INTERNAL BALLISTICS OF GUNS AND ROCKETS

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An exact analytical solution of the equations relating to the internal ballistics of guns and rockets in the non-isothermal model using tubular propellants which burn according to the pressure-index law has been obtained. An approximate solution to a pre-assigned level of accuracy has also been presented.

No exact analytical solution of the equations in non-isothermal model of the internal ballistics of guns and rockets when the propellant burns according to the pressure-index law was known before Kapur[1] who first gave an exact solution of the equations in case of tubular propellant and negligible covolume effect. In the present note we have first given an alternative method of obtaining the exact analytical solution given by Kapur[1]. We have also given an approximate analytical solution in which the approximation can be pushed to any level of accuracy but complexity in calculation increases with the order of approximation. Both the exact and approximate solutions are applicable when specific relations between γ and α hold good, γ and α respectively denoting the ratio of specific heats of gases produced during burning and the index of pressure in the pressure-index law of burning. This is a serious disadvantage and till to-day no propellant is known satisfying the relation between γ and α as required by the exact analytical solution. However, there are three propellants satisfying or very nearly satisfying the specific relation between γ and α as required by the approximate analytical solution.

In the case of rockets using tubular propellants which burn according to the pressure-index law we have obtained an exact analytical solution in the non-isothermal model for the specific case of $\alpha = 0 \cdot 5$. For the solution of rockets, no specific relation between γ and α need be satisfied, unlike the case of orthodox guns.

EXACT SOLUTION FOR ORTHODOX GUNS

With notations of Kapur[2], in the case of a tubular propellant the equations giving the ballistics are

$$z=\zeta\xi + \tfrac{1}{2}(\gamma-1)\frac{\eta^2}{m} \tag{1}$$

$$\eta\frac{d\eta}{d\xi} = m\zeta \tag{2}$$

$$\eta\frac{dz}{d\xi} = \xi^{\alpha} \tag{3}$$

The initial conditions are—

$\xi = 1 \, , \eta = 0 \, , \zeta = \zeta_o = z_o$ at $z = z_o$

From (1) and (2) and with

$$x^{\gamma} = \zeta \xi^{\gamma} \tag{4}$$

we have

$$\gamma \frac{dx}{dz} = \left(\frac{\xi}{x} \right)^{\gamma - 1} = \zeta^{\frac{\gamma - 1}{\gamma}} \tag{5}$$

From (2) and (3) we get

$$\frac{d\eta}{dz} = M \zeta^{1 - a} \tag{6}$$

and equation (6) with (5) and (4) gives

$$\frac{d\eta}{dx} = M \, \gamma \zeta^{2 - a - \frac{1}{\gamma}} \tag{7}$$

From (3), (4) and (5) we obtain

$$\eta = \zeta^{a} \frac{d\xi}{dz} = \frac{\zeta^{a-2}}{\gamma} \left(\zeta - \frac{x}{\gamma} \frac{d\zeta}{dx} \right) \tag{8}$$

Equations (7) and (8) determine η and ζ as a function of x. Equation (4) determines ξ as a function of x, and z is given as a function of x by equation (1). Now we proceed to integrate (7) and (8) with initial conditions

$$\xi = 1, \ \eta = 0, \ \zeta = \zeta_o \ \text{ at } x = x_o = \zeta_o^{\frac{1}{\gamma}}$$

Let us assume that

$$2 = a + \frac{1}{\gamma} \tag{9}$$

Since generally γ is in the neighbourhood of 1·25, (9) implies that a should be about 1·2 which is somewhat greater than the values of a in the case of the usual service propellants.

With (9) equation (7) reduces to

$$\frac{d\eta}{dx} = M\gamma$$

or,

$$\eta = M\gamma \, (x - x_o) \tag{10}$$

Equation (8) we rewrite in the form

$$\zeta^{a-2} \frac{d\zeta}{dx} - \frac{\gamma}{x} \zeta^{a-1} = - \frac{\gamma^2 \eta}{x} \, ,$$

and with

$$\zeta^{a - 1} = \bar{\zeta} \tag{11}$$

this reduces to

$$\frac{d\bar{\zeta}}{dx} - \frac{\gamma(a-1)}{x}\bar{\zeta} = -\frac{\gamma^2(a-1)\eta}{x} \tag{12}$$

Equation (12) is linear and on integration yields

$$\bar{\zeta}\, x^{-\gamma(a-1)} = 1 - \gamma^2(a-1)\int_{x_0}^{x} x^{-1-\gamma(a-1)}\,\eta\, d\, x, \tag{13}$$

substituting the value of η from (10) in the integral in (13) we get

$$\zeta^{a-1} = x^{\gamma(a-1)} - M\gamma^3(a-1)\left[\frac{x - x_0\left(\frac{x}{x_0}\right)^{\gamma(a-1)}}{1-\gamma(a-1)} + x_0\,\frac{1-\left(\frac{x}{x_0}\right)^{\gamma(a-1)}}{\gamma(a-1)}\right] \tag{14}$$

(10) gives η, and (14) ζ. Then ξ can be obtained from (4) and with the determination of ξ, η and ζ we get z from equation (1). The equations are valid till $z = 1$.

APPROXIMATE SOLUTION FOR ORTHODOX GUNS

Let us assume that

$$3 = 2a + \frac{1}{\gamma} \tag{15}$$

Since γ is generally about 1·25, a will be about 1·1. Fortunately for the propellant A, the value of a is 1·11; also for propellants AN and ASN, the values are respectively 1·06 and 1·05 and equation (15) is very nearly true. The solution given below is applicable to these three propellants.

