ONE OPERATIONAL AMPLIFIER SIMULATES THIRD ORDER SYSTEMS WITH A LEAD AND A TIME-CONSTANT

L. K. Wadhwa* and H. S. Rao†

Defence Research and Development Organisation, New Delhi

A method has been developed for the simulation of third order systems with a lead and a time constant using only one operational amplifier. Three basic circuits each consisting of one operational amplifier, three capacitors and six resistors have been analysed and conditions of physical realisability discussed.

In previous communications[1-4] on this subject particular classes of the general third order linear systems were considered for simulation with only one operational amplifier. The purpose of this paper is to consider another particular class of systems, that is, third order systems with a lead and a time-constant, which are characterised by a transfer function of the form

$$F(s) = -\frac{b_1 s\,(b_2 s + 1)}{a_3 s^3 + a_2 s^2 + a_1 s + 1} \tag{1}$$

Of the various possible circuits each employing three capacitors and six resistors, only three will be discussed here. The design formulae and conditions for physical realisability have also been obtained.

SIMULATION OF THIRD ORDER SYSTEMS

A network for the simulation of third order systems is shown in Fig. 1 and its transfer function has been shown[4] to be

$$\frac{E_0}{E_1} = -\frac{\gamma_1\gamma_3\gamma_5}{\gamma_6(\gamma_1+\gamma_2+\gamma_8)(\gamma_3+\gamma_4+\gamma_5+\gamma_7)+\gamma_3\gamma_8(\gamma_4+\gamma_5+\gamma_7)+\gamma_5\gamma_7(\gamma_1+\gamma_2+\gamma_3+\gamma_8)+\gamma_3\gamma_5\gamma_8} \tag{2}$$

Simulation of the system of Eq (1) with the network of Fig. 1 is possible if the admittances (γ's) are properly chosen; and furthermore, it is seen from Eq (2) that at least three of the appropriate admittances will be required to be capacitative. Ten basic circuits each employing three capacitors and six resistors are possible.

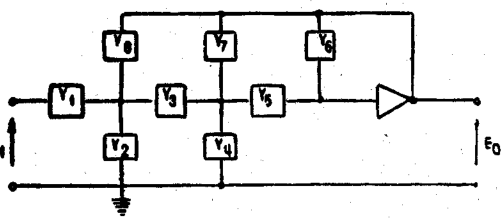


Fig. 1—Network for Simulating third order systems

* At present at the Electronics & Radar Development Establishment, Bangalore.

† At present at the Defence Research & Development Laboratory, Hyderabad.

Each basic circuit can have nine parameters; since the number of constants in the system represented by Eq (1) is five, a choice of resistor values is required so as to reduce the number of circuit parameters to the number of constants in Eq (1). This requirement offers considerable latitude in the choice of intended design values of resistors as a result of which a number of circuits are possible which are essentially a variation of the basic circuit employing three capacitors and six resistors. Each circuit, for its physical realisability, will require satisfaction of a set of conditions which will be different for every circuit. Only three basic circuits each with a certain arbitrary choice of resistor values, have been presented and the conditions of physical realisability discussed and obtained.

(a) γ_3, γ_5 and γ_7 Capacitative

A possible circuit for simulating the system of Eq (1) is shown in Fig. 2 (a), in which

$$\left.\begin{aligned} \gamma_3 &= SC_3;\ \gamma_5 = \left(SC_5 + \frac{1}{\beta R}\right);\ \gamma_7 = SC_7 \\ \gamma_1 &= \gamma_6 = \gamma_8 = \frac{1}{R};\ \gamma_2 = \frac{1}{\alpha R};\ \gamma_4 = \frac{1}{\beta R} \end{aligned}\right\} \quad (3)$$

