ON THE FORM FUNCTION AND PROPERTIES OF MODIFIED NINETEEN-TUBULAR POWDERS II

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A second form of the modified nineteen-tubular powder (with one hole in the centre and the remaining 18 holes in 3 rings, each having 6 holes) has been studied. The form function for such a powder (called the modified nineteen-tubular powder II) has been derived. Tables for various constants occurring in the treatment have been prepared for some sets of values of m and ρ.

Tavernier[1] has studied the form function for heptatubular powders, Kapur and Jain[2] the form function for the modified heptatubular and tri-tubular charges, Patni and Kothari[3] the form function for the modified quadri-tubular powders, and the author [4,5,6], has studied the form functions for (*i*) Bi-tubular Powders (modified and unmodified), (*ii*) Nineteen-tubular Powders and (*iii*) Modified Nineteen-tubular Powders with one hole in the centre, six holes in the first (inner) ring and twelve holes in the second (outer) ring (henceforth to be called the Modified Nineteen-tubular Powders I). In the present paper a second form, called the Modified Nineteen-tubular Powder II (i.e., a charge in which the slivers remaining at the end of the first phase of combustion of the corresponding unmodified form are cut out in the very beginning of combustion and the surface inhibited from burning) has been considered, and the form function for such a powder derived. The cross sections of an unmodified and modified nineteen-tubular grains II will appear as shown in figures 1 to 4. Tables for various constants occurring in the treatment have also been prepared for some sets of values of m and ρ.

Notations

D = The exterior diameter of the charge grain.

d = The diameter of the holes of the grain.

L = The length of the grain.

e = The distance between the hole at the centre and any hole in the first ring or the distance between any hole of the third ring and the curved surface of the grain.

m = The ratio of the exterior diameter of the grain to the diameter of any hole $= \frac{D}{d}$.

ρ = The ratio of the length of the grain to the exterior diameter of the grain $= \frac{L}{D}$

S_o = The initial surface of the grain.

S = The surface of the grain at any instant t.

Z = The fraction of the charge burnt at any instant t.

f = The fraction of the initial thickness (web size e) remaining at any instant t.

δ = The propellant density.

V_0 = The initial volume of the grain.

V = The volume of the grain at any instant t.

FORM FUNCTION

With the above notations we have

$$D = m\, d \tag{1}$$

$$L = \rho\, D = m\, \rho\, d \tag{2}$$

and from fig. 3, we have

$$\frac{D}{2} = 3\, e + \frac{5\, d}{2} \tag{3}$$

$$\therefore\ e = \frac{(m - 5)\, d}{6} \tag{4}$$

The exterior diameter of the grain when the hole at the centre touches the first ring of six holes or the holes of the first, second and third rings touch as shown in figure 2,

