FORM FUNCTION AND VARIATION OF BURNING SURFACE AREA FOR THE HEXA-TUBULAR CHARGE

J. N. KAPUR AND V. K. SRIVASTAVA
Indian Institute of Technology, Kanpur

The problem of combustion of a hexa-tubular charge, which is a cylindrical charge with six holes of equal diameters—one at the centre and the other five symmetrically situated about it, has been considered in this paper. The burning of the charge proceeds in three distinct phases for each of which the form function and variation of burning surface area has been investigated. Equivalent form-factor has also been found. Numerical results for some important cases are tabulated.

The present paper is in continuation of the previous paper on penta-tubular charge[1]. As in the earlier case there are three stages of burning. At the end of the first stage five quadrilateral prisms remain and after the end of the second phase (*i.e.* after rupture) there are ten curvilinear triangles.

The notations used in this paper are the same as in the previous communication[1].

FIRST PHASE OF COMBUSTION

Fig. 1 gives unburnt position in the first phase of combustion.

Initial volume of the grain is given by

$$V_o = \frac{\pi}{4} \, m\rho d^3 \, (m^2 - 6), \tag{1}$$

The volume of grain when a fraction f of D of the grain remains is given by

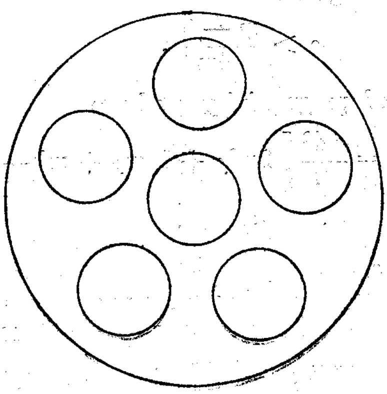

FIG. 1—Unburnt position.

$$V = \left[\pi \left\{ \frac{md}{2} - \frac{D}{2}(1-f) \right\}^2 - 6\pi \left\{ \frac{d}{2} + \frac{D}{2}(1-f) \right\}^2 \right] L, \qquad (2)$$

where the length L is given by

$$L = m\rho d - (1-f)D \qquad (3)$$

Now md and $4D + 3d$ are the two values of the exterior diameter of the grain. Hence

$$\frac{D}{d} = \frac{m-3}{4} \qquad (4)$$

On using equations (1) to (4) we get

$$z = 1 - \frac{V}{V_o} = (1-f)(a - bf - cf^2), \qquad (5)$$

where

$$a = \frac{m-3}{64\, m\rho\,(m^2-6)} \left[3(m+1)^2 + 4m\rho\,(13m+33) \right],$$

$$b = \frac{(m-3)^2}{32\, m\rho\,(m^2-6)} \left[10\, m\rho - 9(m+1) \right],$$

$$c = \frac{5\,(m-3)^3}{64\, m\rho\,(m^2-6)}.$$

At $f=o$, z becomes a which gives the fraction of the grain which is burnt at the end of the first phase of combustion, and they have been tabulated for some sets of values of m and ρ, given in Table 1.

From (3) and (4) we find that the burning will be over before rupture (in a mathematical sense), if

$$\rho \leqslant \frac{m-3}{4m},$$

and the second phase of burning will begin, if

$$\rho > \frac{m-3}{4m}.$$

For finding the function of progressivity we have

$$S_o = \frac{\pi d^2}{2} \left[2m^2\rho + 12m\rho + m^2 - 6 \right] \qquad (6a)$$

TABLE 1

VALUES OF z FOR DIFFERENT SETS OF VALUES OF m AND ρ AT $f=0$

m	4	4	7	10	∞	∞
ρ	1/2	9/4	9/4	9/4	9/4	∞
z	0·58984	0·54427	0·73867	0·77742	0·83333	0·81250

Also the surface S at any instant is given by

$$S = \left[2\pi \left\{ \frac{md}{2} - \frac{D}{2}(1-f) \right\} + 6 \times 2\pi \left\{ \frac{d}{2} + \frac{D}{2}(1-f) \right\} \right] \times$$
$$\left[m\rho d - D(1-f) \right] + 2\left[\pi \left\{ \frac{md}{2} - \frac{D}{2}(1-f) \right\}^2 \right.$$
$$\left. - 6\pi \left\{ \frac{d}{2} + \frac{D}{2}(1-f) \right\}^2 \right], \quad (6b)$$

