STATISTICAL TOLERANCES ON BUILT UP GUNS

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ABSTRACT

The strength of built up guns depends on the shrinkages allowed between cylinders and therefore for a given strength, the assembly is either selective or the nominal sizes are so chosen as to give the required shrinkage for a worst combination. In case the latter method is adopted and tolerances on individual dimensions are made to cater for the worst combination, it can be seen from the analysis that with little disadvantage tolerances can be greatly increased assuming random assemblies of parts whose sizes follow a normal pattern. This can bring down production costs considerably.

Introduction

The usual practice of estimating tolerances that would be obtained in the assembly of machine parts is to add up all the individual tolerances on the parts for the two extreme cases of fits. However, the dimensions of any part will vary within the tolerance zone following a normal curve[1] with a standard deviation σ such that 99·73 percent parts lie between the zone $\pm 3\sigma$ (It can be assumed that this equals the tolerance zone). The clearance/interference between two such parts say a hole and a shaft will follow another normal pattern with a standard deviation equal to $(\sigma_1^2 + \sigma_2^2)^{\frac{1}{2}}$ where σ_1 and σ_2 are the standard deviations of the individual components[2]. Therefore 99·73 percent of the assemblies will have a clearance/interference in the range $\pm 3(\sigma_1^2 + \sigma_2^2)^{\frac{1}{2}}$ and hence the actual fit obtained for any random assembly will be different from the one visualized on the basis of the extreme dimensions of the two mating parts in which case all assemblies lie within the range $\pm 3(\sigma_1 + \sigma_2)$. The above therefore suggests that accepting 0·27 percent rejects on assemblies, to obtain the same class of fit on a random assembly, tolerances on individual components could be increased, thereby reducing the manufacturing costs. The advantage so gained has recently been emphasized by Spotts[3] and in this paper has been applied to the case of built up guns.

Analysis

Consider a built up gun of three cylinders X, Y and Z; the various surface radii on assembly being, a, b, c and d as shown in figure 1. Let the manufacturing specifications for various tubes be as under:—

External radius of cylinder $X = b_x \pm t_1$

Internal radius of cylinder $Y = b_y \pm t_2$

External radius of cylinder $Y = c_y \pm t_3$

Internal radius of cylinder $Z = c_z \pm t_4$

where b_x, b_y, c_y and c_z are nominal sizes and t_1, t_2, t_3 and t_4 are the tolerances on those. Also for shrinkage fits, it is obvious that $b_x > b_y$ and $c_y > c_z$. Further, on assembly the numerical value of interface radii b will lie between b_x and b_y and that of interface c will lie between c_y and c_z.

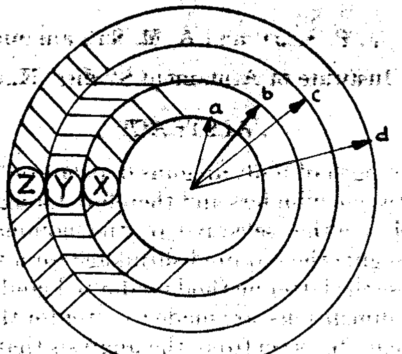

FIG 1

The extreme values of shrinkage between X and Y on the conventional method of estimation are given as $(b_x - b_y) \pm (t_1 + t_2)$ whereas on a statistical basis the values are $(b_x - b_y) \pm (t_1^2 + t_2^2)^{\frac{1}{2}}$ with a normal distribution in random assemblies. The shrink pressure p_{xy} on statistical basis is then given as[4]

$$p_{xy} = \frac{E\,(b^2 - a^2)\,(c^2 - b^2)}{2\,b^3\,(c^2 - a^2)} \left\{(b_x - b_y) \pm (t_1^2 \times t_2^2)^{\frac{1}{2}}\right\} \quad \ldots \quad (1)$$

