ESTIMATION OF SERVICE LIFE OF STORES BY ACTUARIAL METHODS

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A method for estimating the average life of certain military stores has been evolved using survival ratios. The method has been explained with reference to a numerical example.

In many instances one is interested in estimating the average life of service stores like electronic equipments, engines, tyres, clothes, bulbs, items of furniture etc. when there is no record of the failure data, and what is available is only the record of the dates of installation over fairly long period of time. In this paper a method for estimating the average service life of stores has been evolved by using actuarial methods.

NOTATION AND ASSUMPTIONS

Let N be the total number of items for which installation dates are available for a given store at the beginning. Assume that all the items are subject to uniform wear and tear. No item is added or removed except the failed ones which are replaced by new ones. These items are regularly observed at an interval of time t. Let ${}^{o}N^{1}_{x}$ represent the initial frequencies of the store under observation in the class $x-\frac{1}{2}t$ and $x+\frac{1}{2}t$, the class interval being t. ${}^{n}N_{x}$ will denote the number of items observed at the pivotal age x in the nth observation at time nt. Then ${}^{n}N_{x}$ and ${}^{n}N_{x+t}$ are the observations of the items at the pivotal ages x and $x+t$ respectively. Following the usual life table notation (Barclay[1]) let l_x denote the number surviving to age x, L_x the number of years lived by the surviving items between age x and $x+t$, T_x the total number of years lived by the cohort (items installed at the same time initially) after reaching age x and e_x the average number of years lived by the survivors cohorts from exact age x onwards or expectation of life at age x. The expectation of life at birth is the average service life of the item under consideration.

It is also assumed that the ratio $\dfrac{{}^{n}N_{x+t}}{{}^{(n-1)}N_{x}}$ is equal to the survival ratio

$$\frac{l_{x+t}}{l_x} = {}^{t}P_x$$

THE METHOD OF ESTIMATION

The method of computation has been explained by means of a numerical example which has been constructed. In Table 1, column 1 gives the age at intervals of time t starting from zero. In column 2, the total number of N (=890) items are classified into different ages giving the first set of observations $^{\circ}N_x$ at the age x. After time t, second observation is made and the items falling in different ages $^{1}N_{x+t}$ will be the survivors up to age $x+t$ out of the observations $^{\circ}N_x$ at age x. That is, out of 210 items of age zero only 191 survive to the age 1 and out of 181 at age 1 only 164 survive to the age 2 at the second observation and so on. The number of observations at age zero will be those which have been replaced during the time interval between successive observations.

The ratio $\frac{^{1}N_{x+t}}{^{\circ}N_x}$ gives an estimate of the proportion surviving from age x to age $x+t$.

This we can safely assume to be equal to the survival ratio $\frac{l_{x+t}}{l_x} = {}^{t}P_x$ and is given in column 4 of the Table.

In column 5, we give the number of survivors l_x to age x out of the cohort 1000, all of them starting their life simultaneously. This is got from the relation

$$l_{x+t} = {}^{t}P_x . l_x$$

In the present example, the values at age $x+t$ in column 3 have been divided by the corresponding values at age x in column 2 and the result ${}^{t}p_x$ is written in column 4 against

the age $x+t$ *i.e.*,

$$^{1}P_{\circ} = \frac{l_1}{l_{\circ}} = \frac{^{1}N_1}{^{\circ}N_{\circ}} = \frac{191}{210} = 0.9095$$

$$^{1}P_1 = \frac{l_2}{l_1} = \frac{^{1}N_2}{^{\circ}N_1} = \frac{164}{181} = 0.9061$$

and so on.

Since $l_{\circ}=1000$, the values at age $x+t$ in column 5 is got by multiplying the value of ${}^{t}P_x$ against age $x+t$ in column 4 by the value of l_x at age x in column 5, that is

$$l_1 = {}^{1}P_{\circ} . l_{\circ} = 0 \cdot 9096 \times 1000 = 909 \cdot 5$$

$$l_2 = {}^{1}P_1 \cdot l_1 = 0 \cdot 9061 \times 909 \cdot 5 = 824 \cdot 1$$

and so on.