Using equation (4), (6a) and (6b) we get

$$\frac{S}{S_o} = \alpha - \beta f - \gamma f^2 \quad (7)$$

where

$$\left.\begin{aligned} \alpha &= \frac{72m\rho(m+1) + 42m + 57 - 15m^2}{16(2m^2\rho + 12m\rho + m^2 - 6)} \\ \beta &= \frac{(m-3)(20m\rho - 23m - 3)}{8(2m^2\rho + 12m\rho + m^2 - 6)} \\ \gamma &= \frac{15(m-3)^2}{16(2m^2\rho + 12m\rho + m^2 - 6)} \end{aligned}\right\} \quad (8)$$

At the end of the first stage $\frac{S}{S_o} = \alpha$, which gives the ratio of the surface area at rupture to the initial surface area and they have been given for some set of values of m and ρ in Table 2.

TABLE 2

VALUES OF S/S_o AT THE END OF THE FIRST STAGE FOR DIFFERENT SETS OF VALUES OF m AND ρ

m	4	4	7	10	∞	∞
ρ	1/2	9/4	9/4	9/4	9/4	∞
S/S_o	0·88125	1·06086	1·20000	1·28970	1·67005	2·25000

SECOND PHASE OF COMBUSTION

In Fig. 2, O, A and B are the centres of the central and two other holes. OAB is an isosceles triangle ($\angle AOB = 4\pi/10$) of side $2a = D + d$. At the beginning of this phase there are five curvilinear prisms with cross-sections like $H'\,E'\,F'\,G'$ bounded by three circular axes of radii a and the arc $F'\,G'$ of radius $3a$.

We assume burning by parallel layers. As burning proceeds the prism with base $H'\,E'\,F'\,G'$ shrinks into prism with base $HEFG$ made of circular arcs where the radii of the arcs HE, EF and GH are 'r' while that of the arc GF is $4a - r$. Let χ, ω and ϕ be the angles as shown in Fig. 2.

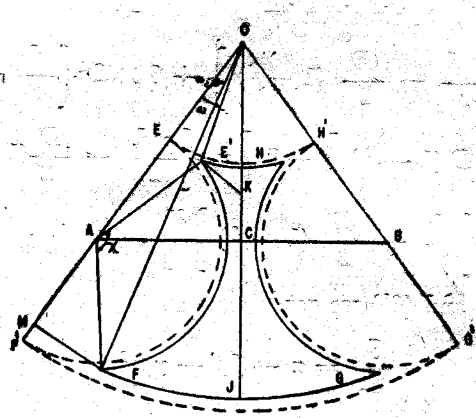

FIG. 2—Second phase of combustion.

From $\triangle\, OEE'$ and $\triangle\, OAF$, we get

$$r = a \sec \omega, \quad \cos \phi = \frac{5 - 2 \sec \omega}{4 - \sec \omega}. \tag{9}$$

End of this phase occurs when the propellant has a rupture at the point C. At this time

$$\omega = \cos^{-1} \frac{a}{2a \cos (3\pi/10)} = 31°46'; \quad \phi = 20°18'30'' \tag{10}$$

Equating the values of FM from $\triangle AFM$ and $\triangle OFM$, we get

$$\sin (\omega + \chi) = (4 \cos \omega - 1) \sin \phi. \tag{11}$$

With the help of equations (9) and (11), χ and ϕ can be obtained in terms of ω.