Substituting Eq (3) into Eq (2) and simplifying

$$\frac{E_o}{E_1} = -\frac{\frac{\alpha}{2(2\alpha+1)} RC_3 S(\beta RC_5 S+1)}{\frac{\alpha\beta}{2(2\alpha+1)} R^3 C_3 C_5 C_7 S^3 + \left[\frac{\alpha\beta}{(2\alpha+1)} R^2 C_3 C_5 + \frac{\beta}{2} R^2 C_5 C_7 + \frac{\alpha(\beta+1)}{2(2\alpha+1)} R^2 C_3 C_7\right] S^2 + \left[\frac{(3\alpha+2\alpha\beta+\beta)}{2(2\alpha+1)} RC_3 + \frac{\beta}{2} RC_5 + \frac{(2\alpha+2\alpha\beta+\beta+1)}{2(2\alpha+1)} RC_7\right] S+1} \quad (4)$$

Equations (1) and (4) will be identical if

$$b_1 = \frac{\alpha}{2(2\alpha+1)} T_3 \quad (5)$$

$$b_2 = \beta T_5 \quad (6)$$

$$a_1 = \frac{(3\alpha+2\alpha\beta+\beta)}{2(2\alpha+1)} T_3 + \frac{\beta}{2} T_5 + \frac{(2\alpha+2\alpha\beta+\beta+1)}{2(2\alpha+1)} T_7 \quad (7)$$

$$a_2 = \frac{\alpha\beta}{(2\alpha+1)} T_3 T_5 + \frac{\beta}{2} T_5 T_7 + \frac{\alpha(\beta+1)}{2(2\alpha+1)} T_3 T_7 \quad (8)$$

$$a_3 = \frac{\alpha\beta}{2(2\alpha+1)} T_3 T_5 T_7 \quad (9)$$

Where

$$T_n = RC_n. \quad (10)$$

Now, simulation of the system of Eq (1) with the network of Fig. 2(a) is possible only if the values of α, β, T_3, T_5, T_7 obtained as the solution of Eq (5) to (9) are real and positive. It is required to determine, therefore, in terms of the given real and positive a's and b's, the values of α, β, T_3, T_5, T_7 and find the conditions, if any, under which these can be real and positive.

Elimination of T_3, T_5 and T_7 from Eq (5), (6), (7), (9) and (5), (6), (8), (9) give the following two equations

$$\beta = \frac{a_1 - \left(\frac{a_3}{2b_1b_2} + 3b_1 + \frac{b_2}{2}\right)}{\left(\frac{b_1}{\alpha} + 2b_1 + \frac{a_3}{2b_1b_2}\right)} \tag{11}$$

and

$$\beta = \frac{b_2}{a_3}\left[a_2 - \left(\frac{a_3}{2b_1} + \frac{a_3}{b_2} + 2b_1b_2\right)\right] \tag{12}$$

which on solution yields

$$\alpha = \frac{4b_1b_2\left[a_2 - \left(\frac{a_3}{2b_1} + \frac{a_3}{b_2} + 2b_1b_2\right)\right]}{\left[\left(\frac{a_3}{b_1} + 4b_1b_2\right)^2 + 2a_3\left(2a_1 - 2b_1 - b_2\right) - 2a_2\left(\frac{a_3}{b_1} + 4b_1b_2\right)\right]} \tag{13}$$

It is evident from Eq (12) and (13) that α and β are real; and these will also be positive if

$$\left\{\frac{1}{2}\left(\frac{a_3}{b_1} + 4b_1b_2\right) + \frac{a_3\,(2a_1 - 2b_1 - b_2)}{\left(\frac{a_3}{b_1} + 4b_1b_2\right)}\right\} > a_2 > \left\{a_3\left(\frac{1}{2b_1} + \frac{\;}{b_2}\right) + 2b_1b_2\right\} \tag{14}$$

If the inequalities of (14) are satisfied then α and β will be real and positive; and as seen from Eq (5), (6) and (9) the corresponding T_3, T_5, T_7 will also be real and positive.