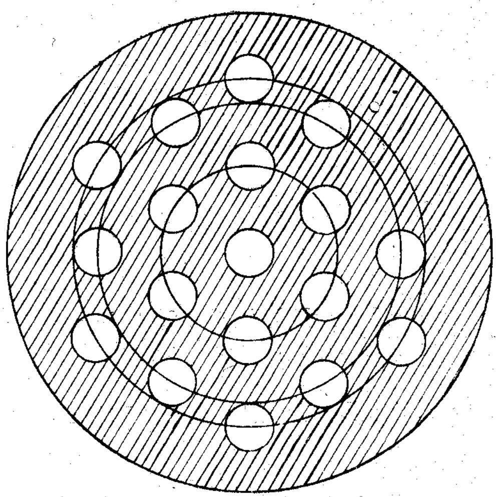

Fig. 1—Unmodified nineteen-tubular charge II

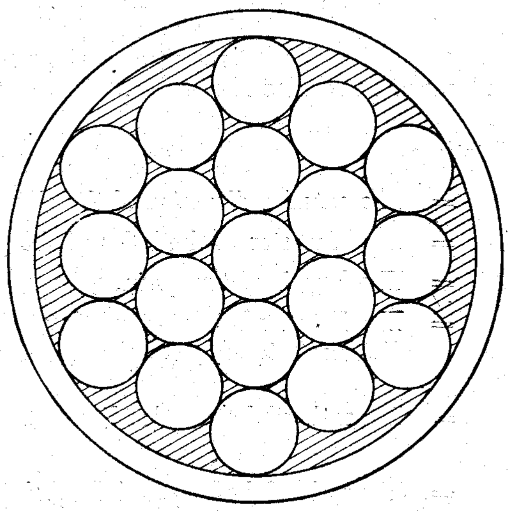

Fig. 2—Unmodified nineteen-tubular charge II showing the position of slivers at the end of first stage of burning

$$= D - e$$

$$= \frac{5}{6} (m + 1) d \tag{5}$$

The area of the base of the thirty slivers that remain at the end of the first phase of combustion of the corresponding unmodified form of the grain i.e., the area of the base of the thirty slivers cut out in the very beginning of combustion of the modified nineteen-tubular grain II

$$= \pi \left[\left(\frac{D-e}{2} \right)^2 - 19 \left(\frac{d + e}{2} \right)^2 \right]$$

$$= \frac{(m + 1)^2 d^2 \pi}{24} \tag{6}$$

The initial volume V_o of the modified nineteen-tubular grain II is given as

$$V_o = \left[\pi \left(\frac{D}{2} \right)^2 - \frac{(m + 1)^2 d^2 \pi}{24} - 19 \pi \left(\frac{d}{2} \right)^2 \right] \times L$$

$$= \frac{\pi m (m - 5) (5m + 23) \rho d^3}{24} \tag{7}$$

The volume V of the modified nineteen-tubular grain II at any instant t is given as

$$V = \left[\pi \left\{ \frac{D}{2} - \frac{e(1-f)}{2} \right\}^2 - 19 \pi \left\{ \frac{d}{2} + \frac{e(1-f)}{2} \right\}^2 - \frac{(m + 1)^2 d^2 \pi}{24} \right] \times \left[\rho m d - e (1 - f) \right]$$

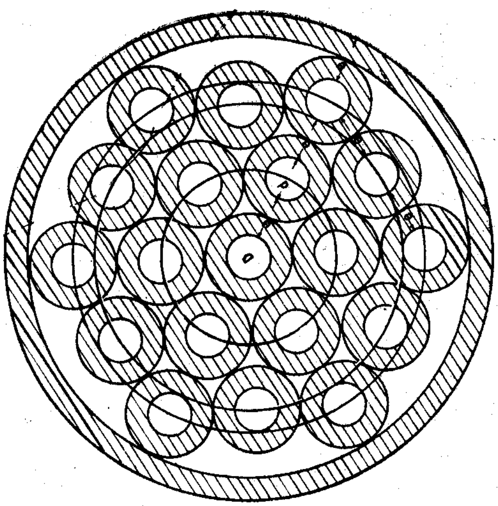

Fig. 3—Modified nineteen-tubular charge II

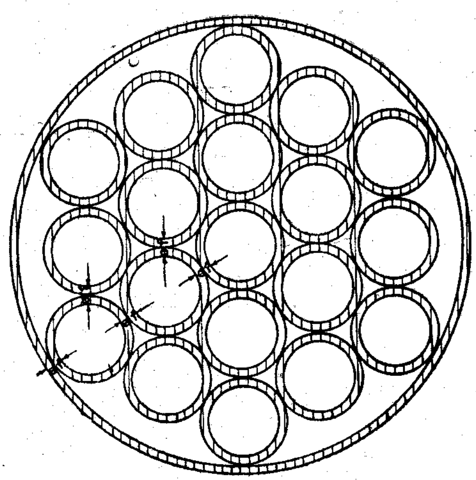

Fig. 4—Modified nineteen-tubular charge II when a fraction f of e remains

$$= \frac{\pi (m - 5) d^3}{24} \left[(5m + 23) - 3 (m - 5) (1 - f)^2 - 2 (1 - f) (m + 19) \right] \times \left[\rho m - \frac{(m - 5) (1 - f)}{6} \right] \quad (8)$$