As r and ω increase, the length of the prism is given by

$$L = (m\rho d - D) - 2a(\sec \omega - 1) \tag{12}$$

Now area $E'FGH$

$$= 2[\text{Sector } OJF + \triangle OFA - \text{Sector } E'AF - \text{Sector } KOE' - \triangle OE'A], \tag{13}$$

$$= a^2\, H(\omega), \qquad 0 \leqslant \omega \leqslant 31°43' \tag{14}$$

where

$$H(\omega) = \Big[16 \left(\frac{\pi}{5} - \phi\right) + 8 \sin \phi - 2 \tan \omega - 2 \left\{ 4 \left(\frac{\pi}{5} - \phi\right) - \sin \phi \right\} \sec \omega - (\phi + \chi - \omega) \sec^2 \omega \Big] \tag{15}$$

Hence the volume of the prism is given by

$$V = 5\, a^2\, H(\omega).\, L. \tag{16}$$

From equation (12)

$$L = \frac{d}{4}\left[\ 4m\rho + 4 - (m+1)\sec\omega\ \right] \tag{17}$$

$$z = 1 - \frac{5}{64}\ \frac{(m+1)^2\,[4m\rho + 4 - (m+1)\ \sec\omega]}{4m\rho(m^2-6)}\ H(\omega) \tag{18}$$

Function of Progressivity

Length of the circumference of $E'FGH$

$$= 2r\left(\frac{\pi}{5} - \omega\right) + 2(4a - r)\left(\frac{\pi}{5} - \phi\right) + 2\,r\chi$$

$$= 2a\left[\ (\phi + \chi - \omega)\ \sec\omega + 4\left(\frac{\pi}{5} - \phi\right)\right] \tag{19}$$

Hence the surface exposed to cumbustion during this phase is given as

$S =$ five times the length of the circumference E'FGH $\times$ length of the grain at the instant+twice$\times$five times the area $E'FGH$.

$$= 10a\ \left[(\phi + \chi - \omega) + 4\left(\frac{\pi}{5} - \phi\right)\right] \times L + 10a^2\,H(\omega),$$

$$= \frac{5}{10}\ d^2\,(m+1)\,[4m\rho + 4 - (m+1)\sec\omega] \times$$

$$\left[(\phi + \chi - \omega)\ \sec\omega + 4\left(\frac{\pi}{5} - \phi\right) + \frac{5}{32}\,(m+1)^2 H(\omega)\right],$$

$$o \leqslant \omega \leqslant 31°43' \tag{20}$$

Using equation (6) and (20) we get

$$\frac{S}{S_o} = \frac{5\,(m+1)\left[\ 2\left\{(\phi + \chi - \omega)\ \sec\omega + 4\left(\frac{\pi}{5} - \phi\right)\right\}\times \left\{4m\rho + 4 - (m+1)\ \sec\omega\right\} + \frac{1}{6}\,(m+1)\,H(\omega)\right]}{16\pi\,(2m^2\rho + 12m\rho + m^2 - 6)}\ .\quad o \leqslant \omega \leqslant 31°43 \tag{21}$$

Also the fraction f of D is given by

$$f = \frac{2a - 2r}{D} = \left(\frac{m+1}{m-3}\right)\,(1 - \sec\omega). \tag{22}$$

THIRD PHASE OF COMBUSTION

During this phase of combustion the powder grain consists of ten curvilinear triangular prisms. Five of these have bases of the area LMN and the other five bases of the area PQR. All of them have the length

$$L = m\,\rho\,d - D - 2\,(\,R - a\,) \tag{23}$$

Let ξ, η, ζ, α and β be the angles as shown in Fig. 3. From $\triangle\ ACP$ we have

$$R = 2\,a\ \cos\frac{3\pi}{10}\ \sec\zeta = 2\,a\,k\sec\zeta. \tag{24}$$

where $k = \cos 3\pi/10$ and α will range from 31° 43′ to 36° *i.e.* ξ varies from 20° 18′ to 36°

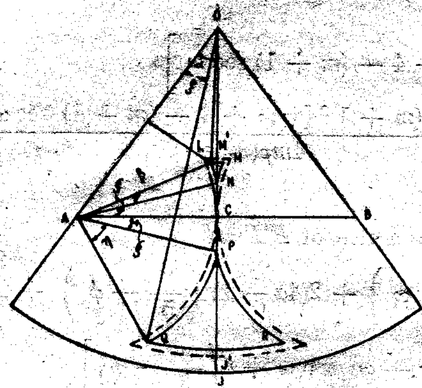

FIG. 3—Third phase of combustion

Also from $\triangle\ AOQ$

$$\cos \xi = \frac{5 - 4\, k \sec \zeta}{4 - 2\, k \sec \zeta}. \tag{25}$$

Complete combustion will occur at $\xi = 36°$, and $\zeta = 37°\ 28'$ From $\triangle$ AOQ.