Therefore, the circuit of Fig 2 (a) for simulating the given system of Eq (1) is physically realisable provided the condition (14) is satisfied. The circuit component values may then be determined with the aid of Eq (12), (13), (5), (6) and (9). Having thus determined α, β, T_3, T_5, T_7 and choosing arbitrarily a convenient value for any one of the capacitors, the remaining component values may then be obtained with the aid of Eq (3) and (10).

(*b*) γ_3, γ_5, *and* γ_8 *Capacitative.*

Another possible circuit for simulating the system of Eq (1) with

$$\left.\begin{aligned} \gamma_3 &= SC_3;\ \gamma_5 = \left(SC_5 + \frac{1}{\alpha R}\right);\ \gamma_8 = SC_8 \\ \gamma_4 &= \gamma_6 = \gamma_7 = \frac{1}{R};\ \gamma_1 = \gamma_2 = \frac{1}{\beta R} \end{aligned}\right\} \tag{15}$$

is shown in Fig. 2(b).

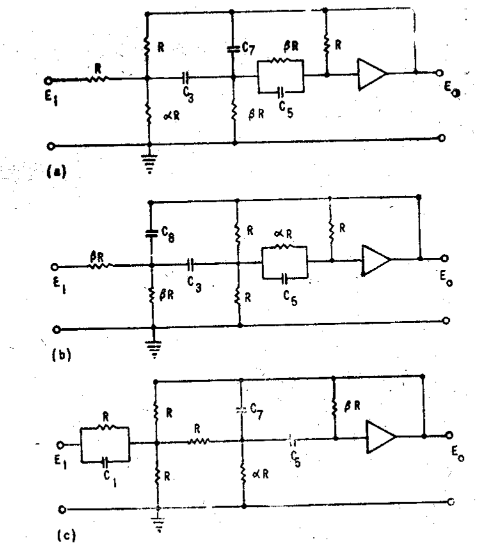


FIG. 2—Net works for the simulation of

$$\frac{E_0}{E_1} = -\frac{b_1 s\,(b_2 s + I)}{a_3 s^3 + a_2 s^2 + a_1 5 + I}$$

Substituting Eq (15) into Eq (2) and simplifying

$$\frac{E_o}{E_1} = - \frac{\frac{1}{4(\alpha+1)} RC_3S\,(\alpha RC_5S + 1)}{\frac{\alpha\beta}{4(\alpha+1)} R^3C_3C_5C_8S^3 + \left[\frac{\alpha\beta}{2(\alpha+1)} R^2C_3C_5 + \frac{\alpha\beta}{2(\alpha+1)} R^2C_5C_8 + \frac{\beta}{4} R^2C_3C_8\right]S^2 + \left[\frac{(\alpha+\alpha\beta+\beta)}{2(\alpha+1)} RC_3 + \left(\frac{\alpha}{\alpha+1}\right)RC_5 + \frac{\beta}{2}RC_8\right]S + 1} \tag{16}$$

Equations (1) and (16) will be identical if

$$b_1 = \frac{1}{4(\alpha+1)}\, T_3 \tag{17}$$

$$b_2 = \alpha\, T_5 \tag{18}$$

$$a_1 = \frac{(\alpha+\alpha\beta+\beta)}{2(\alpha+1)} T_3 + \left(\frac{\alpha}{\alpha+1}\right) T_5 + \frac{\beta}{2}\, T_8 \tag{19}$$

$$a_2 = \frac{\alpha\beta}{2(\alpha+1)} T_3\,T_5 + \frac{\alpha\beta}{2(\alpha+1)}\, T_5\,T_8 + \frac{\beta}{4}\, T_3T_8 \tag{20}$$

$$a_3 = \frac{\alpha\beta}{4\,(\alpha+1)}\, T_3\,T_5\,T_8 \tag{21}$$

where

$$T_n = RC_n. \tag{22}$$

Elimination of β, T_3, T_5, and T_8 from Eq (17) through (21) gives a cubic

$$a_3\alpha^3 - (a_2b_2 + 2b_1b_2^2 - 3a_3)\alpha^2 + (a_1b_2^2 + 3a_3 - 2a_2b_2 - 2b_1b_2^2)\alpha - (a_2b_2 - a_1b_2^2 - a_3 - b_2^3) = 0 \tag{23}$$

which will yield at least one real and positive α, as shown in the Appendix, corresponding to which a set of real positive β, T_3, T_5, and T_8 exists; provided any one set of the conditions listed in any one of the following three cases is satisfied

