VARIATION OF BURNING SURFACE AREA FOR MULTITUBULAR CHARGES

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ABSTRACT

In continuation of the earlier work already published[2,3] the variation of burning surface area for multi-tubular charges is discussed in this paper.

Introduction

The variation of the function of progressivity $\frac{S}{S_o}$ (the ratio of the burning surface area at any time to the initial burning surface area) is an important characteristic for any particular charge shape. An expression for $\frac{S}{S_o}$ can be easily obtained if the form-function i.e. the relation between z and f is known. Conversely from a knowledge of $\frac{S}{S_o}$ as a function of f, we can deduce the form-function. In any particular case, the decision as to which of the two functions $\frac{S}{S_o}$ or z has to be calculated first depends upon the relative ease with which either can be computed.

For a hepta-tubular charge, Tavernier[1] had deduced the expression for $\frac{S}{S_o}$ from that of z. For the second stage of burning it leads to very complicated differentiation since z is not known explicitly as a function of f. In fact he has expressed $\frac{S}{S_o}$ in terms of $K(w)$ and $I(w)$ both of which contain $H'(\omega)$. We have obtained in this paper an explicit expression for $K(w)$ without involving any differentiation and given the Tables for the same. Incidentally we have been able to find an explicit expression for $H'(w)$ also.

We have also examined the variation of $\left(\frac{S}{S_o}\right)_{max}$ both with m and ρ for tri-tubular, quadra-tubular and hepta-tubular charges. It appears that for a given shape this value in general increases both with m and ρ and the behaviour for a fixed value of m is similar for all values of ρ. The study which has been done for the above three charges should help us in finding out suitable values of m and ρ for choosing a shape of a particular type with given values of $\left(\frac{S}{S_o}\right)_{max}$.

Notations

The following notations have been used :

C = Mass of the grain.

δ = Propellant density.

d = The diameter of each hole of the grain.

D = The distance between any two holes or between any hole and the curved surface of the grain.

L = The length of the grain.

m = Ratio of the exterior diameter of the grain to the diameter of the holes.

$$\left.\begin{array}{lll} m > \left(1+\frac{2}{\sqrt{3}}\right) & \text{for the tri-tubular charge.} \\ \quad > (1+\sqrt{2}) & \text{for the quadra-tubular charge.} \\ \quad > \ 3 & \text{for the hepta-tubular charge.} \end{array}\right\}$$

ρ = Ratio of the length of the grain to the exterior diameter of the grain.

$$\left.\begin{array}{lll} \rho > \left[\frac{(m-1)\sqrt{3}-2}{2(\sqrt{3}+1)\,m}\right] & \text{for the tri-tubular charge.} \\ \quad > \left[\frac{m(\sqrt{2}-1)-1}{m\sqrt{2}}\right] & \text{for the quadra-tubular charge.} \\ \quad > \left[\frac{m-3}{4m}\right] & \text{for the hepta-tubular charge.} \end{array}\right\}$$

z = Fraction of the charge burnt at the instant 't'.

f = Fraction of the initial thickness (web-size) remaining at the instant 't'.

The function of progressivity for the first stage

In this section we shall find $\frac{S}{S_0}$ from first principles without first finding the form-function. In fact we have deduced the form-function for each charge from the expression for $\frac{S}{S_0}$ as a function of :

(*a*) Tri-tubular charge.

The appearance of the end section of a grain is shown in the following figs 1, 2 and 3.

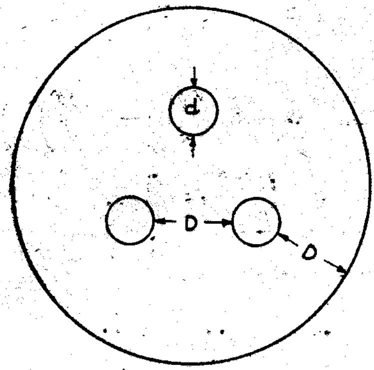

FIG 1

Unburnt position

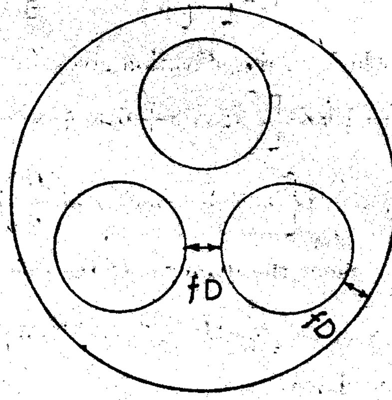

FIG 2

Position when a fraction f of D remains.

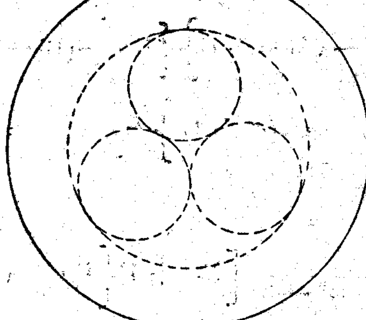

FIG 3

Position of the four slivers at the end of the first stage of burning

The initial surface area is given by

$$S = \left\{ 2\pi\left(\frac{md}{2}\right) + 3.2\pi\left(\frac{d}{2}\right)\right\}\rho md + 2\left\{\pi\left(\frac{md}{2}\right)^2 - 3\pi\left(\frac{d}{2}\right)^2\right\}$$

$$= \tfrac{1}{2}\pi d^2 (2m^2\rho + 6m\rho + m^2 - 3) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Also the surface area at any instant is given by

$$S=\left[2\pi\left\{\left(\frac{md}{2}\right)-\frac{D(1-f)}{2}\right\}+3.2\pi\left\{\frac{d}{2}+\frac{D(1-f)}{2}\right\}\right][\rho md-D(1-f)]$$
$$+2\left[\pi\left\{\frac{md}{2}-\frac{D(1-f)}{2}\right\}^2-3\pi\left\{\frac{d}{2}+\frac{D(1-f)}{2}\right\}^2\right] \quad (2a)$$

which on simplification gives

$$S=\tfrac{1}{2}\pi d^2\left[\left\{(2m^2\rho+6m\rho+m^2-3)+4(m\rho-m-3)\left(\frac{D}{d}\right)-6\left(\frac{D}{d}\right)^2\right\}\right.$$
$$\left.+4\left\{(m+3-m\rho)+3\left(\frac{D}{d}\right)\right\}\left(\frac{D}{d}\right)f-6\left(\frac{D}{d}\right)^2 f^2\right] \quad \ldots \quad (2b)$$

Since the diameter of the cylinder equals md i.e.

$$\frac{2}{\sqrt{3}}(D+d)+2D+d=md \quad \ldots \quad \ldots \quad \ldots \quad (3a)$$

we have

$$\frac{D}{d}=\tfrac{1}{4}[(3m-1)-\sqrt{3}(m+1)] \quad \ldots \quad (3b)$$

Substituting for $\frac{D}{d}$ from (3*b*) in (2*b*) we get after some simplification

$$S=\tfrac{1}{2}\pi d^2\left[\tfrac{1}{4}(m+1)\left\{4(5-\sqrt{3})m\rho-13(2-\sqrt{3})m+3(3\sqrt{3}-2)\right\}\right.$$
$$-\tfrac{1}{4}\left\{(3m-1)-\sqrt{3}(m+1)\right\}\left\{4m\rho-(13-3\sqrt{3})m-3(3-\sqrt{3})\right\}f$$
$$\left.-\tfrac{3}{8}\left\{(3m-1)-\sqrt{3}(m+1)\right\}^2 f^2\right] \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Dividing (4) by (1)

$$\frac{S}{S_o}=\frac{1}{2m^2\rho+6m\rho+m^2-3}\left[\tfrac{1}{4}(m+1)\left\{4(5-\sqrt{3})m\rho-2(13m+3)\right.\right.$$
$$\left.+\sqrt{3}(13m+9)\right\}-\tfrac{1}{4}\left\{(3m-1)-\sqrt{3}(m+1)\right\}$$
$$\left\{4m\rho-(13m+9)+3\sqrt{3}(m+1)\right\}f$$
$$\left.-\tfrac{3}{8}\left\{(3m-1)-\sqrt{3}(m+1)\right\}^2 f^2\right] \quad \ldots \quad (5a)$$

which is of the form

$$\frac{S}{S_o}=\alpha-\beta f-\gamma f^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5b)$$

where

$$\left.\begin{aligned} a &= \frac{(m+1)[4(5-\sqrt{3})m\rho - 2(13m+3) + \sqrt{3}(13m+9)]}{4(2m^2\rho + 6m\rho + m^2 - 3)} \\ \beta &= \frac{[(3m-1) - \sqrt{3}(m+1)][4m\rho - (13m+9) + 3\sqrt{3}(m+1)]}{4(2m^2\rho + 6m\rho + m^2 - 3)} \\ \gamma &= \frac{3[\,(3m-1) - \sqrt{3}(m+1)\,]^2}{8(2m^2\rho + 6m\rho + m^2 - 3)} \end{aligned}\right\} \quad .. \quad (6)$$

(*b*) Quadra-tubular Charge.

The appearance of the end section of a grain is shown in the following figs. 4, 5 and 6.

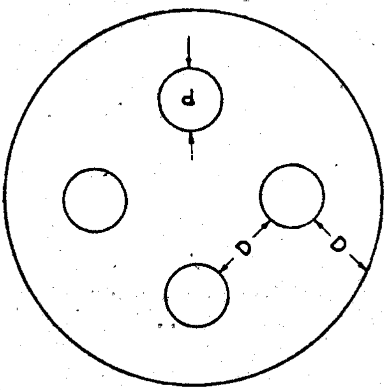

FIG 4
Unburnt position

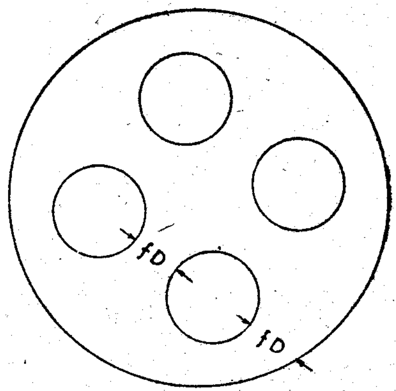

FIG 5
Position when a fraction *f* of *D* remains

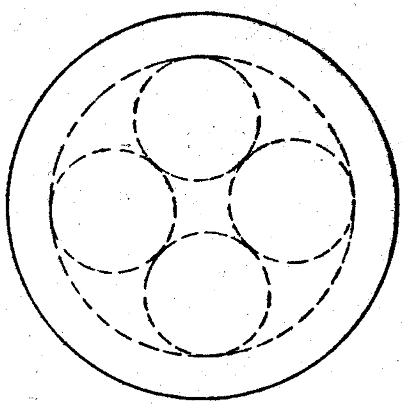

FIG 6
Position of the five slivers at the end of the first stage of burning.