The fraction Z of the charge burnt at any instant t is then given as

$$Z = \frac{V_\circ \delta - V\delta}{V_\circ \delta} = 1 - \frac{V}{V_\circ}. \quad (9)$$

Using (7) and (8) in (9) and simplifying we have

$$Z = (1 - f) (1 - Bf - Cf^2) \quad (10)$$

where

$$B = \frac{(m - 5) \{9m\rho - 4 (m + 1\}}{3 (5m + 23) m\rho} \quad (11)$$

$$C = \frac{(m - 5)^2}{2 (5m + 23) m\rho}. \quad (12)$$

Thus (10) gives the form function for the modified nineteen-tubular charge II.

Now

$$\frac{dZ}{df} = - (1 + B) + 2 (B - C) f + 3C f^2 \quad (13)$$

and

$$\left(\frac{dZ}{df}\right)_{f=1} = (- 1 + B + C) \quad (14)$$

$$\therefore \frac{S}{S_\circ} = \frac{\left(\frac{dZ}{df}\right)}{\left(\frac{dZ}{df}\right)_{=1}} = \alpha' - \beta' f - \gamma' f^2 \quad (15)$$

where

$$'\alpha = \frac{(1 + B)}{(1 - B - C)}$$

$$= \frac{8 (m + 1) (6m\rho - m + 5)}{\{12m\rho (m + 19) + (m - 5) (5m + 23)\}} \quad (16)$$

$$\beta' = \frac{2 (B - C)}{(1 - B - C)}$$

$$= \frac{2 (m - 5) \{18m\rho - 11m + 7\}}{\{12m\rho (m + 19) + (m - 5) (5m + 23)\}} \quad (17)$$

and

$$\gamma' = \frac{3\,C}{(1 - B - C)}$$

$$= \frac{9\,(m-5)^2}{\{12m\rho\,(m+19) + (m-5)\,(5m+23)\}} \qquad (18)$$

The values of B, C, α', β' and γ' for some sets of values of m and ρ are given in Tables 1 to 4 while Table 5 gives the values of Z and $\frac{S}{S_o}$ for various values of f.

TABLE 1

VALUES OF B, C, α', β' AND γ' FOR VARIOUS VALUES OF ρ WHEN $m=6$.

ρ	½	1	9/4	5	20	∞
B	—0·00210	0·02725	0·04356	0·05073	0·05514	0·05660
C	0·00314	0·00157	0·00070	0·00031	0·00008	0·00000
α'	0·99895	1·05774	1·09188	1·10726	1·11680	1·12000
β'	—0·01049	0·05289	0·08969	0·10626	0·11655	0·12000
γ'	0·00944	0·00486	0·00219	0·00099	0·00025	0·00000

TABLE 2

$m=7$

ρ	½	1	9/4	5	20	∞
B	—0·00164	0·05090	0·08009	0·09294	0·10082	0·10345
C	0·00985	0·00493	0·00219	0·00098	0·00025	0·00000
α'	1·00662	1·11304	1·17694	1·20623	1·22459	1·23077
β'	—0·02318	0·09739	0·16978	0·20297	0·22376	0·23077
γ'	0·02980	0·01565	0·00716	0·00326	0·00082	0·00000

TABLE 3

$m=10$

ρ	½	1	9/4	5	20	∞
B	0·00457	0·10502	0·16083	0·18539	0·20046	0·20548
C	0·03425	0·01712	0·00761	0·00342	0·00086	0·00000
α'	1·04513	1·25878	1·39597	1·46130	1·50304	1·51724
β'	—0·06176	0·20026	0·36852	0·44863	0·49982	0·51724
γ'	0·10689	0·05852	0·02746	0·01266	0·00322	0·00000

TABLE 4

$m=\infty$

ρ	$\frac{1}{2}$	1	9/4	5	20	∞
B	0·06667	0·33333	0·48148	0·54667	0·58667	0·60000
C	0·20000	0·10000	0·04444	0·02000	0·00500	0·00000
α'	1·45454	2·35294	3·12500	3·56923	3·88571	4·00000
β'	—0·36364	0·82353	1·84375	2·43077	2·84898	3·00000
γ'	0·81818	0·52941	0·28125	0·13846	0·03673	0·00000