(*i*) $A_1 > o,\ B_1 \gtrless o$; *and* $A_2 > o,\ B_2 \gtrless o$

Either $OL > OK,\ OB > OA$ (24)

or $OK > OL,\ OA > OB$ (24a)

(*ii*) $A_1 > o,\ B_1 \gtrless o$; *and* $A_2 < o,\ B_2 > o$

$$\left.\begin{aligned} &\Delta \equiv (\,B_2^2 + 8A_2b_1\,) > o. \\ &OB > OA > OD \end{aligned}\right\} \tag{25}$$

(*iii*) $A_1 > o,\ B_1 \gtrless o;\ A_2 = o,\ B_2 > o$

$$OE > OA. \tag{26}$$

where

$$OL \equiv \frac{A_1}{2b_1} \equiv \frac{1}{2b_1}\left(a_1 + b_2 - \frac{a_3}{2b_1b_2}\right)$$

$$OK \equiv \frac{a_3}{4b_1^2b_2^2} A_2 \equiv \frac{a_3}{4b_1^2b_2^2}\left(\frac{2a_2b_1b_2}{a_3} - 2b_1 - b_2\right)$$

$$OA = \frac{B_1 + \sqrt{B_1^2 + 8A_1b_1}}{4b_1}, \quad OD = \frac{B_2 - \sqrt{B_2^2 + 8A_2b_1}}{4b_1}$$

$$OB = \frac{B_2 + \sqrt{B_2^2 + 8A_2b_1}}{4b_1}, \qquad OE = \frac{B_2}{2b_1}$$

$$B_1 = \left(a_1 - \frac{a_3}{2b_1b_2} - 2b_1\right), \qquad B_2 = \left(\frac{2a_2b_1b_2}{a_3} - 4b_1\right)$$

The procedure for design would be to compute the values of A_1, A_2, B_1, B_2 and see if any one of the three cases mentioned above is applicable ; and then check and see if the set of conditions listed under that case are satisfied. The fulfilment of these conditions signifies that the circuit of Fig. 2(*b*) for simulating the given system is physically realisable. The next step then would be to solve the cubic of Eq (23) and obtain α. Substitution of the real and positive value (s) of α into Eq (17) and (18) gives T_3 and T_5 and T_8 may then be obtained by solving Eq (19) and (21). The circuit component values may be calculated with the aid of Eq (15) and (22).

(*c*) γ_1, γ_5 *and* γ_7 *Capacitative*

Another possible circuit is shown in Fig. 2 (*c*), in which

$$\left.\begin{aligned} \gamma_1 &= \left(SC_1 + \frac{1}{R}\right), \quad \gamma_5 = SC_5, \quad \gamma_7 = SC_7 \\ \gamma_2 &= \gamma_3 = \gamma_8 = \frac{1}{R}, \quad \gamma_4 = \frac{1}{\alpha R}, \quad \gamma_6 = \frac{1}{\beta R} \end{aligned}\right\} \tag{27}$$

Substituting Eq (27) into Eq (2) and simplifying

$$\frac{E_0}{E_1} = -\frac{\frac{\alpha\beta}{(3\alpha+4)} RC_5 S (RC_1S+1)}{\left(\frac{\alpha\beta}{3\alpha+4}\right) R^3C_1C_5C_7S^3 + \left(\frac{\alpha}{3\alpha+4}\right)\left[R^2C_1C_5 + 4\beta R^2C_5C_7 + R^2C_1C_7\right]S^2 + \left[\left(\frac{\alpha+1}{3\alpha+4}\right) RC_1 + \frac{\alpha(\beta+4)}{(3\alpha+4)} RC_5 + \left(\frac{4\alpha}{3\alpha+4}\right) RC_7\right] S + 1} \tag{28}$$