In this case

$$S_{\circ} = \left\{2\pi\left(\frac{md}{2}\right) + 4.\ 2\pi\left(\frac{d}{2}\right)\right\}\rho md + 2\left\{\pi\left(\frac{md}{2}\right)^2 - 4\pi\left(\frac{d}{2}\right)^2\right\}$$

$$= \tfrac{1}{2}\pi d^2(2m^2\rho + 8m\rho + m^2 - 4) \quad \ldots \quad (7)$$

Also

$$S = \left[2\pi\left\{\left(\frac{md}{2}\right) - \frac{D(1-f)}{2}\right\} + 8\pi\left\{\frac{d}{2} + \frac{D(1-f)}{2}\right\}\right]$$
$$\left[\rho md - D(1-f)\right] + 2\left[\pi\left\{\frac{md}{2} - \frac{D(1-f)}{2}\right\}^2\right.$$
$$\left. - 3\pi\left\{\frac{d}{2} + \frac{D(1-f)}{2}\right\}^2\right] \quad \ldots \quad (8a)$$

which on simplification gives

$$S = \tfrac{1}{2}\pi d^2\left[\left\{(2m^2\rho + 8m\rho + m^2 - 4) + 2(3m\rho - 2m - 8)\left(\frac{D}{d}\right)\right.\right.$$
$$\left. - 9\left(\frac{D}{d}\right)^2\right\} + 2\left\{2m + 8 - 3m\rho + 9\left(\frac{D}{d}\right)\right\}\left(\frac{D}{d}\right)f$$
$$\left. - 9\left(\frac{D}{d}\right)^2 f^2\right] \quad \ldots \quad (8b)$$

Since the diameter of the cylinder equals md i.e.

$$(D + d)\sqrt{2} + d + 2D = md \quad \ldots \quad (9a)$$

we have

$$\frac{D}{d} = \frac{m\sqrt{2} - (m+1)}{\sqrt{2}} \quad \ldots \quad (9b)$$

Substituting for $\frac{D}{d}$ from (9b) in (8b) we obtain after a little simplification,

$$S = \tfrac{1}{2}\pi d^2\left[\tfrac{1}{2}(m+1)\left\{2(8 - 3\sqrt{2})m\rho - (33m+17) + 2\sqrt{2}(11m+8)\right\}\right.$$
$$- \left\{m\sqrt{2} - (m+1)\right\}\left\{3\sqrt{2}m\rho + 9(m+1) - \sqrt{2}(11m+8)\right\}f$$
$$\left. - \frac{9}{2}\left\{m\sqrt{2} - (m+1)\right\}^2 f^2\right] \quad \ldots \quad (10)$$

Dividing (10) by (7), we get

$$\frac{S}{S_{\circ}} = \frac{1}{2m^2\rho + 8m\rho + m^2 - 4}\left[\tfrac{1}{2}(m+1)\left\{2(8 - 3\sqrt{2})m\rho - (33m+17)\right.\right.$$
$$\left. + 2\sqrt{2}(11m+8)\right\} - \left\{m\sqrt{2} - (m+1)\right\}$$
$$\left\{3\sqrt{2}m\rho + 9(m+1) - \sqrt{2}(11m+8)\right\}f$$
$$\left. - \frac{9}{2}\left\{m\sqrt{2} - (m+1)\right\}^2 f^2\right] \quad \ldots \quad (11a)$$

which is of the form

$$\frac{S}{S_0} = a - \beta f - \gamma f^2 \qquad \ldots (11b)$$

where

$$\left.\begin{aligned} a &= \frac{(m+1)[2(8-3\sqrt{2})m\rho - (33m+17) + 2\sqrt{2}(11m+8)]}{2(2m^2\rho + 8m\rho + m^2 - 4)} \\ \beta &= \frac{[m\sqrt{2} - (m+1)][3\sqrt{2}m\rho + 9(m+1) - \sqrt{2}(11m+8)]}{(2m^2\rho + 8m\rho + m^2 - 4)} \\ \gamma &= \frac{9[m\sqrt{2} - (m+1)]^2}{2(2m^2\rho + 8m\rho + m^2 - 4)} \end{aligned}\right\} \qquad (12)$$

(c) Hepta-tubular charge:

The appearance of the end section of a grain is shown in the following figs 7, 8 and 9.

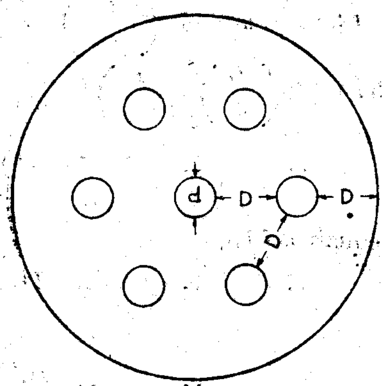

FIG 7
Unburnt position

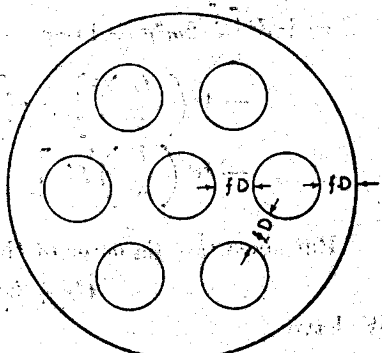

FIG 8
Position when a fraction f of D remains.

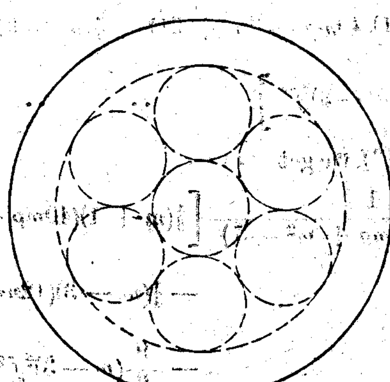

FIG 9
Position of the twelve slivers at the end of the first stage of burning.

In this case

$$S_o = \left[\left\{2\pi\left(\frac{md}{2}\right)+14\pi\left(\frac{d}{2}\right)\right\}\rho md + 2\left\{\pi\left(\frac{md}{2}\right)^2-7\pi\left(\frac{d}{2}\right)^2\right\}\right]$$

$$= \tfrac{1}{2}\pi d^2(2m^2\rho + 14m\rho + m^2 - 7) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Also

$$S = \left[2\pi\left\{\left(\frac{md}{2}\right)-\frac{D(1-f)}{2}\right\}+14\pi\left\{\frac{d}{2}+\frac{D(1-f)}{2}\right\}\right]\left[\rho md - D(1-f)\right]+2\left[\pi\left\{\left(\frac{md}{2}\right)-\frac{D(1-f)}{2}\right\}^2-7\pi\left\{\frac{d}{2}+\frac{D(1-f)}{2}\right\}^2\right] \quad \ldots \quad (14a)$$

which on simplification gives

$$S = \tfrac{1}{2}\pi d^2\left[\left\{(2m^2\rho + 14m\rho + m^2 - 7) + 4(3m\rho - m - 7)\left(\frac{D}{d}\right) - 18\left(\frac{D}{d}\right)^2\right\} + 4\left\{m + 7 - 3m\rho + 9\left(\frac{D}{d}\right)\right\}\left(\frac{D}{d}\right)f - 18\left(\frac{D}{d}\right)^2 f^2\right] \quad \ldots \quad (14b)$$

But since the diameter of the cylinder equals md i.e.

$$4D + 3d = md \quad \ldots \quad \ldots \quad \ldots \quad (15a$$

We have

$$\frac{D}{d} = \frac{m-3}{4} \quad \ldots \quad \ldots \quad \ldots \quad (15b)$$

Substituting for $\dfrac{D}{d}$ from (15*b*), in (14*b*), we obtain after simplification,

$$S = \tfrac{1}{2}\pi d^2\left[\tfrac{1}{8}(m+1)(40m\rho - 9m + 31) - \tfrac{1}{4}(m-3)(12m\rho - 13m - 1)f - \frac{9}{8}(m-3)^2 f^2\right] \quad \ldots \quad (16)$$

Dividing (16) by (13), we get

$$\frac{S}{S_o} = \frac{1}{(2m^2\rho + 14m\rho + m^2 - 7)}\left[\tfrac{1}{8}(m+1)(40m\rho - 9m + 31) - \tfrac{1}{4}(m-3)(12m\rho - 13m - 1)f - \frac{9}{8}(m-3)^2 f^2\right] \quad \ldots \quad (17a)$$

which is of the form

$$\frac{S}{S_o} = \alpha - \beta f - \gamma f^2 \quad \ldots \quad \ldots \quad \ldots \quad (17b)$$

where

$$\left.\begin{aligned} \alpha &= \frac{(m+1)\ (40\ m\rho - 9m + 31)}{8(2m^2\rho + 14m\rho + m^2 - 7)} \\ \beta &= \frac{(m-3)\ (12m\rho - 13m - 1)}{4(2m^2\rho + 14m\rho + m^2 - 7)} \\ \gamma &= \frac{9}{8}\ \frac{(m-3)^2}{(2m^2\rho + 14m\rho + m^2 - 7)} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (18)$$

We shall now obtain the (z, f) relation for the three charges. Since S is clearly proportional to $\frac{dz}{df}$ and since initially $S=S_o$ when $f=1$ we have,

$$\frac{S}{So} = \frac{\frac{dz}{df}}{\left[\frac{dz}{df}\right]_{f=1}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19a)$$

Also

$$\frac{S}{So} = \alpha - \beta f - \gamma f^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19b)$$

Hence

$$\frac{dz}{df} = K\ (\alpha - \beta f - \gamma f^2) \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

where

$$K = \left[\frac{dz}{df}\right]_{f=1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

Integrating (20) and using the condition that intially $z=o$ when $f=1$, we obtain

$$\begin{aligned} z &= -K\ \left[\alpha\,(1-f) - \frac{\beta}{2}\,(1-f^2) - \frac{\gamma}{3}\,(1-f^3)\right] \\ &= -K\,(1-f)\left[(\alpha - \tfrac{1}{2}\beta - \tfrac{1}{3}\gamma) - (\tfrac{1}{2}\beta + \tfrac{1}{3}\gamma)f - \tfrac{1}{3}\gamma f^2\right] \end{aligned} \quad (22a)$$

which is the (z, f) relation of the form

$$z = (1-f)\ (a - bf - cf^2) \quad \ldots \quad \ldots \quad \ldots \quad (22b)$$

where

$$\left.\begin{aligned} a &= -K\,(\alpha - \tfrac{1}{2}\beta - \tfrac{1}{3}\gamma) \\ b &= -K\,(\tfrac{1}{2}\beta + \tfrac{1}{3}\gamma) \\ c &= -K\,(\tfrac{1}{3}\gamma) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

We shall now obtain the value of $K \left(= \left[\frac{dz}{df}\right]_{f=1}\right)$ from the equation of burning viz.

$$c\ (dz) = -S_o\,\frac{D}{2}\ (df)\ \delta \quad \ldots \quad \ldots \quad \ldots \quad (24a)$$

whence

$$\left[\frac{dz}{df}\right]_{f=1} = -\frac{D\,S_o\,\delta}{2c} = -\frac{D\,S_o}{2\,V_o} \quad \ldots \quad (24b)$$

where V_o is original volume of the grain and can be calculated for each charge. In fact

$$V_o = \left[\pi\left(\frac{md}{2}\right)^2 - n\pi\left(\frac{d}{2}\right)^2\right]\rho md$$
$$= \tfrac{1}{4}\,\pi\rho md^3\,(m^2 - n) \quad \ldots \quad (25)$$

where $n=3$, 4 and 7 for the tri-tubular, quadra-tubular and hepta-tubular charges respectively.