PROGRESSIVE AND DEGRESSIVE NATURE OF THE BURNING SURFACE

Differentiating (15), we have

$$\frac{d}{df}\left(\frac{S}{S_o}\right) = -\beta' - 2\gamma' f \qquad (19)$$

and

$$\frac{d^2}{df^2}\left(\frac{S}{S_o}\right) = -2\gamma' \qquad (20)$$

From (18) we find that γ' is always positive and therefore $\frac{d^2}{df^2}\left(\frac{S}{S_o}\right)$ is always negative. Hence $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is a decreasing function of f.

When $f=1$, i.e., at the beginning of the combustion of the grain, we have

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=1} = -\beta' - 2\gamma' = \frac{4\,(m-5)\,(m+19-9m\rho)}{\{12m\rho\,(m+19)+(m-5)\,(5m+23)\}} \qquad (21)$$

and

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=0} = -\beta' = \frac{2\,(m-5)\,(11m-7-18m\rho)}{\{12m\rho\,(m+19)+(m-5)\,(5m+23)\}} \qquad (22)$$

For $\frac{S}{S_o}$ to have a maximum value at any point between the beginning and end of combustion, the values of $\frac{d}{df}\left(\frac{S}{S_o}\right)$ should have opposite signs at $f=1$ and $f=0$, that is we should have

TABLE 5

VALUES OF Z AND S/S_o FOR VARIOUS VALUES OF f.

	f	1	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$m = 6$	Z	0	0·0999	0·1999	0·3000	0·4000	0·5001	0·6002	0·7002	0·8002	0·9002	1
$\rho = \frac{1}{2}$	S/S_o	1·0	1·0007	1·0013	1·0017	1·0018	1·0018	1·0016	1·0012	1·0007	0·9999	0·9989
	f	1	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$m = 6$	Z	0	0·0974	0·1954	0·2940	0·3932	0·4930	0·5933	0·6942	0·7956	0·8975	1
$\rho = 1$	S/S_o	1·0	1·0062	1·0123	1·0183	1·0242	1·0301	1·0358	1·0414	1·0470	1·0524	1·0577
	f	1	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$m = 6$	Z	0	0·0960	0·1929	0·2907	0·3894	0·4890	0·5895	0·6908	0·7930	0·8961	1
$\rho = 9/4$	S/S^o	1·0	1·0093	1·0187	1·0280	1·0373	1·0465	1·0556	1·0648	1·0738	1·0829	1·0919
	f	1	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$m = 7$	Z	0	0·0926	0·1869	0·2829	0·3805	0·4797	0·5806	0·6830	0·7871	0·8929	1
$\rho = 9/4$	S/S_o	1·0	1·0183	1·0365	1·0546	1·0725	1·0903	1·1079	1·1254	1·1427	1·1599	1·1769
	f	1	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$m = 10$	Z	0	0·0894	0·1733	0·2651	0·3603	0·4588	0·5607	0·6657	0·7740	0·8854	1
$\rho = 9/4$	S/S_o	1·0	1·0421	1·0836	1·1245	1·1650	1·2048	1·2442	1·2829	1·3212	1·3588	1·3960
	f	1	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$m = \infty$	Z	0	0·0531	0·1173	0·1923	0·2780	0·3741	0·4802	0·5961	0·7215	0·8563	1
$\rho = 9/4$	S/S_o	1·0	1·2378	1·4700	1·6966	1·9175	2·1328	2·3425	2·5466	2·7450	2·9378	3·1250
	f	1	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$m = \infty$	Z	0	0·0460	0·1040	0·1740	0·2560	0·3500	0·4560	0·5740	0·7040	0·8460	1
$\rho = \infty$	S/S_o	1·0	1·3000	1·6000	1·9000	2·2000	2·5000	2·8000	3·1000	3·4000	3·7000	4·0000