$$\cos \left(\frac{3\,\pi}{10} + \eta + \zeta \right) = \frac{4\, k \sec \zeta - 3}{2\, k \sec \zeta}, \tag{26}$$

The equation (25) and (26) will give the value of η, ξ and ζ. Also in $\triangle\ OEL$

$$a = R \cos \alpha \quad \text{or} \quad R = a \sec \alpha \tag{27}$$

and

$$\alpha + \beta + \zeta = 3\pi/10 \tag{28}$$

On equating the values of R from (24) and (28) we get

$$\cos \zeta = 2 \cos \frac{3\,\pi}{10} \cos \alpha \tag{29}$$

Equations (28) and (29) will give the values of α, β for arbitrary value of ζ.

Area of the sliver LMN

$$= 2\,[\,\triangle\ ANO - \text{Sector } LM'O - \triangle\ ALO - \text{Sector } LNA\,],$$

$$= a^2 K(\alpha), \qquad 31°\ 43' \leqslant \alpha \leqslant 36° \tag{30}$$

where

$$K(\alpha) = 4\, k \sec \zeta \left[\sin \left(3\pi/10 - \zeta \right) \sin \alpha - k \left(\pi/5 + \beta - \alpha \right) \sec \zeta \right].$$

Area of the sliver PQR

$$= 2\,[\,\text{Sector } OQJ' + \triangle\ OAQ - \text{Sector } PAQ - \triangle\ APQ\,]$$

$$= a^2 K(\zeta), \qquad 0 \leqslant \zeta \leqslant 37°\ 28' \tag{31}$$

where

$$K(\zeta) = 4\left[4\left(\frac{\pi}{5} - \xi\right) + 2\sin\xi - \left\{4\left(\frac{\pi}{5} - \xi\right) + \sin\xi + \sin\left(\frac{3\pi}{10} + \zeta\right)\right\} k\sec\zeta - \left(\xi + \eta - \frac{\pi}{5}\right) k^2 \sec^2\zeta\right].$$

$$z = 1 - \frac{V}{V_\circ} = 1 - \frac{5\,a^2\,[K(\alpha) + K(\zeta)]}{\frac{\pi}{4}\, m\,\rho\, d^3\,(m^2 - 6)} \cdot L.$$

where

$$L = m\rho d - D - 2(R - a)$$

$$= \frac{d}{4}\,[4m\rho + 4 - 2(m+1)\,k\sec\zeta] \qquad (32)$$

$$z = 1 - \frac{5(m+1)\,[K(\alpha) + K(\zeta)]\,[4m\rho + 4 - 2(m+1)\,k\sec\zeta]}{64\,(m^2 - 6)\,m\rho} \qquad (33)$$

Function of Progressivity

Perimeters of the curvilinear triangles LMN and PQR are respectively

$$l_1 = 4a\left(\frac{\pi}{5} + \beta - \alpha\right) k\sec\zeta,$$

and

$$l_2 = 4a\left[2\left(\frac{\pi}{5} - \xi\right) + \left(\eta + \xi - \frac{\pi}{5}\right) k\sec\zeta\right]$$