Substituting the values of D, S_o, and V_o for the three charges, we obtain the corresponding values of K for the three charges. Thus

$$K = -\frac{(2m^2\rho+6m\rho+m^2-3)}{4m\rho\,(m^2-3)}\left[(3m-1)-\sqrt{3}(m+1)\right] \text{ for the tri-tubular charge.}$$

$$= -\frac{(2m^2\rho+8m\rho+m^2-4)}{\sqrt{2}m\rho\,(m^2-4)}\left[m\sqrt{2}-(m+1)\right] \text{ for the quadra-tubular charge.}$$

$$= -\frac{(2m^2\rho+14m\rho+m^2-7)}{4m\rho\,(m^2-7)}\left[m-3\right] \text{ for the hepta-tubular charge.}$$

Now substituting the values of α, β, γ and K for the three charges we obtain, after some simplification, the corresponding expressions for a, b, c and consequently the $(z\text{-}f)$ relation

$$z = (1-f)\,(a - bf - cf^2) \quad \ldots \quad (26)$$

for the three charges. Thus

for the tri-tubular charge;

$$\left.\begin{aligned} a &= \frac{[(3m-1)-\sqrt{3}\,(m+1)]}{8m\rho\,(m^2-3)}\left[(7-\sqrt{3})\,m^2\rho + (11-\sqrt{3})\,m\rho + 2\,(5\sqrt{3}-8)\left(\frac{m+1}{2}\right)^2\right] \\ b &= \frac{[(3m-1)-\sqrt{3}\,(m+1)]^2}{16m\rho\,(m^2-3)}\left[2m\rho-(5-\sqrt{3})\,(m+1)\right] \\ c &= \frac{[(3m-1)-\sqrt{3}\,(m+1)]^3}{32\,m\rho\,(m^2-3)} \end{aligned}\right\} \quad (27)$$

for the quadra-tubular charge;

$$\left.\begin{aligned} a &= \frac{[m\sqrt{2}-(m+1)]}{2\sqrt{2}m\rho\,(m^2-4)}\left[(10-3\sqrt{2})\,m^2\rho + (16-3\sqrt{2})\,m\rho + (8\sqrt{2}-11)\,(m+1)^2\right] \\ b &= \frac{[m\sqrt{2}-(m+1)]^2}{2m\rho\,(m^2-4)}\left[3\,m\rho-(8-3\sqrt{2})\,(m+1)\right] \\ c &= \frac{3\,[m\sqrt{2}-(m+1)]^3}{2\sqrt{2}\,m\rho\,(m^2-4)} \end{aligned}\right\} \quad (28)$$

and for the hepta-tubular charge:

$$\left.\begin{aligned} a &= \frac{(m-3)}{8\,m\rho\,(m^2-7)}\left[7\,m^2\rho + 19m\rho + \left(\frac{m+1}{2}\right)^2\right] \\ b &= \frac{(m-3)^2}{16\,m\rho\,(m^2-7)}\left[6\,m\rho - 5\,m - 5\right] \\ c &= \frac{3\,(m-3)^3}{32\,m\rho\,(m^2-7)} \end{aligned}\right\} \quad (29)$$

The function of progressivity for the second stage

(a) Tri-tubular charge:[2]

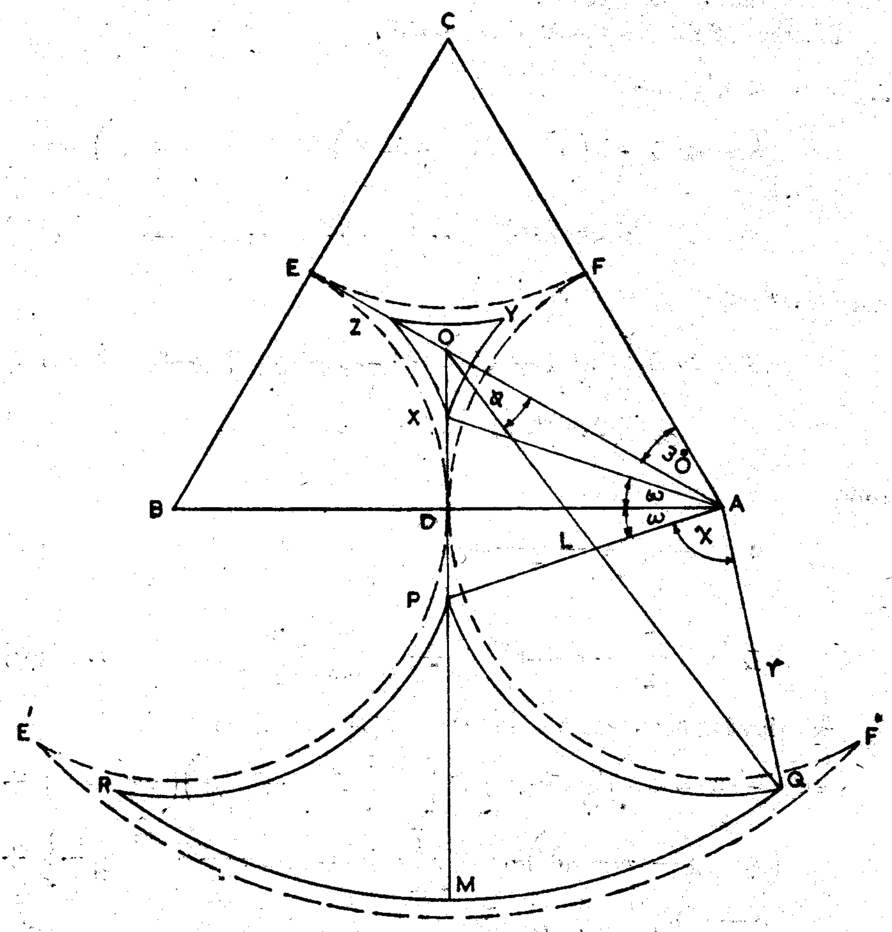

FIG 10

Length of the circumference of the $\triangle$ *PQR* is

$$= 2\,r\,\chi + \frac{a}{\sqrt{3}}\left\{2\,(1+\sqrt{3}) - \sqrt{3}\,\sec\omega\right\}(120^\circ - 2\phi)$$

$$= 2a\left[\frac{2}{\sqrt{3}}\,(1+\sqrt{3})\,(60^\circ - \phi) + (\phi + \chi - 60^\circ)\,\sec\omega\right] \quad (30)$$

and that of *XYZ* is

$$= 3\,r\,(60^\circ - 2\,\omega)$$

$$= 6\,a\,(30^\circ - w)\,\sec\omega \quad \ldots \quad (31)$$

Now the surface of the combustion at the instant '*t*' is given by
S=(thrice the length of the circumference of the $\triangle PQR$ + that of the $\triangle XYZ$) × height of the grain + twice (thrice the area of the $\triangle PQR$ + that of the $\triangle XYZ$)
But the $\triangle XYZ$ disappears when ω=30°.

$\therefore$ for $0^\circ \leqslant \omega \leqslant 30^\circ$

$$S = 6a\,L\left\{\frac{2}{\sqrt{3}}\left(1+\sqrt{3}\right)\left(60^\circ - \phi\right) + \left(\phi + \chi - \omega - 30^\circ\right)\sec\omega\right\}$$

$$+ 2a^2\,H\,(\omega) \quad \ldots \quad (32a)$$

and for $30^\circ \leqslant \omega \leqslant 47^\circ\,35'$

$$S = 6a\,L\left\{\frac{2}{\sqrt{3}}\left(1+\sqrt{3}\right)\left(60^\circ - \varphi\right) + \left(\varphi + \chi - 60^\circ\right)\sec\omega\right\}$$

$$+ 2a^2\,H\,(\omega) \quad \ldots \quad (32b)$$

Now

$$a = (3 - \sqrt{3})\left(\frac{m+1}{8}\right)d \quad \ldots \quad (33)$$

and

$$L = d\left[1 + \rho m - \left(\frac{3-\sqrt{3}}{4}\right)\frac{m+1}{\cos\omega}\right] \quad \ldots \quad (34)$$

Substituting for a and L from (33) and (34) in (32) we obtain,

$$S = \frac{1}{8}\left(3-\sqrt{3}\right)\left(m+1\right)d^2\left[6\left\{2\left(1+\frac{1}{\sqrt{3}}\right)\left(60^\circ - \varphi\right) + \left(\varphi + \chi - \omega - 30^\circ\right)\sec\omega\right\}\left\{1 + m\rho - \frac{3-\sqrt{3}}{4}\,\frac{m+1}{\cos\omega}\right\} + \frac{3-\sqrt{3}}{4}\,(m+1)\,H(\omega)\right] \quad 0^\circ \leqslant \omega \leqslant 30^\circ \quad \ldots \quad (35a)$$

and

$$S = \frac{1}{8}\left(3-\sqrt{3}\right)\left(m+1\right)d^2\left[6\left\{2\left(1+\frac{1}{\sqrt{3}}\right)\left(60^\circ-\varphi\right)+\left(\varphi+\chi-60^\circ\right)\sec\omega\right\}\left\{1+m\rho-\frac{3-\sqrt{3}}{4}\ \frac{m+1}{\cos\omega}\right\}+\frac{3-\sqrt{3}}{4}(m+1)H(\omega)\right] \qquad 30^\circ \leqslant \omega \leqslant 47^\circ\,35' \quad .. \quad (35b)$$

Also the surface of the powder initially exposed to the combustion is given by

$$S_\circ = \tfrac{1}{2}\pi d^2(2m^2\rho + 6m\rho + m^2 - 3) \qquad .. \qquad .. \qquad (36)$$