Equations (1) and (28) will be identical if

$$b_1 = \left(\frac{\alpha\beta}{3\alpha+4}\right) T_5 \tag{29}$$

$$b_2 = T_1 \tag{30}$$

$$a_1 = \left(\frac{\alpha+1}{3\alpha+4}\right) T_1 + \frac{\alpha(\beta+4)}{(3\alpha+4)} T_5 + \left(\frac{4\alpha}{3\alpha+4}\right) T_7 \tag{31}$$

$$a_2 = \left(\frac{\alpha}{3\alpha+4}\right)\left[T_1 T_5 + 4\beta T_5 T_7 + T_1 T_7\right] \tag{32}$$

$$a_3 = \left(\frac{\alpha\beta}{3\alpha+4}\right) T_1 T_5 T_7 \tag{33}$$

Where

$$T_n = RC_n. \tag{34}$$

Elimination of T_1, T_5 and T_7 from Eq (29), (30), (31), (33) and (29), (30), (32), (33) gives the following two equations

$$\frac{4b_1}{\beta} = \left(a_1 - b_1 - \frac{b_2}{3} - \frac{4a_3}{3b_1b_2}\right) + \frac{\left(\frac{16a_3}{b_1b_2} + b_2\right)}{3(3\alpha+4)} \tag{35}$$

and $$\frac{b_1 b_2}{\beta} = \left(a_2 - \frac{a_3}{3b_1} - \frac{4a_3}{b_2}\right) + \frac{4a_3}{3b_1(3\alpha+4)} \quad (36)$$

which on solution yields

$$\alpha = \frac{4}{3}\left[\frac{(a_1 b_2^2 + 16a_3) - \left(4a_2b_2 + b_1 b_2^2 + \frac{b_2^3}{4}\right)}{\left(4a_2b_2 + b_1b_2^2 + \frac{b_2^3}{3}\right) - (a_1 b_2^2 + 16a_3)}\right] \quad (37)$$

and

$$\beta = \frac{b_1^2 b_2^4}{4a_3}\left[\frac{1}{4a_2b_2 + \left(\frac{a_2b_2}{4a_3}\right)b_1b_2^2 + \frac{b_2^3}{4} - (a_1b_2^2 + 16a_3)}\right] \quad (38)$$

Now, it will be seen from Eq (37) and (38) that α and β are real. These will be also positive if

$$\text{Min}\left[\left(4a_2b_2 + b_1b_2^2 + \frac{b_2^3}{3}\right), \left\{4a_2 b_2 + \left(\frac{a_2b_2}{4a_3}\right)b_1b_2^2 + \frac{b_2^3}{4}\right\}\right] > (a_1 b_2^2 + 16a_3) > \left(4a_2b_2 + b_1b_2^2 + \frac{b_2^3}{4}\right) \quad (39)$$

Therefore, if the inequalities of (39) are satisfied then the corresponding T_1 T_5 and T_7 as seen from (29), (30), (33) are also real and positive and the circuit of Fig. 2 (c) for simulating the system of (1) is physically realisable. The circuit component values may be then obtained with the aid of (37), (38), (29), (30), (33), (34) and (27).