The solution follows Piccard's method of successive approximation. In equation (7) we replace ζ by its initial value and on integrating we get

$$\eta = M\gamma\zeta_0^{\,2-a-\frac{1}{\gamma}}(x - x_0) \tag{16}$$

Putting this value of η in the integral of (13) we obtain

$$\zeta^{a-1} = x^{\gamma(a-1)} - M\gamma^3(a-1)\zeta_0^{\,2-a-\frac{1}{\gamma}}\left[\frac{x - x_0\left(\frac{x}{x_0}\right)^{\gamma(a-1)}}{1-\gamma(a-1)} + x_0\,\frac{1-\left(\frac{x}{x_0}\right)^{\gamma(a-1)}}{\gamma(a-1)}\right] \tag{17}$$

(15) gives

$$2 - a - \frac{1}{\gamma} = a - 1,$$

so we can replace $\zeta^{2-\alpha-\frac{1}{\gamma}}$ in (7) by the value of ζ^{a-1} as given by (17) and on integration we get a modified value of η. Then replacing η in (13) by this modified value we get modified value of ζ^{a-1}. We may go on repeating this cycle of operations till the pre-assigned level of accuracy has been obtained. This becomes possible for the integrations involved are those of powers of x.

EXACT SOLUTION FOR ROCKETS

We have the following equations of internal ballistics of rockets using tubular propellant as deduced from those of recoilless gun given by Corner[3] by putting $W=\infty$ $X = 0$,

$$P\left(U-\frac{C}{\delta}\right) = NCRT \tag{18}$$

$$D\frac{d\phi}{dt} = \beta P^{a} \tag{19}$$

$$\frac{dN}{dt} = \frac{d\phi}{dt} - \frac{x\,SP}{C\,(RT)^{\frac{1}{2}}} \tag{20}$$

$$\frac{d\,(NT)}{dt} = T_{\circ}\frac{d\phi}{dt} - \frac{\gamma\psi\,SP\,(RT)^{\frac{1}{2}}}{CR} \tag{21}$$

These equations are to be integrated with initial conditions

$$P = 0,\ T = T_{\circ},\ N = 0,\ \phi = 0 \text{ at } t = t_{\circ}$$

Now, from the above equations with the following substitution

$$Al = U - \frac{C}{\delta} \tag{22}$$

$$\zeta = \frac{Al\ P}{CRT_{\circ}} \tag{23}$$

$$T' = \frac{T}{T_{\circ}} \tag{24}$$

$$\Psi = \frac{\psi\,SD}{\beta C\,(RT_{\circ})^{\frac{1}{2}}} \times \left(\frac{CRT_{\circ}}{Al}\right)^{1-a} \tag{25}$$

we obtain the following equations

$$\zeta = NT' \tag{26}$$

$$\frac{dN}{d\phi} = 1 - \Psi\,\zeta^{1-a}\,(T')^{-\frac{1}{2}} \tag{27}$$

$$\frac{d}{d\phi}(NT') = 1 - \gamma\,\Psi\,\zeta^{1-a}\,(T')^{\frac{1}{2}} \tag{28}$$

Equations (26), (27) and (28) give the ballistics when integrated with initial conditions

$\zeta=0, N=0, T'=1$ at $\phi = 0$

From (26) and (27) we obtain

$$\frac{dN}{d\phi} = 1 - \Psi N^{1-\alpha} (T')^{1-\alpha-\frac{1}{2}}$$

Now we take $\alpha = 0{\cdot}5$. Therefore the above equation reduces to

$$\frac{dN}{d\phi} = 1 - \Psi\sqrt{N}$$

Integrating the above equation subject to initial conditions we get

$$\phi = -\frac{2}{\Psi^2}\left[\Psi\sqrt{N} + \log\left(1 - \Psi\sqrt{N}\right)\right] \tag{29}$$

From (26), (27) and (28) with $\alpha = 0{\cdot}5$ we get

$$\frac{dT'}{dN} + \frac{T'}{N}\, 1 + \left(\frac{\gamma\Psi\sqrt{N}}{1-\Psi\sqrt{N}}\right) = \frac{1}{N\left(1-\Psi\sqrt{N}\right)}.$$

This equation with

$$\sqrt{N} = N' \tag{30}$$

yields

$$\frac{dT'}{dN'} + \frac{2T'}{N'}\left(1 + \frac{\gamma\Psi N'}{1-\Psi N'}\right) = \frac{2}{N'\left(1-\Psi N'\right)}.$$

The equation is linear and only one, on integration, subject to initial conditions, yields

$$T'N'^2\left(1 - \Psi N'\right)^{-2\gamma} = 2\int_0^{N'} N'\left(1-\Psi N'\right)^{-(2\gamma+1)} dN'.$$

Therefore, we have

$$T' = \frac{1}{\gamma\Psi N}\left[\sqrt{N} - \frac{\left(1-\Psi\sqrt{N}\right) - \left(1-\Psi\sqrt{N}\right)^{2\gamma}}{(2\gamma-1)\,\Psi}\right] \quad (N \neq 0) \tag{31}$$

and from (26)

$$\zeta = \frac{1}{\gamma\Psi}\left[\sqrt{N} - \frac{\left(1-\Psi\sqrt{N}\right) - \left(1-\Psi\sqrt{N}\right)^{2\gamma}}{(2\gamma-1)\,\Psi}\right]. \tag{32}$$

Equations (29), (31) and (32) determine ϕ, T' and ζ respectively all in terms of N. The equations are valid so long as $\phi \leq 1$.

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