The error term due to variations of a, b and c has been neglected for the strains are very small in comparison to b_x, b_y etc. and therefore the term $\frac{(b^2 - a^2)\,(c^2 - b^2)}{2b^3\,(c^2 - a^2)}$ is assumed to be a constant for the various assemblies. The hoop stresses at the inner layers of X and Y are then[4]:

$$(\sigma_t)_x = -\frac{E\,(c^2 - b^2)}{b\,(c^2 - a^2)} \left\{(b_x - b_y) \pm (t_1^2 + t_2^2)^{\frac{1}{2}}\right\} \quad \ldots \quad (2)$$

$$= -\left\{P_1 \pm Q_1\,(t_1^2 + t_2^2)^{\frac{1}{2}}\right\} \quad \ldots \quad (2a)$$

$$(\sigma)_y = \frac{E\,(b^2 + c^2)\,(b^2 - a^2)}{2\,b^3\,(c^2 - a^2)} \left\{(b_x - b_y) \pm (t_1^2 + t_2^2)^{\frac{1}{2}}\right\} \quad \ldots \quad (3)$$

$$= P_2 \pm Q_2\,(t_1^2 + t_2^2)^{\frac{1}{2}} \quad \ldots \quad (3a)$$

where P_1, P_2 and Q_1, Q_2 are constants and the negative sign in equations (2) and 2(a) indicates a compressive stress. The stretch at the external radius of Y is then[4]:

$$(w_c)_y = \frac{c\,(b^2 - a^2)}{b\,(c^2 - a^2)} \left\{(b_x - b_y) \pm (t_1^2 + t_2^2)^{\frac{1}{2}}\right\} \quad \ldots \quad (4)$$

$$= R \pm S\,(t_1^2 + t_2^2)^{\frac{1}{2}} \quad \ldots \quad (4a)$$

where R and S are constants.

The external diameter of cylinder Y should therefore statistically be

$$c_y + R \pm [S^2 (t_1^2 + t_2^2) + t_3^2]^{\frac{1}{2}}$$

Therefore considering a normal spread in the manufactured size of cylinder Z, the shrinkage between Y and Z will be given by:

$$\partial = (c_y - c_z + R) \pm [S^2 (t_1^2 + t_2^2) + t_3^2 + t_4^2)]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (5)$$

which gives the hoop stress at the inner layer of Z as

$$(\sigma_t)_z \pm \frac{E (c^2 + d^2)(c^2 - a^2)}{2 c^3 (d^2 - a^2)} \Big\{(c_y - c_z + R) \pm [S^2 (t_1^2 + t_2^2) + t_3^2 + t_4^2)]^{\frac{1}{2}} \Big\} \quad \ldots \quad (6)$$

$$\pm K \pm L [S^2 (t_1^2 + t_2^2) \pm t_3^2 + t_4^2]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots (6a)$$

By the conventional method, the worst cases of ∂ and $(\sigma_t)_z$ would have been estimated as:

$$\partial' = (c_y - c_z + R) \pm S (t_1 + t_2) + (t_3 + t_4) \quad \ldots \quad (7)$$

and $(\sigma_t)'_z = K \pm L [S (t_1 + t_2) + t_3 + t_4] \quad \ldots \quad \ldots \quad \ldots (8)$

By comparing equations (6a) and (8) it is clear that the estimated values of maximum hoop stress at the inner layer of cylinder Z, on the conventional basis are larger than those expected on random assemblies and therefore for a given value of maximum hoop stress the tolerances on individual cylinder radii could be increased. If we denote the conventional and statistical tolerances by suffixes c and s, we have:

$$S^2 (t_{s1}^2 + t_{s2}^2) + t_{s3}^2 + t_{s4}^2 = [S (t_{c1} + t_{c2}) + t_{c3} + t_{c4}]^2 \quad \ldots \quad (9)$$

If tolerances on all radii are equal i.e. $t_{s1} = t_{s2} = t_{s3} = t_{s4} = t_s$ and $t_{c1} = t_{c2} = t_{c3} = t_{c4} = t_c$ we get

$$t_s / t_c = \sqrt{2} (S + 1) / (S^2 + 1)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (10)$$

The theoretical maximum value of t_s / t_c (=2) occurs for S=1 and the variation of t_s / t_c with S is given in fig. 2.