Dividing (35) by (36), we obtain

For $0^\circ \leqslant \omega \leqslant 30^\circ$,

$$\frac{S}{S_\circ} = \frac{(3-\sqrt{3})\ (m+1)}{16\pi(2m^2\rho+6m\rho+m^2-3)}\left[6\left\{2\left(1+\frac{1}{\sqrt{3}}\right)\left(60^\circ-\varphi\right)+\left(\varphi+\chi-\omega-30^\circ\right)\sec\omega\right\}\left\{4+4m\rho-(3-\sqrt{3})\left(\frac{m+1}{\cos\omega}\right)\right\}+(3-\sqrt{3})\ (m+1)H(\omega)\right] \quad .. \quad .. \quad .. \quad .. \quad (37a)$$

and for $30^\circ \leqslant \omega \leqslant 47^\circ\,35'$,

$$\frac{S}{S_\circ} = \frac{(3-\sqrt{3})\ (m+1)}{16\pi(2m^2\rho+6m\rho+m^2-3)}\left[6\left\{2\left(1+\frac{1}{\sqrt{3}}\right)\left(60^\circ-\varphi\right)+(\varphi+\chi-60^\circ)\sec\omega\right\}\left\{4+4m\rho-(3-\sqrt{3})\ \frac{m+1}{\cos\omega}\right\}+(3-\sqrt{3})\ (m+1)H(\omega)\right] \quad .. \quad .. \quad .. \quad .. \quad (37b)$$

When $\omega = 0^\circ$, (37a) gives

$$\frac{S}{S_\circ} = \frac{(3-\sqrt{3})\,(m+1)}{24(2m^2\rho+6m\rho+m^2-3)}\left[6\left(2+\frac{1}{\sqrt{3}}\right)\left\{4+4m\rho-(3-\sqrt{3})(m+1)\right\}+(7\sqrt{3}-9)(m+1)\right]$$

which can be put in the form

$$\frac{S}{S_\circ} = \frac{m+1}{4(2m^2\rho+6m\rho+m^2-3)}\left[4(5-\sqrt{3})m\rho-2(13m+3)+\sqrt{3}(13m+9)\right]$$

This is the value of α at the end of the first period of burning.

The values of $\frac{S}{S_0}$ as given from (37) are tabulated below for different values of ω ranging from 0° to 47° 35′, for some set of values of m and ρ.

TABLE 1

m	ρ	ω: 0°	5°	10°	15°	20°	25°	30°	32° 30′	35°	37° 30′	40°	42° 30′	45°	47° 35′
4	$\frac{1}{2}$	0·665	0·607	0·543	0·474	0·398	0·318	0·236	0·199	0·161	0·124	0·089	0·055	0·025	0
4	1	0·869	0·790	0·708	0·623	0·533	0·438	0·338	0·294	0·247	0·200	0·151	0·102	0·052	0
4	9/4	1·019	0·925	0·830	0·733	0·632	0·526	0·414	0·364	0·310	0·255	0·198	0·136	0·072	0
7	9/4	1·059	0·961	0·862	0·762	0·657	0·547	0·430	0·379	0·323	0·266	0·206	0·142	0·075	0
10	9/4	1·084	0·984	0·883	0·780	0·673	0·561	0·441	0·388	0·332	0·273	0·211	0·146	0·078	0
∞	9/4	1·179	1·070	0·960	0·848	0·732	0·610	0·480	0·423	0·361	0·298	0·231	0·160	0·084	0
∞	∞	1·634	1·481	1·330	1·179	1·022	0·859	0·683	0·606	0·522	0·434	0·340	0·238	0·127	0

The relationship between $\frac{S}{S_o}$ & f, and $\frac{S}{S_o}$ & z is illustrated in the following figs (11) and (12) respectively.

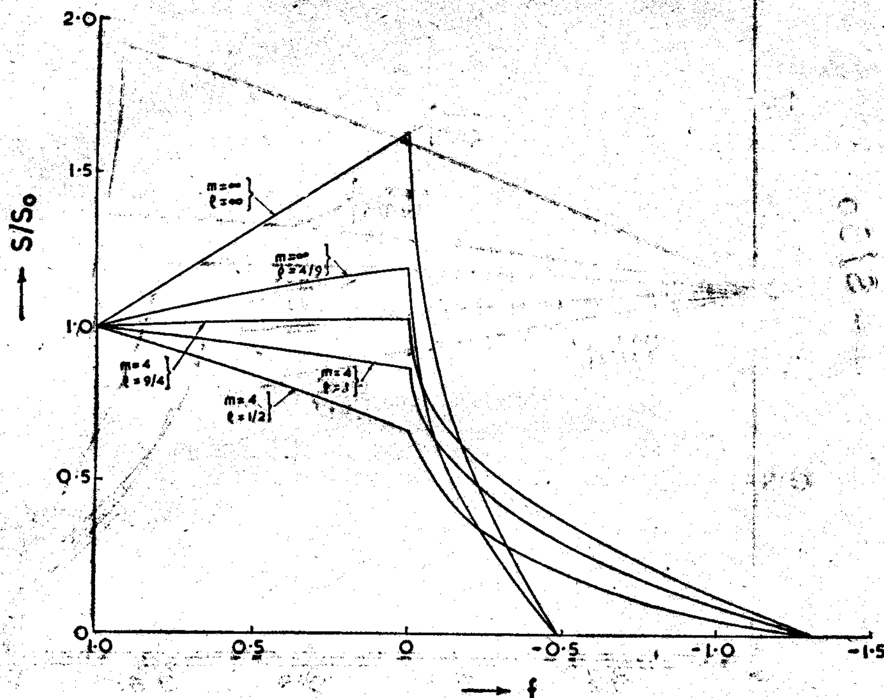

Fig 11

Relation between $\frac{S}{S_o}$ & f for the tri-tubular charge for some set of values of m & ρ

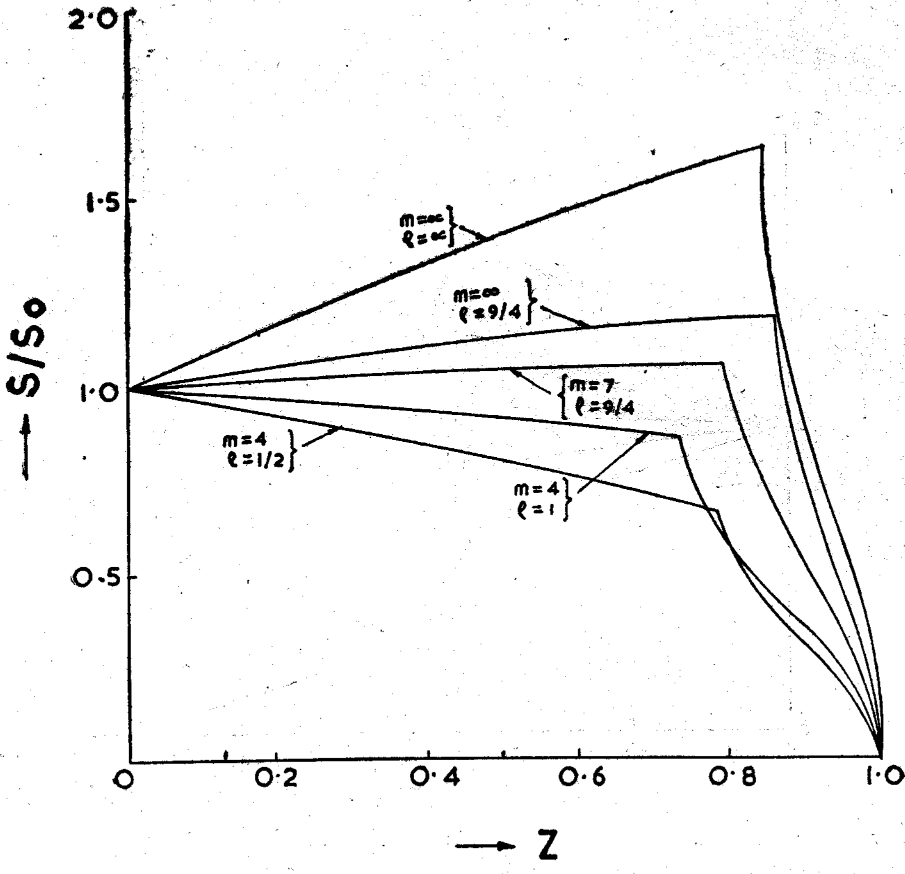

FIG 12

Relation between $\frac{S}{S_0}$ & z for the tri-tubular charge

for some set of values of m & ρ

(*b*) Quadra-tubular charge[3]:

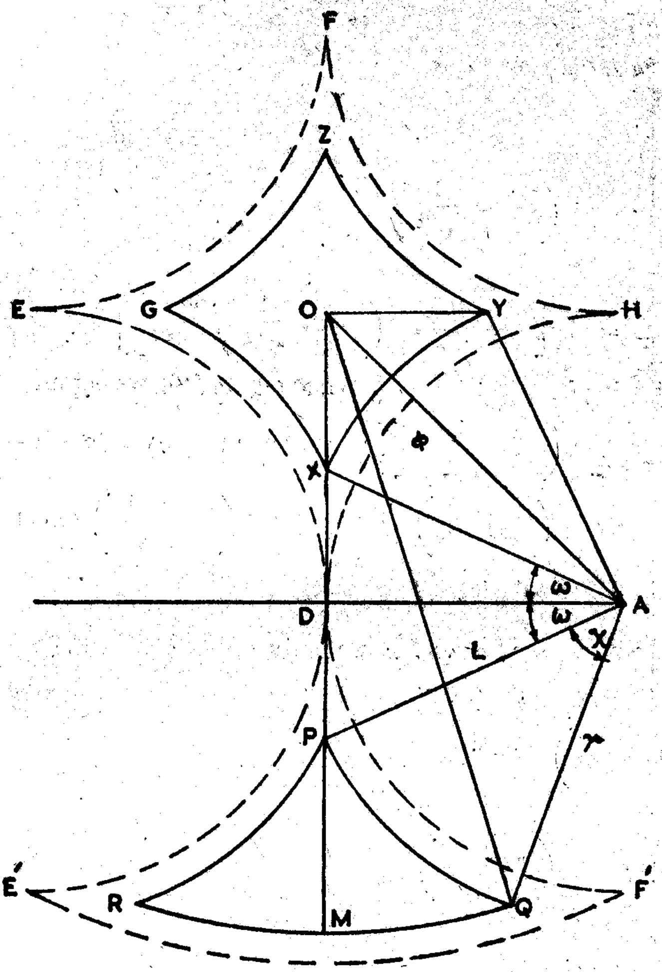

FIG 13

ngth of the circumference of the $\triangle PQR$ is

$$= 2r\chi + a\,(2 + \sqrt{2} - \sec\omega)\,(90^\circ - 2\varphi)$$

$$= 2a[(2 + \sqrt{2})\,(45^\circ - \varphi) + (\varphi + \chi - 45^\circ)\sec\omega] \,.. \qquad (38)$$

d that of XYO is

$$= r\,(90^\circ - 2\omega)$$

$$= 2a\,(45^\circ - \omega)\sec\omega \quad .. \quad .. \quad .. \quad .. \qquad (39)$$