Now the surface of combustion at instant 't' is given by

$$S = 5\,(l_1 + l_2)\,L + 10a\,[K(\alpha) + K(\zeta)].\ L \qquad (34)$$

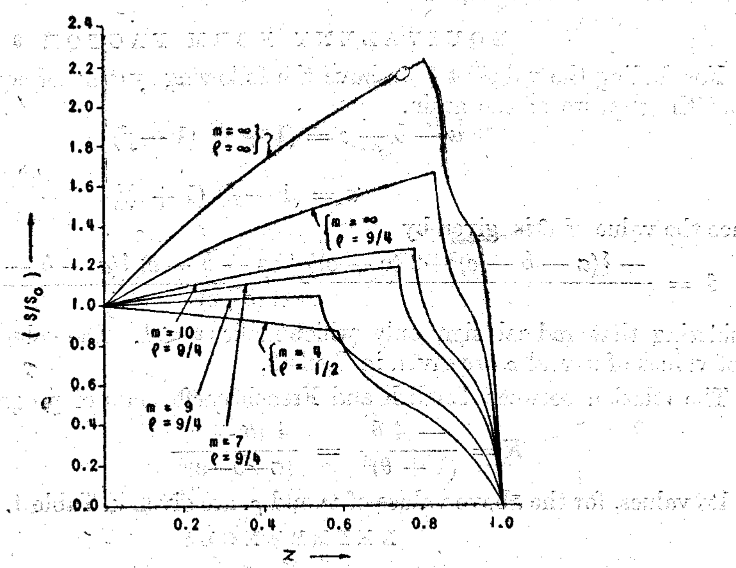

FIG. 4—Relation between $S/S_\circ$ and Z for different set of values of m and ρ

TABLE 3

EQUIVALENT FORM FACTOR BEFORE THE RUPTURE OF THE GRAIN

m \ ρ	1/2	1	9/4	5	20	∞
4	0·12950	0·00765	—0·05305	—0·07944	—0·18056	—0·24448
7	0·07592	—0·02295	—0·05092	—0·16602	—0·19315	—0·31712
10	0·15239	—0·04432	—0·15987	—0·21263	—0·24427	—0·35098
∞	0·11696	—0·11785	—0·29439	—0·36379	—0·40033	—0·42021

TABLE 4

RELATION BETWEEN ENGLISH AND FRENCH COEFFICIENTS

m \ ρ	½	1	9/5	5	20	∞
4	—0·30878	—0·02478	0·23599	0·3748	0·46410	1·4486
7	—0·4526	0·09342	0·31714	0·95449	0·18618	2·72958
10	—0·45896	0·19436	0·90605	1·3708	1·71051	3·33299
∞	—0·375	0·60554	2·37183	3·59504	4·50697	5·40000

Hence equations (6*a*) and (34) give

$$\frac{S}{S_o} = \frac{5}{16}\,\frac{(m+1)\left[4\left\{2\left(\frac{\pi}{5}-\xi\right)+k\sec\zeta\left(\beta+\frac{\pi}{5}-\alpha+\eta+\xi-\frac{\pi}{5}\right)\right\}\right]\times[4m\rho+4-2\,(m+1)\,k\sec\zeta]+[K\,(\alpha)+K\,(\zeta)]\,(m+1)}{\pi\,(2m^2\rho+12m\rho+m^2-6)} \tag{35}$$

Also the fraction f of D is given by

$$f = \frac{2a-2R}{D} = \frac{m+1}{m-3}\,(1-k\sec\zeta). \tag{36}$$

The equations (5), (10) and (33) give the values of z in terms of f. Equations (7), (21) and (35) give the value of (S/So) in terms of 'f'. These have been calculated, and their relations are shown in Fig. 4.

EQUIVALENT FORM FACTOR θ

For finding the value of θ we have the following system of equations[2] for the period before the rupture of the grain.

$$a-b-c = (1+\theta)\,(1-f)$$

and

$$a = (1-f)\,(1+\theta f) \tag{37}$$

Hence the value of θ is given by

$$\theta = \frac{-\{(a-b-c)^2+2\,(b+c)\}\pm(a-b-c)\,\{(a-b-c)^2+4\,(b+c)\}^{\frac{1}{2}}}{2\,(b+c)} \tag{38}$$

considering that radical sign only which makes $\theta < 1$. The values of θ for the different set of values of m and ρ are given in Table 3.

The relation between English and French coefficients of progressivity is,

$$K = \frac{-4\,\theta}{(1+\theta)^2} = \frac{4\,(b+c)}{(a-b-c)^2} \tag{39}$$

Its values, for the above values of m and ρ, are given in Table 4.

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1. KAPUR, J. N. & SRIVASTAVA, V. K., *Def. Sci. J.*, 14 (1964), 71.
2. TAVERNIER, P., *Mem. Art. franc.* 4° (1956), 1015.