CONDITIONS UNDER WHICH THE CIRCUIT OF FIGURE 2(b) IS PHYSICALLY REALISABLE

Simulation of the system represented by Eq (1) with the network of Fig. 2 (b) is possible only if the values of α, β, T_3, T_5 and T_8 are obtained as the solution of equations

$$b_1 = \frac{1}{4(\alpha+1)} T_3 \quad (1\cdot1)$$

$$b_2 = \alpha\, T_5 \quad (1\cdot2)$$

$$a_1 = \frac{(\alpha+\alpha\beta+\beta)}{2(\alpha+1)} T_3 + \left(\frac{\alpha}{\alpha+1}\right)T_5 + \frac{\beta}{2} T_8 \quad (1\cdot3)$$

$$a_2 = \frac{\alpha\beta}{2(\alpha+1)} T_3 T_5 + \frac{\alpha\beta}{2(\alpha+1)} T_5 T_8 + \frac{\beta}{4} T_3 T_8 \quad (1\cdot4)$$

$$a_3 = \frac{\alpha\beta}{4(\alpha+1)} T_3 T_5 T_8 \quad (1\cdot5)$$

are real and positive; where a's and b's are real and positive constants.

It is, therefore, required to determine the conditions under which α, β, T_3, T_5, and T_8 can be real and positive; and graphical methods may be perhaps a convenient means of obtaining these.

Simulation of T_3, T_5 and T_8 from Eq (1·1), (1·2), (1·3), (1·5) and (1·1), (1·2), (1·4), (1·5) give the following two equations

$$\beta = \frac{-2b_1\alpha^2 + B_1\alpha + A_1}{2b_1(\alpha+1)^2} \quad (1\cdot6)$$

$$\beta = \frac{a_3[-2b_1\alpha^2 + B_2\alpha + A_2]}{4b_1^2b_2^2(\alpha+1)} \quad (1\cdot7)$$

The intersection of the curves of Eq (1·6) and (1·7) in the first quadrant of the α—β plane will give both α and β as real and positive. It is seen from Eq (1·1), (1·2) and (1·5) that the corresponding T_3, T_5 and T_8 will be also real and positive. It is clear, therefore, that only the portion of the curves lying on the right of the β—axis are of interest.

The curve of Eq (1·6) will cut the α-axis (i.e. $\beta=0$) at two points (A, A′) whose co-ordinates may be obtained by equating to zero the right hand side of Eq (1·6) and solving the resulting quadratic

$$2b_1\alpha^2 - B_1\alpha - A_1 = o. \tag{1·8}$$

the roots of which are

$$\alpha_{(A,A')} = \frac{B_1 \pm \sqrt{B_1^2 + 8A_1b_1}}{4b_1} \tag{1·9}$$

where

$$\left.\begin{aligned} A_1 &= \left(a_1 + b_2 - \frac{a_3}{2b_1b_2}\right) \\ B_1 &= \left(a_1 - 2b_1 - \frac{a^3}{2b_1b_2}\right) \end{aligned}\right\} \tag{1·10}$$

Now, if

and

$$\left.\begin{aligned} &A_1 > o \\ &B_1 \gtrless o \end{aligned}\right\} \tag{1·11}$$

then Eq (1·8) will have one positive and one negative real root. The other two cases, that is, $A_1 < o$ and $B_1 < o$ give two negative and perhaps complex roots; while $A_1 < o$ and $B_1 > o$ is not possible, in view of the fact that a's and b's are real and positive constants.

Similarly Eq (1·7) will cut the α—axis at two points (B, D) whose α—coordinates are

$$\alpha_{(B,D)} = \frac{B_2 \pm \sqrt{B^2{}_2 + 8\,A_2\,b_1}}{4b_1} \tag{1·12}$$

where

$$\left.\begin{aligned} A_2 &= \left(\frac{2\,a_2\,b_1b_2}{a_3} - 2b_1 - b_2\right) \\ B_2 &= \left(\frac{2\,a_2\,b_1b_2}{a_3} - 4b_1\right) \end{aligned}\right\} \tag{1·13}$$

