. Hennequin, E. Influence of the edge effects on the initial velocity of fragments from a warhead. *In Proceedings of 9th International Symposium on Ballistics*, Shrivenham, UK, 1986. pp. 215-23.
3. Flis, W.J. & Clark, W.J. A modern comprehensive model of fragment projection by explosive warheads. *In Proceedings of 3rd Ballistics Symposium on Classified and Controlled Topics*, Johns Hopkins University, Applied Physics Laboratory. Laurel, MD, Nov 1995.
4. Shao, B.H.; Zhang, D.X.; Li, G.H. & Zhou, Z.H. Motion of the flyer plate under glancing detonation. *Explo. Shock Waves*, 1985, **5**(21), 1-12.
5. Vesty, P. Automated simulation model building from energy and power function. Raytheon, Tewksbury, MA, July 1997.
6. Fan, Y.F. & Duan, Z.P. Strength effect correction of Gurney formula. *J. Ballistics*, 2009, **21**(1), 27-30.
7. Lioyd, R.M. Conventional warhead system physics and engineering design. American Institute of Aeronautics and Astronautics, 1998, **179**, pp. 75.
8. Li, S.C. The design of XX warhead. Beijing Institute of Technology, Beijing, China, 1992. MsD Thesis.
9. Jin, Jiabo & Wang, Shushan. The Gurney equation of cylindrical charge. *In The Theory, Method, and Engineering Application of Detonation Mechanics*, edited by Ning, J.G., Huang, F.L. & Qin, C.S. Beijing Institute of Technology Press, Beijing, China, 2002. pp. 507-09.
10. Zhang, Zhiwei; Wang, Shushan & Ma, Feng. Gurney equation of the hollow-charge. *In Theory and Practice of Energetic materials (VI)*. *Science*, USA, 2005. pp. 1117-120.

Contributors



Dr Wang Shushan obtained his PhD from Beijing Institute of Technology. Presently, he is working as Professor at State Key Laboratory of Explosion Science and Technology (BIT). His research areas are terminal effects and evaluation of damage effectiveness.



Mr Li Chuanzeng obtained his Masters from Shenyang Institute of Technology in 2008. He is pursuing his PhD at State Key Laboratory of Explosion Science and Technology (BIT). He is also working as an engineer at the Beijing North Hongqi Mechanical & Electronic Co., Ltd. (CNGC). His research areas are fuze and warhead coordination.