In this case

S = Four times (the length of the circumference of the $\triangle PQR$ + that of XYO) $\times$ height of the grain + eight times (the area of the $\triangle PQR$ + that of XYO)

$$= 8a\, L\, [(2+\sqrt{2})\,(45^\circ - \varphi) + (\varphi + \chi - \omega)\sec\omega] + 8a^2\, H(\omega)$$
$$0^\circ \leqslant \omega \leqslant 45^\circ \qquad \ldots \qquad (40)$$

Now

$$a = \tfrac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)\left(m + 1\right)d \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

and

$$L = d\left[1 + \rho m - \left(1 - \frac{1}{\sqrt{2}}\right)\frac{m+1}{\cos\omega}\right] \quad \ldots \quad \ldots \quad (42)$$

Substituting for a and L from (41) and (42) in (40), we obtain

$$S = 4\left(1 - \frac{1}{\sqrt{2}}\right)\left(m + 1\right)d^2\left[\left\{(2+\sqrt{2})\,(45^\circ - \varphi) + (\varphi + \chi - \omega)\sec\omega\right\}\right.$$
$$\left.\left\{1 + \rho m - \left(1 - \frac{1}{\sqrt{2}}\right)\frac{m+1}{\cos\omega}\right\} + \tfrac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)(m+1)\, H(\omega)\right]$$
$$\ldots \quad \ldots \quad (43)$$

Also

$$S_\circ = \tfrac{1}{2}\,\pi\, d^2\,(2m^2\rho + 8m\rho + m^2 - 4)$$

Dividing (43) by (44), the function of the progressivity of the powder is given by

$$\frac{S}{S_\circ} = \frac{8\left(1 - \frac{1}{\sqrt{2}}\right)\quad(m+1)}{\pi(2m^2\rho + 8m\rho + m^2 - 4)}\left[\left\{(2+\sqrt{2})(45^\circ - \varphi) + (\varphi + \chi - \omega)\sec\omega\right\}\right.$$
$$\left.\left\{1 + \rho m - \left(1 - \frac{1}{\sqrt{2}}\right)\frac{m+1}{\cos\omega}\right\} + \tfrac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)(m+1)\, H(\omega)\right]$$
$$0^\circ \leqslant \omega \leqslant 45^\circ \quad \ldots \quad \ldots \quad (45)$$

For $\omega = 0^\circ$, (45) gives

$$\frac{S}{S_\circ} = \frac{2\left(1 - \frac{1}{\sqrt{2}}\right)\quad(m+1)}{(2m^2\rho + 8m\rho + m^2 - 4)}\left[(5+\sqrt{2})\left\{1 + \rho m - \left(1 - \frac{1}{\sqrt{2}}\right)(m+1)\right\}\right.$$
$$\left. + \tfrac{1}{4}(5\sqrt{2} - 6)\,(m+1)\right]$$

which can be put in the form

$$\frac{S}{S_\circ} = \frac{m+1}{2(2m^2\rho + 8m\rho + m^2 - 4)}\left[2\,(8 - 3\sqrt{2})\,\rho m - (33m + 17)\right.$$
$$\left. + 2\sqrt{2}\,(11m + 8)\right]$$

This is the value of α at the end of the first period of burning.

The following table gives the values of $\frac{S}{S_o}$ as a function of ω for some set of values of m and ρ.

TABLE 2

m \ ρ		ω												
		0°	5°	10°	15°	20°	25°	30°	32° 30′	35°	37′ 30′	40°	42° 30′	45°
4	½	0·745	0·677	0·602	0·522	0·438	0·349	0·257	0·211	0·165	0·120	0·076	0·005	0
4	1	0·926	0·833	0·748	0·654	0·557	0·456	0·349	0·294	0·237	0·179	0·119	0·060	0
4	9/4	1·053	0·952	0·850	0·748	0·642	0·531	0·414	0·352	0·288	0·221	0·150	0·078	0
7	9/4	1·132	1·023	0·914	0·803	0·690	0·571	0·445	0·379	0·310	0·239	0·162	0·084	0
10	9/4	1·181	1·067	0·953	0·838	0·720	0·596	0·465	0·396	0·324	0·249	0·169	0·088	0
∞	9/4	1·366	1·234	1·103	0·970	0·833	0·690	0·539	0·459	0·376	0·289	0·196	0·102	0
∞	∞	1·879	1·696	1·517	1·338	1·155	0·964	0·761	0·652	0·538	0·418	0·287	0·150	0

The relationship between $S/S_{\circ}$ & f, and $S/S_{\circ}$ & z is illustrated below in the figures (14) and (15) respectively.

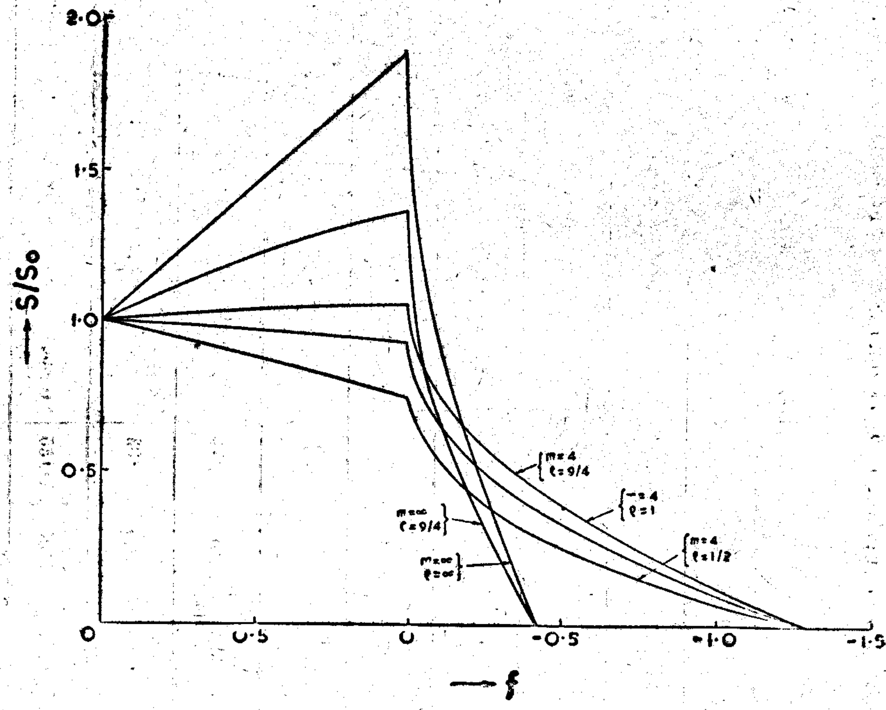

FIG 14

Relation between $\dfrac{S}{S_{\circ}}$ & f for the quadra-tubular charge for some set of values of m & ρ

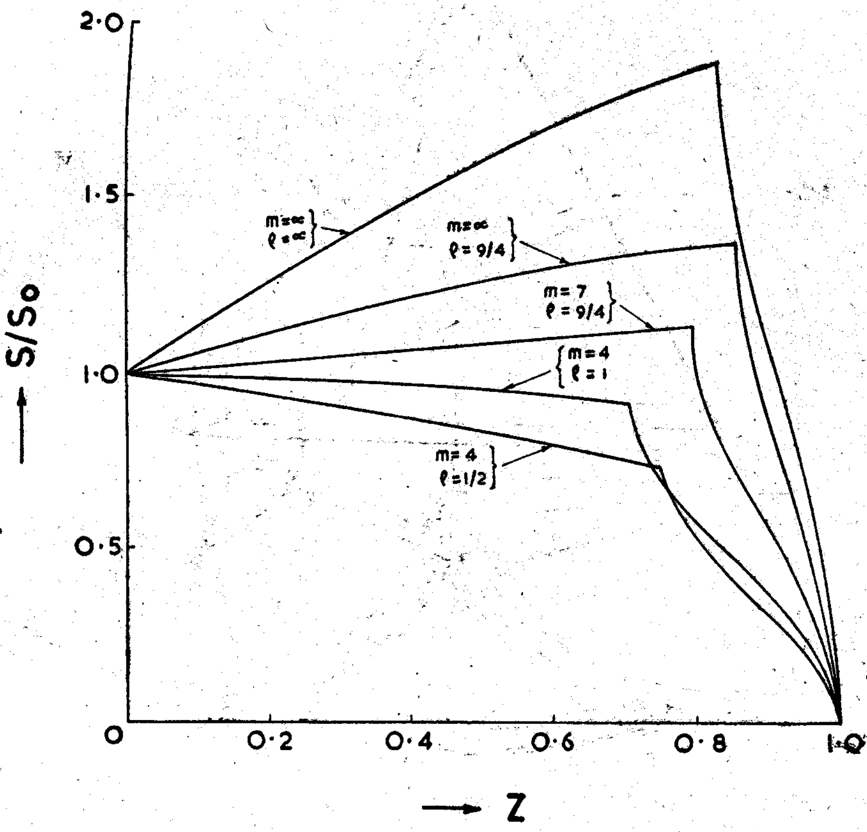

FIG 15

Relation between $\frac{S}{S_o}$ & z for the quadra-tubular charge for some set of values of m & ρ.

(c) Hepta-tubular charge[4]:

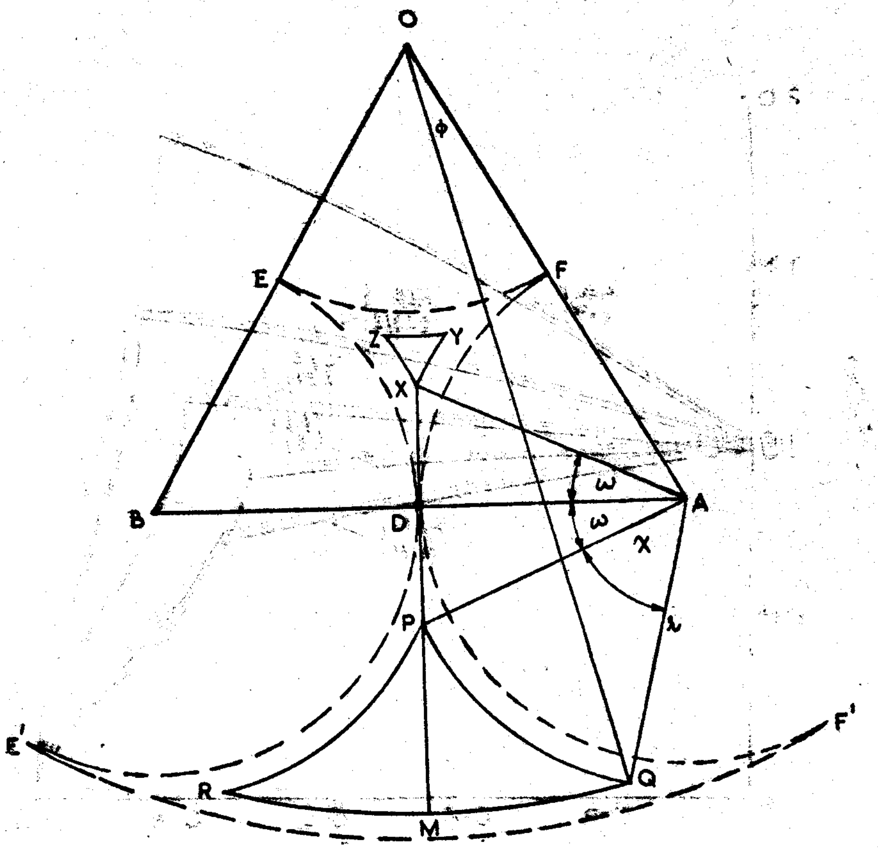

FIG 16

Length of the circumference of the $\triangle PQR$ is

$$= 2r\chi + (4a - r)\,(60^\circ - 2\varphi)$$

$$= 2a\,[4\,(30^\circ - \varphi) + (\varphi + \chi - 30^\circ)\sec\omega] \quad \ldots \quad \ldots (46)$$

and that of the $\triangle XYZ$ is

$$= 3r\,(60^\circ - 2\omega)$$

$$= 6a\,(30^\circ - \omega)\sec\omega \quad \ldots \quad \ldots \quad \ldots \quad \ldots (47)$$