$$\left[\frac{d}{df} \left(\frac{S}{S_o} \right) \right]_{f=1} \left[\frac{d}{df} \left(\frac{S}{S_o} \right) \right]_{f=0} < 0 \qquad (23)$$

i.e.,

$$\{m + 19 - 9m\rho\} \{11m - 7 - 18m\rho\} < 0 \qquad (24)$$

The first member of this inequality becomes zero for

$$\rho_1 = \frac{m + 19}{9m} \qquad (25)$$

and the second member for

$$\rho_2 = \frac{11m - 7}{18m} \qquad (26)$$

so that

$$\rho_2 - \rho_1 = \frac{m - 5}{2m} \qquad (27)$$

but from (4), $m > 5$ as e is positive, therefore in every case $\rho_2 > \rho_1$. Again from (2) and (4), we have

$$L = \frac{6\, m\, \rho\, e}{(m - 5)} \qquad (28)$$

and since L should be greater than or equal to e,

$$\rho \geqslant \frac{m - 5}{6m} \qquad (29)$$

Let us write

$$\rho_{min} = \frac{m - 5}{6m} \qquad (30)$$

The values of ρ_1, ρ_2 and ρ_{min} when m takes the values 6, 7, 8, 9, 10 and ∞ are given in Table 6.

TABLE 6

VALUES OF ρ_1, ρ_2 AND ρmin WHEN m=6, 7, 8, 9, 10 AND ∞.

m	6	7	8	9	10	∞
ρ_1	0·4629	0·4126	0·3750	0·3456	0·3222	0·1111
ρ_2	0·5463	0·5555	0·5625	0·5679	0·5722	0·6111
ρ_{min}	0·0277	0·0476	0·0625	0·0741	0·0833	0·1667

When we give to ρ a value lying between ρ_1 and ρ_2, the inequality (24) will be satisfied and the maximum of $\frac{S}{S_o}$ will occur when

$$\frac{d}{df}\left(\frac{S}{S_\circ}\right) = -\beta' - 2\gamma' f = O \tag{31}$$

or

$$f = -\frac{\beta'}{2\gamma'}$$

$$= \frac{(11m - 7 - 18m\rho)}{9\,(m - 5)} \tag{32}$$

which can be written as

$$f = 1 - \frac{\rho - \rho_1}{3\rho_{min}} \tag{33}$$

With the help of (25) and (26), (21) and (22) can be written as

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=1} = \frac{36m\,(m - 5)\,(\rho_1 - \rho)}{\{12m\rho\,(m + 19) + (m - 5)\,(5m + 23)\}} \tag{34}$$

and

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=0} = \frac{36m\,(m - 5)\,(\rho_2 - \rho)}{\{12m\rho\,(m + 19) + (m - 5)\,(5m + 23)\}} \tag{35}$$

Hence

(*i*) For $\rho_{min.} \geqslant \rho \leqslant \rho_1$, $\frac{d}{df}\left(\frac{S}{S_\circ}\right)$ is always positive and the powder is degressive throughout.

(*ii*) For $\rho_1 < \rho < \rho_2$, $\frac{d}{df}\left(\frac{S}{S_\circ}\right)$ is negative in the beginning and then positive so that the powder is first progressive and then degressive.

(*iii*) For $\rho \geqslant \rho_2$, $\frac{d}{df}\left(\frac{S}{S_\circ}\right)$ is always negative so that the powder is progressive throughout.

The results obtained above are given in Table 7.

These results obtained above are identical to those obtained by Tavernier[1] for the hepta-tubular powders and by the author[5,6] for (*i*) nineteen-tubular powders and (*ii*) modified nineteen-tubular powders I.

Table 8 which gives the maximum values of $\frac{S}{S_\circ}$ for some of the modified multitubular powders for some sets of values of m and ρ, will be useful for having a comparative study of these powders.

TABLE 7

S/S_o WHEN ρ LIES BETWEEN ρ_1 AND ρ_2.