Now, if

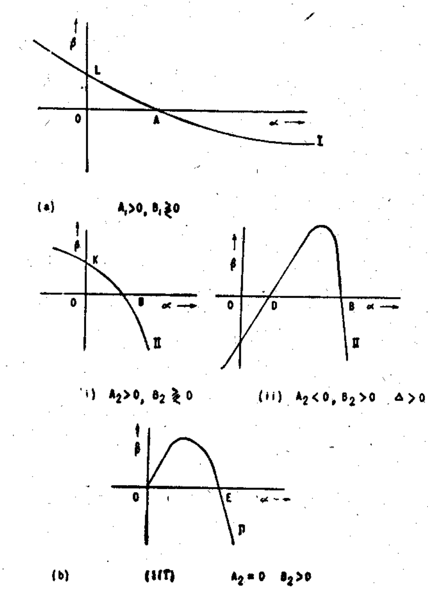
$$\left. A_2 > o \quad B_2 \gtrless o \right\} \tag{1·14a}$$

then Eq (1·12) will give one positive and one negative real roots; but if

Either $\quad A_2 < o, \quad B_2 > o \quad$ and $\triangle = (B_2^2 + 8\,A_2\,b_1) > o$ (1·14b)

or $\quad A_2 = o, \quad B_2 > o$ (1·14c)

then Eq (1·12) will give two positive real roots. The other case wherein $A_2 < o$ and $B_2 < o$ is not of interest as under these conditions the pair of roots will be negative and perhaps complex.



I $\beta = \dfrac{-2b_1 \propto^2 + \beta_1 \propto + A_1}{2b_1 (\propto + 1)^2}$

II $\beta = \dfrac{a_3 (-b_1 \propto^2 + \beta_2 \propto + A_2)}{4b_1^2 \, b_2^2 \, (\propto + 1)}$

FIG. 3—Sketches of the curves for positive $\propto$

Therefore, if the conditions as expressed in (1·11) and either in (1·14*a*) or (1·14*b*) or (1·14*c*) are satisfied then it is possible for a portion of each curve of Eq (1·6) and (1·7) to exist in the first quadrant, and it may be possible, under certain conditions for these to intersect each other at one or more points in that region. The sketches of the portions of the curves lying on the right of the β—axis are shown in Fig. 3.

It may be evident from the sketches of Fig. 3 that α and β will be real and positive if the set of conditions listed in any one of the following three cases are satisfied.

(*i*) $A_1 > o$, $B_1 \gtrless o$; and $A_2 > o$, $B_2 \gtrless o$

Either $OL > OK, \quad OB > OA$ (1·15*a*)

or $OK > OL, \quad OA > OB$ (1·15*b*)

(*ii*) $A_1 > o$, $B_1 \gtrless o$; and $A_2 < o$, $B_2 > o$

$$\left.\begin{array}{l} \triangle = (B_2^2 + 8\, A_2\, b_1) > o \\ \text{and} \quad OB > OA > OD \end{array}\right\} \quad (1{\cdot}16)$$

(*iii*) $A_1 > o$, $B_1 \gtrless o$; and $A_2 = o$, $B_2 > o$

$OE > OA$ (1·17)

Where

$$OL \equiv \frac{A_1}{2b_1} \equiv \frac{1}{2b_1}\left(a_1 + b_2 - \frac{a_3}{2b_1 b_2}\right)$$

$$OK \equiv \frac{a_3}{4b_1^2\, b_2^2} \quad A_2 \equiv \frac{a_3}{4b_1^2\, b_2^2}\left(\frac{2a_2\, b_1 b_2}{a_3} - 2b_1 - b_2\right)$$

$$OA = \frac{B_1 + \sqrt{B_1^2 + 8A_1 b_1}}{4b_1}, \quad OD = \frac{B_2 - \sqrt{B_1^2 + 8\,A_2\,b_1}}{4b^1}$$

$$OB = \frac{B_2 + \sqrt{B_1^2 \times 8A_2 b_1}}{4b_1}, \quad OE = \frac{B_2}{2b_1}$$

Therefore, if the set of conditions listed in any one of the above three cases are satisfied then it is possible to simulate the system of Eq (1) with the circuit of Fig. 2 (*b*).

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