Now

$S =$ six times (the length of the circumference of the $\triangle PQR$ + that of the $\triangle XYZ$) $\times$ height of the grain + twelve times (the area of the $\triangle PQR$ + that of the $\triangle XYZ$).

But the $\triangle XYZ$ disappears when $\omega = 30°$.

$\therefore$ For $0° \leqslant \omega \leqslant 30°$

$$S = 12aL \left\{ 4(30° - \varphi) + (60° + \varphi + \chi - 3\omega) \sec \omega \right\} + 12\, a^2 H(\omega) \quad (48a)$$

and for $30° \leqslant \omega \leqslant 42° 25'$

$$S = 12aL \left\{ 4(30° - \varphi) + (\varphi + \chi - 30°) \sec \omega \right\} + 12a^2 H(\omega) \quad .. \quad (48b)$$

Now

$$a = \left(\frac{m+1}{8} \right) d \quad .. \quad .. \quad .. \quad .. \quad .. \quad (49)$$

and

$$L = \left[1 + \rho m - \frac{m+1}{4 \cos \omega} \right] d \quad .. \quad .. \quad .. \quad .. \quad (50)$$

Substituting for a and L from (49) and (50) in (48), we obtain

$$S = \frac{3}{16}(m+1)\, d^2 \Bigg[8 \left\{ 4\,(30° - \varphi) + (60° + \varphi + \chi - 3\omega) \sec \omega \right\}$$
$$\left(1 + m\rho - \frac{m+1}{4 \cos \omega} \right) + (m+1)\, H(\omega) \Bigg] \quad 0° \leqslant \omega \leqslant 30° \quad .. \quad (51a)$$

and

$$S = \frac{3}{16}(m+1)\, d^2 \Bigg[8 \left\{ 4\,(30° - \varphi) + (\varphi + \chi - 30°) \sec \omega \right\}$$
$$\left(1 + m\rho - \frac{m+1}{4 \cos \omega} \right) + (m+1)\, H(\omega) \Bigg] \quad 30° \leqslant \omega \leqslant 42° 25'$$
$$.. \quad .. \quad (51b)$$

Also

$$S_{\circ} = \tfrac{1}{2} \pi d^2 (2m^2\rho + 14m\rho + m^2 - 7) \quad .. \quad (52)$$

Dividing (51) by (52)

$$\frac{S}{S_{\circ}} = \frac{3\,(m+1)}{8\pi\,(2m^2\rho + 14m\rho + m^2 - 7)} \Bigg[2 \left\{ 4\,(30° - \varphi) + (60° + \varphi + \chi - 3\omega) \right.$$
$$\left. \sec \omega \right\} \left(4 + 4m\rho - \frac{m+1}{\cos \omega} \right) + (m+1)\, H(\omega) \Bigg] \quad 0° \leqslant \omega \leqslant 30°$$
$$.. \quad .. \quad (53a)$$

and

$$\frac{S}{S_{\circ}} = \frac{3\,(m+1)}{8\pi\,(2m^2\rho + 14m\rho + m^2 - 7)} \Bigg[2 \left\{ 4(30° - \varphi) + (\varphi + \chi - 30°) \sec \omega \right\}$$
$$\left(4 + 4\,m\rho - \frac{m+1}{\cos \omega} \right) + (m+1)\, H(\omega) \Bigg] \quad 30° \leqslant \omega \leqslant 42° 25'$$
$$(53b)$$

when $\omega = 0°$, (53a) gives

$$\frac{S}{S_{\circ}} = \frac{(m+1)}{8\,(2m^2\rho + 14\,m\rho + m^2 - 7)} \left(40m\rho - 9m + 31 \right)$$

This is the value of a at the end of the first period of burning.

In following table are shown the values of $\frac{S}{S_o}$, the values within the brackets correspond to those obtained by Tavernier [1].

TABLE 3

m	ρ	ω: 0°	5°	10°	15°	20°	25°	30°	32° 30′	35°	37° 30′	40°	42° 25′
4	½	0·884 (0·884)	0·779 (0·737)	0·669 (0·645)	0·555 (0·546)	0·438 (0·441)	0·317 (0·331)	0·194 (0·211)	0·154 (0·172)	0·114 (0·131)	0·075 (0·088)	0·036 (0·044)	0 (0)
4	1	0·999 (0·999)	0·878 (0·829)	0·756 (0·728)	0·632 (0·621)	0·504 (0·507)	0·371 (0·389)	0·233 (0·254)	0·188 (0·211)	0·142 (0·164)	0·096 (0·113)	0·048 (0·058)	0 (0)
4	9/4	1·072 (1·072)	0·941 (0·888)	0·811 (0·781)	0·680 (0·668)	0·546 (0·550)	0·406 (0·425)	0·258 (0·282)	0·210 (0·236)	0·160 (0·185)	0·109 (0·129)	0·055 (0·068)	0 (0)
7	9/4	1·238 (1·238)	1·087 (1·026)	0·937 (0·902)	0·786 (0·772)	0·631 (0·635)	0·469 (0·492)	0·298 (0·326)	0·243 (0·273)	0·186 (0·214)	0·126 (0·149)	0·064 (0·078)	0 (0)
10	9/4	1·348 (1·348)	1·184 (1·116)	1·020 (0·982)	0·856 (0·840)	0·687 (0·692)	0·511 (0·536)	0·324 (0·355)	0·265 (0·297)	0·202 (0·234)	0·138 (0·163)	0·069 (0·085)	0 (0)
∞	9/4	1·841 (1·841)	1·617 (1·525)	1·394 (1·341)	1·169 (1·148)	0·938 (0·945)	0·698 (0·732)	0·444 (0·485)	0·362 (0·406)	0·277 (0·320)	0·188 (0·223)	0·095 (0·117)	0 (0)
∞	∞	2·5 (2·5)	2·194 (2·067)	1·893 (1·820)	1·591 (1·562)	1·283 (1·293)	0·960 (1·007)	0·615 (0·673)	0·504 (0·567)	0·388 (0·449)	0·266 (0·315)	0·135 (0·617)	0 (0)

The relationship between $\frac{S}{S_o}$ & f, and $\frac{S}{S_o}$ & z is illustrated in the following figures (17) and (18) respectively.

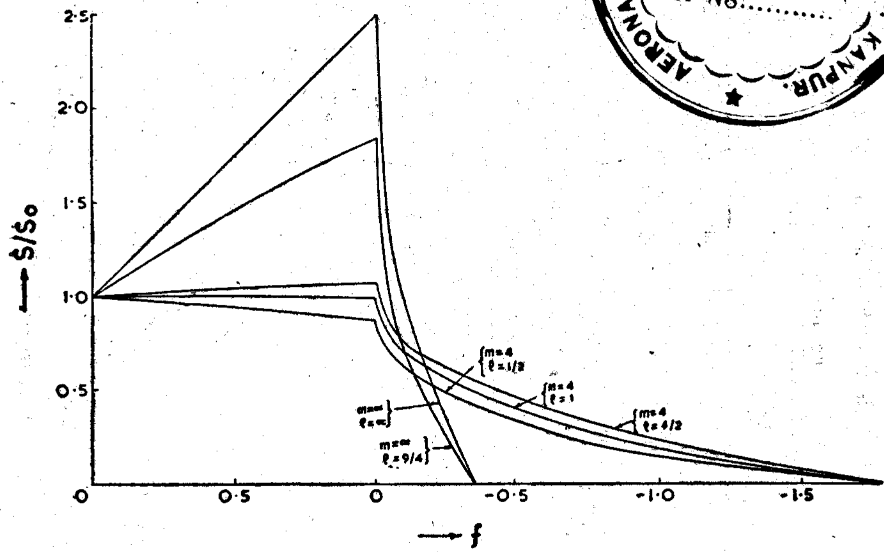

FIG 17

Relation between $\frac{S}{S_o}$ & f for the hepta-tubular charge for some set of values m & ρ

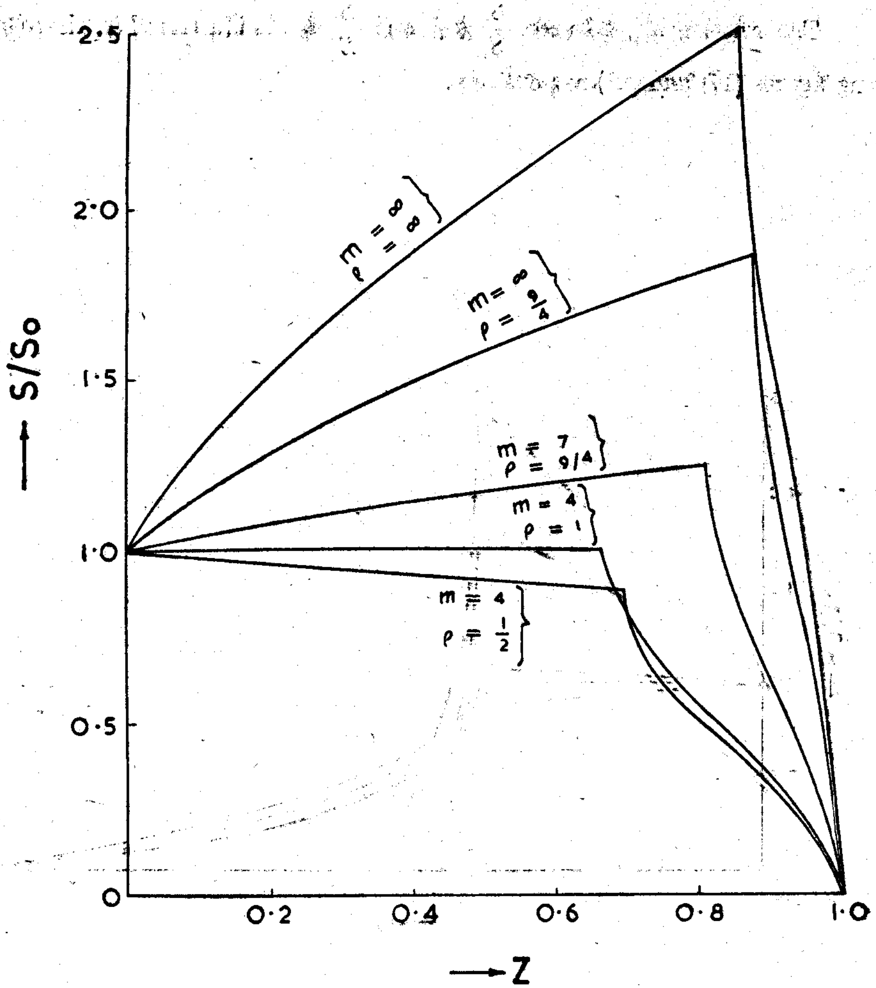

FIG 18

Relation between $\frac{S}{S_o}$ & z for the hepta-tubular charge for some set of values of m & ρ