Discussion

It may be noted that S is a fraction and is a measure of the stretch that occurs on the external radius of the middle cylinder. When $S = 0$, the interface stress between the second and third cylinder arises out of interference fits based on their sizes only. For any positive value of S the stress obtained between the second and third cylinder depends not only on the fit based on their sizes but also is governed by the stretch experienced by the second cylinder. The value of S is a maximum at unity when $b = c$ i.e. when the intermediate cylinder becomes paper thin and therefore will have the maximum possible stretch. If b is a geometric mean of a and c and $c=2a$, S becomes $\sqrt{2/3}$ which gives $t_s / t_c = 1{\cdot}9$ therefore suggesting that statistical tolerances are 90 percent more than those based on conventional method where aggregated effect of tolerances for worst assembly is considered.

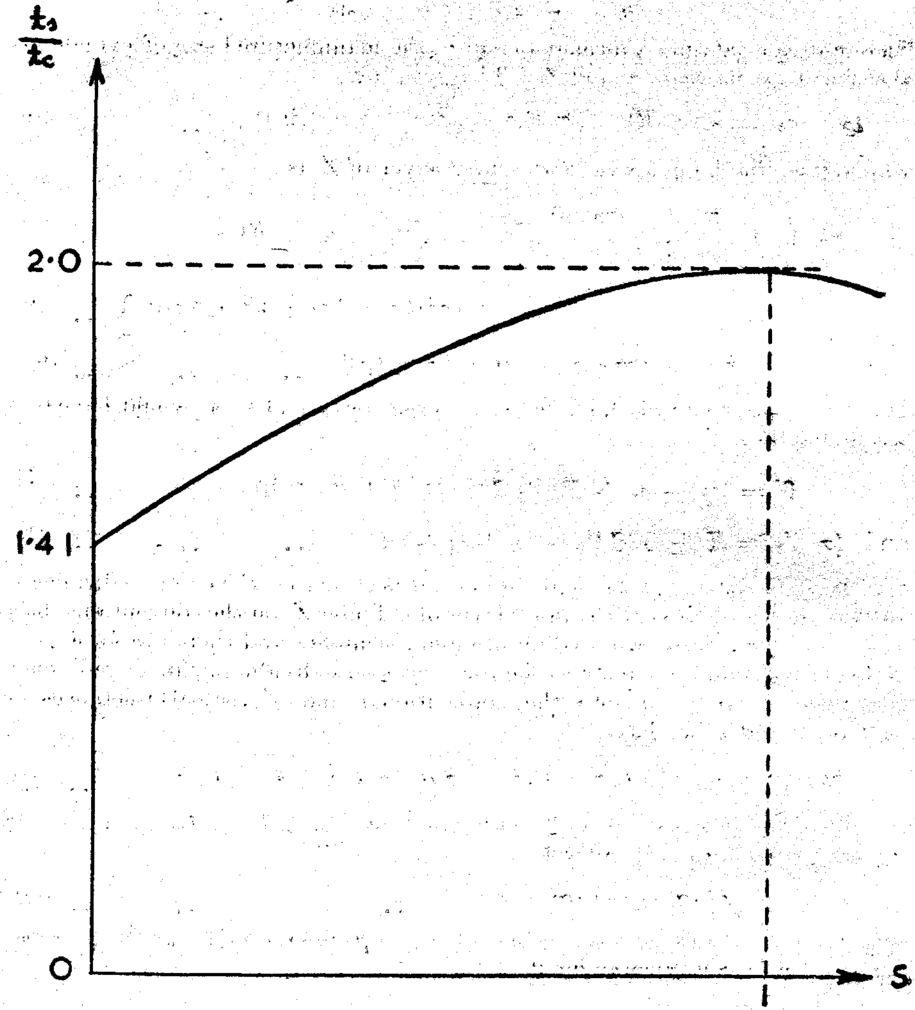

Fig 2

References

1. Conway—*Engineering Tolerances.*
2. Faires—*Design of machine elements.*
3. Spotts—An application of statistics for dimensioning of machine parts—*Trans ASME series B* Nov. 1959.
4. Timoshenko—*Strength of materials* pt II.