There is no harm in keeping the decimals at this stage since this is only an intermediate step to calculate the average life. The error introduced by the rounding off will be negligible if half of them are rounded off upwards and half downwards.

Column 6 of the Table gives L_x the total number of years lived by the cohort after reaching age x and before reaching age $x+t$. This is calculated by making use of the relation

$$L_x = \frac{t(l_x + l_{x+t})}{2}$$

Since $t=1$ in our case, $L_x = l_x + p_{x+1}$ and is calculated by taking the average of successive 2 values of column 5; that is

$$L_{\circ} = \frac{l_{\circ} + l_1}{2} = \frac{1000 + 909.5}{2} = 954.7$$

$$L_1 = \frac{l_1 + l_2}{2} = \frac{909.7 + 824.1}{2} = 866.8$$

and so on.

The cumulated sum of the values of L_x in column 6 from age x to the end of the Table is given in column 7 which represents T_x, the total number of years lived by the cohort after reaching age x till all of them go out of service.

Lastly, the ratio $\frac{T_x}{l_x} = e_x$ gives the average life expected for an item to be in service after it has served for x years. This is called the 'Expectation of Life' at age x in the usual life table terminology. The average service life of an item is the same as the life expectancy at birth and is given by $e_\circ = \frac{T_\circ}{l_\circ}$. In our present example, $e_\circ = \frac{T_\circ}{l_\circ} = \frac{5627.9}{1000}$ $= 5\cdot 6$ years.

TABLE 1

LIFE TABLE OF THE EQUIPMENT UNDER STUDY

Age in years	Number of units of various ages (initially corresponding to time 0)	Units at second observation after time t=1 year	Survival ratio	If 1000 units are put into service		
				Number surviving to age	$L_x =$	$T_x =$
x	$^\circ N_x$	$^1 N_x$	$^t p_x$	l_x	$t\,\frac{(l_x + l_{x+t})}{2}$	$\sum_x^\infty L_x$
1	2	3	4	5	6	7
0	210	126		1000·0	954·7	5627·9
1	181	191	0·9095	909·5	866·8	4673·2
2	123	164	0·9061	824·1	757·1	3806·4
3	126	103	0·8374	690·1	629·9	3049·3
4	80	104	0·8254	569·6	516·2	2419·4
5	42	65	0·8125	462·8	424·2	1903·2
6	38	35	0·8333	385·7	350·2	1479·0
7	30	31	0·8158	314·7	243·8	1128·8
8	25	26	0·8667	272·8	256·4	835·0
9	16	22	0·8800	240·1	217·6	578·6
10	8	13	0·8125	195·1	170·6	361·0
11	5	6	0·7500	146·3	117·1	190·3
12	3	3	0·6000	87·8	58·5	72·3
13	2	1	0·3333	29·3	14·7	14·7
14	1	0	0·0000	0·0000	0·0	0·0000

Average service life of the store $\frac{T_\circ}{l_\circ} = \frac{5627.9}{1000} = 5\cdot 6$ years

It may be observed that $\frac{T_o}{l_o} = \frac{\sum_{x=o}^{\infty} x d_x}{l_o}$, where d_x is the number of failures in the cohort between ages x and $x+t$. Obviously $\frac{\sum_{x=o}^{\infty} x d_x}{l_o}$, represents the average life.

The average life can also be calculated in terms of other characteristics like mileages, hours of burning etc. to suit the particular item under study.

An isolated value of e_o may not give a complete picture of the changes in economic, social and psychological behaviour of persons using the stores under study as the items are replaced according to these preferences. For this one must take a series of observations over a longer period at interval of time t and e_o values are to be calculated for each of the observations. The trend in the values of e_o, thus calculated, will throw light on these changes and will be of great help to frame the inventory policy.

REFERENCES

1. Barclay, G.W., "Techniques of Population Analysis" John Wiley & Sons, Inc. New York (1958).