The expression for $\frac{S}{S_o}$ as obtained by Tavernier[1] is

$$\frac{S}{S_o} = \frac{-3(m+1)}{8\pi(2m^2\rho + 14m\rho + m^2 - 7)} \left[\left(4m\rho + 4 - \frac{m+1}{\cos\omega} \right) H'(\omega) \frac{\cos^2\omega}{\sin\omega} - (m+1)\, H(\omega) \right] \quad .. \quad (54a)$$

$$= -\frac{3(m+1)}{8\pi(2m^2\rho + 14m\rho + m^2 - 7)} \left[4(\rho m + 1)\, K(\omega) - (m+1)\, I(\omega) \right] \quad (54b)$$

where

$$I(\omega) = H'(\omega)\cot\omega + H(\omega) \quad \ldots \quad (55a)$$

and

$$K(\omega) = H'(\omega)\frac{\cos^2\omega}{\sin\omega} \quad \ldots \quad (55b)$$

Comparing the above expression for $\frac{S}{S_\circ}$ with ours we see that

$$K(\omega) = -2\left[4\left(\frac{\pi}{6}-\phi\right)+\left(\frac{\pi}{3}+\phi+\chi-3\omega\right)\sec\omega\right] \quad 0^\circ \leqslant \omega \leqslant 30^\circ \quad (56a)$$

$$= -2\left[4\left(\frac{\pi}{6}-\phi\right)+\left(\phi+\chi-\frac{\pi}{6}\right)\sec\omega\right] \quad 30^\circ \leqslant \omega \leqslant 42^\circ 25' \quad (56b)$$

We have calculated and collected the values of $K(\omega)$ in the following table, the values written within the brackets correspond to those obtained by Tavernier [1].

TABLE 4

ω					
0°	5°	10°	15°	20°	25°
—10·4720	—9·1884	—7·9280	—6·6648	—5·3738	—4·0224
(—10·4720)	(—8·6590)	(—7·6225)	(—6·5432)	(—5·4140)	(—4·2195)
ω					
30°	32° 30′	35°	37° 30′	40°	42° 25′
—2·5744	—2·1122	—1·6260	—1·1142	—0·5672	0
(—2·8194)	(—2·3728)	(—1·8794)	(—1·3181)	(—0·6975)	(0)

Variation of $(S/S_\circ)_{max}$ for the tritubular charge[2]

It has been shown[2] that for a tritubular charge, $\frac{S}{S_\circ}$ is maximum for

$$f = \frac{(13m+9) - 3\sqrt{3}(m+1) - 4mp}{3[(3m-1) - \sqrt{3}(m+1)]} \quad \ldots \quad (57)$$

with

$$\left.\begin{aligned} \rho_1 &= \frac{m+3}{m} \\ \rho_2 &= \frac{(13m+9) - 3\sqrt{3}(m+1)}{4} \end{aligned}\right\} \quad \ldots \quad (58)$$

also

$$\left(\frac{S}{S_\circ}\right)_{max} = 1 \qquad \text{for} \quad \rho < \rho_1$$

$$= \alpha + \frac{\beta^2}{4\gamma} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

$$= \alpha \qquad \rho > \rho_2$$

We shall now consider the following four cases :

Case I. Firstly we take $m=4$. Substituting for α, β and γ we obtain

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{8\rho\,(4\rho + 7) + 137}{3(56\rho + 13)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{7}{4} \text{ and } \rho_2 = \frac{61 - 15\sqrt{3}}{16}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 5

ρ	1·75 ($=\rho_1$)	2	2·189 ($=\rho_2$)	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·005	1·015	1·019	1·053	1·080	1·097	1·149	1·167

Case II. Secondly we take $m=7$. In this case

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{7\rho(7\rho + 10) + 169}{3(70\rho + 23)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{10}{7} \text{ and } \rho_2 = \frac{25 - 6\sqrt{3}}{7}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 6

ρ	1·429 ($=\rho_1$)	1·8	2·087 ($=\rho_2$)	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·015	1·042	1·059	1·115	1·159	1·187	1·276	1·307

Case III. Thirdly we take $m=10$. In this case

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{20\rho(10\rho + 13) + 629}{3(260\rho + 97)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{13}{10} \quad \text{and} \quad \rho_2 = \frac{139 - 33\sqrt{3}}{40}$$

We tabulate below the values of $\left(\frac{S}{S_o}\right)_{max}$ for some values of ρ.

TABLE 7

ρ	1·3 $(=\rho_1)$	2·046 $(=\rho_2)$	2·25	3	4	5	20	∞
(S/S_o) max	1	1·058	1·084	1·151	1·204	1·237	1·344	1·383

Case IV. Fourthly we take $m = \infty$. In this case

$$\left(\frac{S}{S_o}\right)_{max} = \frac{2\rho(\rho + 1) + 5}{3(2\rho + 1)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = 1 \quad \text{and} \quad \rho_2 = \tfrac{1}{4}(13 - 3\sqrt{3})$$

We tabulate below the values of $\left(\frac{S}{S_o}\right)_{max}$ for some values of ρ.

TABLE 8

ρ	1 $(=\rho_1)$	1·5	1·951 $(=\rho_2)$	2·25	3	4	5	20	∞
(S/S_o) max	1	1·042	1·123	1·180	1·277	1·356	1·407	1·573	1·634

The results of the above four cases are illustrated below in Fig 19.

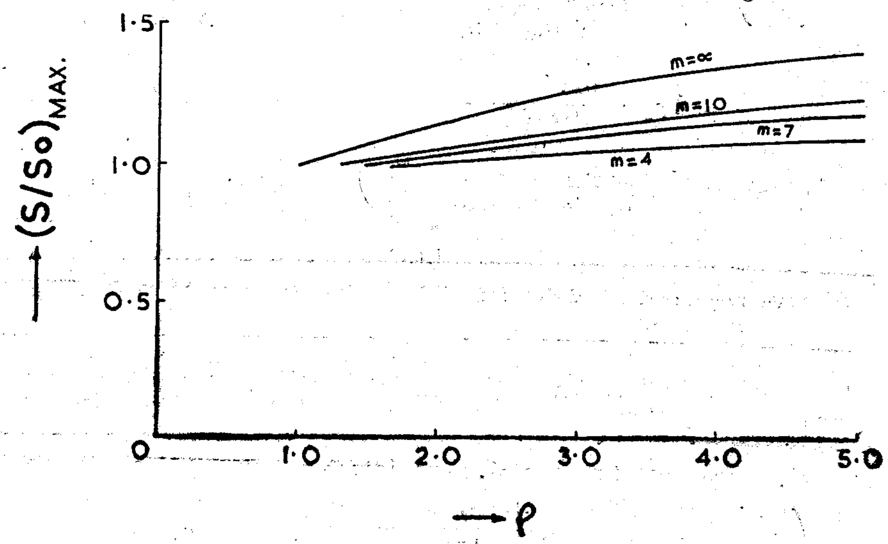

FIG 19

Variation of $\left(\frac{S}{S_o}\right)_{max}$ with ρ for the tri-tubular charges for some values of m

Variation of $\left(\frac{S}{S_\circ}\right)_{max}$ for the quadra-tubular charge

It has also been shown[3] that for a quadra-tubular charge, $\frac{S}{S_\circ}$ is maximum for

$$f = \frac{(11m + 8)\sqrt{2} - 9(m + 1) - 3\sqrt{2}m\rho}{9[m\sqrt{2} - (m + 1)]} \quad \ldots \quad (59)$$

with

$$\left.\begin{aligned} \rho_1 &= \frac{2}{3}\,\frac{m+4}{m} \\ \rho_2 &= \frac{(11m + 8)\sqrt{2} - 9(m + 1)}{3\sqrt{2}\,m} \end{aligned}\right\} \quad \ldots \quad (60)$$

Also

$$\left(\frac{S}{S_\circ}\right)_{max} = 1 \qquad \rho < \rho_1$$

$$= \alpha + \frac{\beta^2}{4\gamma} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

$$= \alpha \qquad \rho > \rho_2$$

We shall now consider the following four cases:

Case I. Firstly we take $m=4$. Then substituting for α, β and γ we obtain

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{12\rho(3\rho + 4) + 91}{9(16\rho + 3)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{4}{3} \text{ and } \rho_2 = \frac{52\sqrt{2} - 45}{12\sqrt{2}}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 9

ρ	1·333 ($=\rho_1$)	1·682 ($=\rho_2$)	2	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·016	1·039	1·053	1·082	1·104	1·117	1·160	1·174

Case II. Secondly we take $m=7$. For this case

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{7}{9}\,\frac{3\rho(21\rho + 22) + 127}{(154\rho + 45)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{22}{21} \text{ and } \rho_2 = \frac{85\sqrt{2} - 72}{21\sqrt{2}}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 10

ρ	1·048 $(=\rho_1)$	1·623 $(=\rho_2)$	2	2·25	3		5	20	∞
$(S/S_\circ)_{max}$	1	1·055	1·106	1·132	1·185	1·227	1·254	1·337	1·366

Case III. Thirdly we take $m=10$. For this case

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{15\rho\,(15\rho + 14) + 412}{18\,(35\rho + 12)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{14}{15} \text{ and } \rho_2 = \frac{118\sqrt{2} - 99}{30\sqrt{2}}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 11

ρ	0·933 $(=\rho_1)$	1·6 $(=\rho_2)$	2	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·082	1·149	1·181	1·247	1·3	1·333	1·438	1·476

Case IV. Fourthly we take $m=\infty$. In this case

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{3\rho(3\rho + 2) + 13}{9(2\rho + 1)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{2}{3} \text{ and } \rho_2 = \frac{11\sqrt{2} - 9}{3\sqrt{2}}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 12

ρ	0·667 $(=\rho_1)$	1	1·545 $(=\rho_2)$	2	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·037	1·189	1·314	1·366	1·475	1·565	1·622	1·810	1·879

The results of the above four cases are illustrated below in Fig. 20.