	Case (*i*)	Case (*ii*)	Case (*iii*)
ρ	$\rho_{min} \leq \rho \leq \rho_1$	$\rho_1 < \rho < \rho_2$	$\rho \geq \rho_2$
$\frac{S}{S_o}$	Decreasing function of Z.	Increasing function of Z in the beginning then decreasing function of Z.	Increasing function of Z.
	The powder is degressive.	The powder is first progressive and then degressive.	The powder is progressive.

TABLE 8

VALUES of $\left(\frac{S}{S_c}\right)_{max.}$

	$m = 7$, $\rho = \frac{9}{4}$	$m = 10$, $\rho = \frac{9}{4}$	$m = \infty$, $\rho = \frac{9}{4}$	$m = \infty$, $\rho = \infty$
Modified tri-tubular[3]	1·0600	1·0700	1·1800	1·6300
Modified quadri tubular[4]	1·1352	1·1853	1·376	1·8785
Modified heptatubular[3]	1·2400	1·3500	1·8600	2·5000
Modified nineteen-tubular[1] I	1·1922	1·4135	3·1549	4·0697
Modified nineteen-tubular II	1·1769	1·3960	3·1250	4·0000

THE EQUIVALENT FORM FACTOR θ FOR THE MODIFIED NINETEEN-TUBULAR POWDER II

Following Kapur and Jain[2], the equivalent form factor θ for the modified nineteen-tubular powder II by the method of least squares is given by

$$\theta = -\left(B + \frac{C}{2}\right) \tag{36}$$

or

$$\theta = -\frac{(m - 5)(36m\rho - 13m - 31)}{12m\rho\ (5m + 23)} \tag{37}$$

which will be negative if

$$\rho > \frac{13m + 31}{36m} \tag{38}$$

Table 9 gives the values of θ for the modified nineteen-tubular charge II.

TABLE 9

VALUES OF θ FOR MODIFIED 19 TUBULAR CHARGE II.

m \ ρ	½	1	9/4	5	20	∞
6	0·0005	—0·0280	—0·0439	—0·0509	—0·0552	—0·0566
7	—0·0033	—0·0524	—0·0812	—0·0934	—0·1009	—0·1034
10	—0·0217	—0·1136	—0·1646	—0·1871	—0·2009	—0·2055
∞	—0·1667	—0·3833	—0·5037	—0·5567	—0·5892	—0·6000

MODIFIED NINETEEN TUBULAR POWDER II WITH A GIVEN VALUE OF THE EQUIVALENT FORM FACTOR θ

From (37), we have the ratio of the length to the exterior diameter of the grain as

$$\rho = \frac{(m - 5)(13m + 31)}{12m\{(5m + 23)\theta + 3(m - 5)\}} \tag{39}$$

The values of ρ for some values of m and θ are given in Table 10.

TABLE 10

VALUES OF ρ

θ \ m	6	7	10	20	∞
—0·5	..	..	..	..	2·167
—0·4	..	..	..	..	1·083
—0·3	..	..	..	2·245	0·722
—0·2	..	..	16·771	0·891	0·542
—0·1	..	14·524	0·871	0·556	0·433
0·0	0·505	0·484	0·447	0·404	0·361
0·1	0·182	0·246	0·301	0·317	0·309
0·2	0·111	0·165	0·227	0·261	0·271
0·3	0·080	0·124	0·182	0·222	0·241
0·4	0·063	0·099	0·152	0·193	0·217
0·5	0·051	0·083	0·130	0·171	0·197

Table 10 gives the value of ρ when the values of m and θ are given.

When ρ is infinite, (39) gives on simplification

$$m = \frac{15 - 23\theta}{3 + 5\theta} \tag{40}$$

Table 11 gives the values of m for $\rho = \infty$, when θ takes up the values from —0·1 to —0·5.

TABLE 11

VALUES OF m FOR $\rho = \infty$ WHEN θ TAKES DIFFERENT VALUES FROM —0·1 TO —0·5.

θ	—0·1	—0·2	—0·3	—0·4	—0·5
m	6·92	9·80	14·60	24·20	53·00

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