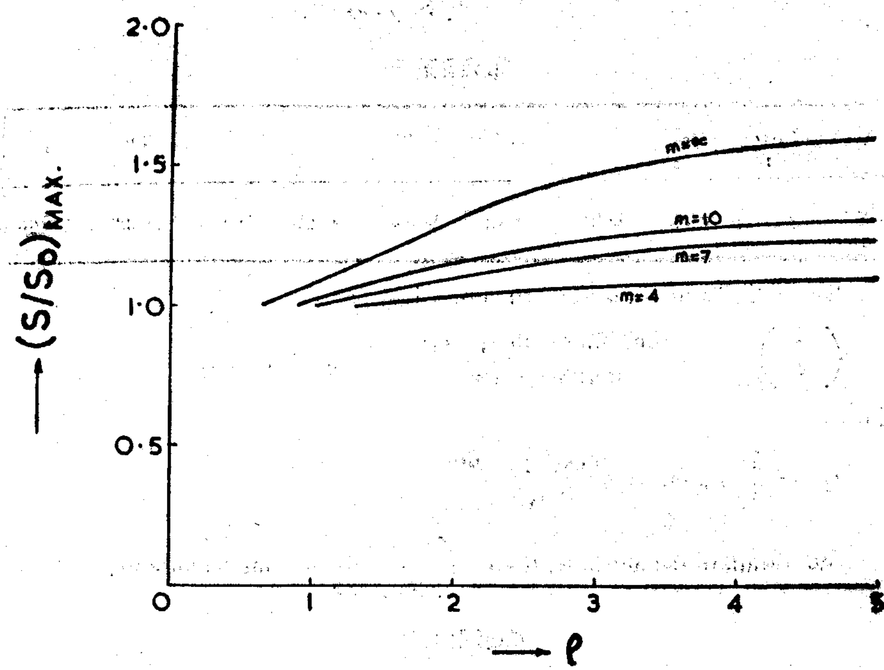

FIG 20

Variation of $\left(\frac{S}{S_o}\right)_{max}$ with ρ for quadra-tubular charge for some values of m

Variation of $\left(\frac{S}{S_o}\right)_{max}$ for the hepta-tubular charge

Tavernier[1] has shown that $\frac{S}{S_o}$ is maximum for

$$f = \frac{13\,m + 1 - 12\,m\rho}{9\,(m - 3)} \qquad \ldots \quad (61)$$

with

$$\left.\begin{aligned} \rho_1 &= \frac{m + 7}{3m} \\ \rho_2 &= \frac{13m + 1}{12m} \end{aligned}\right\} \qquad \ldots \quad (62)$$

Thus

$$\begin{aligned} \left(\frac{S}{S_o}\right)_{max} &= 1 & \rho &< \rho_1 \\ &= \alpha + \frac{\beta^2}{4\gamma} & \rho_1 \leqslant \rho &\leqslant \rho_2 \\ &= \alpha & \rho &> \rho_2 \end{aligned}$$

We shall now consider the following four cases:

Case I. Firstly we take $m = 4$. Then substituting for α, β and γ, we obtain

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{24\rho\,(12\rho + 11) + 323}{9\,(88\rho + 9)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{11}{12} \text{ and } \rho_2 = \frac{53}{48}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 13

ρ	0·917 $(=\rho_1)$	1·104 $(=\rho_2)$	2	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·011	1·064	1·072	1·087	1·099	1·107	1·129	1·136

Case II. Secondly we take, $m = 7$. In this case

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{21\rho\,(3\rho + 2) + 55}{9\,(14\rho + 3)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{2}{3} \text{ and } \rho_2 = \frac{23}{21}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 14

ρ	0·667 $(=\rho_1)$	0·9	1·095 $(=\rho_2)$	1·5	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·024	1·070	1·155	1·238	1·283	1·317	1·339	1·405	1·429

Case III. Thirdly we take $m=10$. In this case

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{60\rho\,(30\rho + 17) + 1415}{9\,(340\rho + 93)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with

$$\rho_1 = \frac{17}{30} \text{ and } \rho_2 = \frac{131}{120}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 15

ρ	0·567 ($=\rho_1$)	0·7	0·9	1·092 ($=\rho_2$)	1·5	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·011	1·056	1·119	1·234	1·348	1·410	1·458	1·489	1·584	1·615

Case IV. Fourthly we take $m=\infty$. In this case

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{6\rho\,(3\rho+1)+11}{9\,(2\rho+1)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2.$$

with

$$\rho_1 = \frac{1}{3} \text{ and } \rho_2 = \frac{13}{12}$$

We tabulate below the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for some values of ρ.

TABLE 16

ρ	0·333 ($=\rho_1$)	0·9	1·083 ($=\rho_2$)	2	2·25	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·229	1·355	1·775	1·841	1·982	2·097	2·171	2·412	2·5

The results of the above four cases are illustrated in the following figure 21.

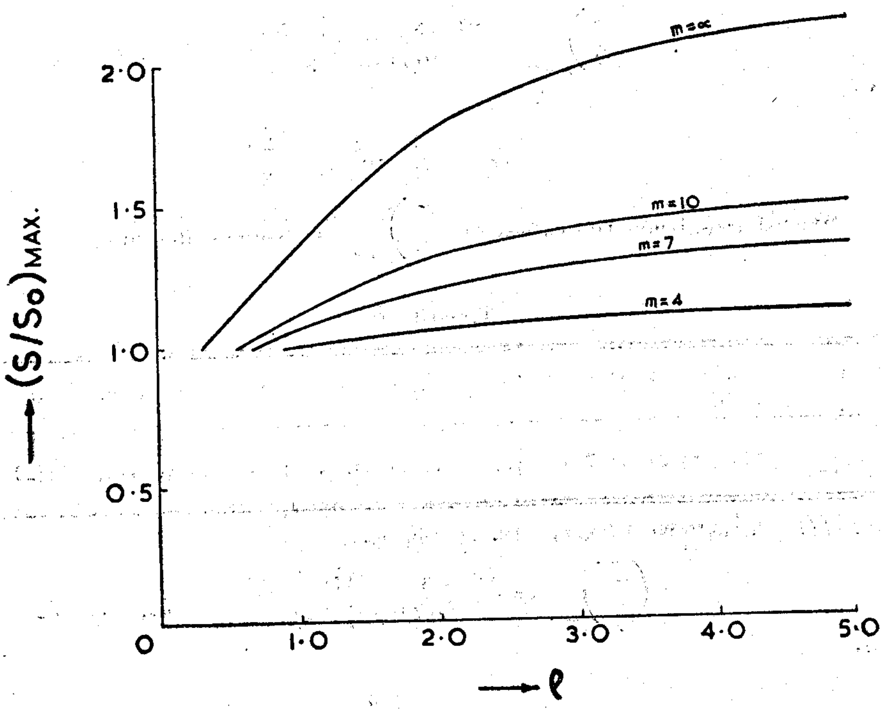

FIG 21

Variation $\left(\frac{S}{S_\circ}\right)_{max}$ with ρ for hepta-tubular charge for some values of m

We shall now examine the behavour of $\left(\frac{S}{S_\circ}\right)_{max}$ when ρ is infinite, for the three charges. It is found that

$$\left(\frac{S}{S_\circ}\right)_{max} \text{(for the tri-tubular charge)} = a = \frac{5-\sqrt{3}}{2}\left(\frac{m+1}{m+3}\right)$$

which is always $< \frac{5-\sqrt{3}}{2}$

$$\left(\frac{S}{S_\circ}\right)_{max} \text{(for the quadra-tubular charge)}$$

$$= a = \frac{8-3\sqrt{2}}{2}\left(\frac{m+1}{m+4}\right)$$

which is always $< \frac{8-3\sqrt{2}}{2}$

$$\left(\frac{S}{S_\circ}\right)_{max} \text{(for the hepta-tubular charge)}$$

$$= a = \frac{5}{2}\left(\frac{m+1}{m+7}\right)$$

which is always $< \frac{5}{2}$

In the following table are exhibited the values of $\left(\frac{S}{S_\circ}\right)_{max}$ for the three charges for some values of m.

TABLE 17

$(S/S_\circ)_{max}$ \ m	3	4	5	6	7	8	9	10	20	∞
Tri-tubular ..	1·089	1·167	1·225	1·271	1·307	1·337	1·362	1·383	1·492	1·634
Quadra-tubular	1·073	1·174	1·252	1·315	1·366	1·409	1·445	1·476	1·644	1·879
Hepta-tubular	1	1·136	1·25	1·346	1·429	1·5	1·563	1·618	1·944	2·5

The above results are illustrated below in fig. 22.

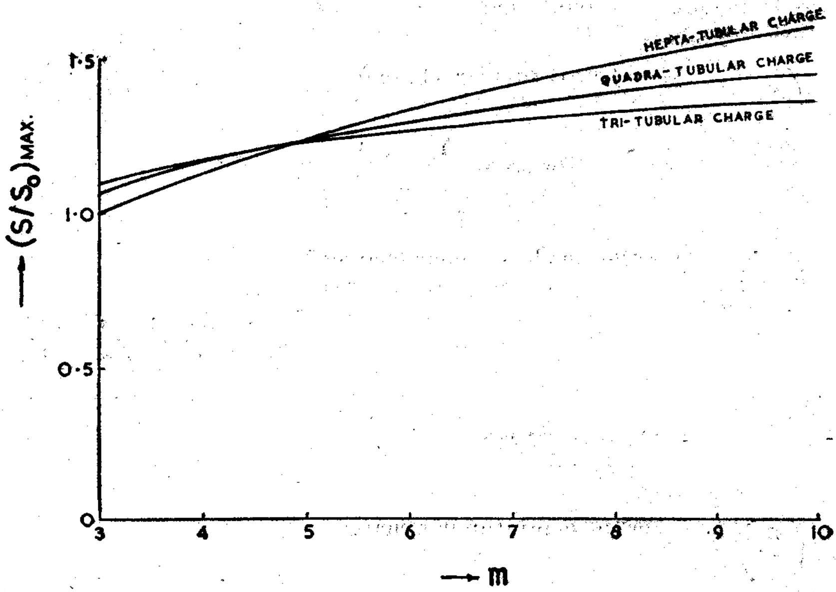

FIG 22

Variation of $(S/S_{\circ})_{max}$ with m for the three charges in the case when ρ is infinite

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