A STUDY ON THE EFFECT OF A GAUZE IN THE FIELD OF WEAK AXISYMMETRIC TURBULENCE

C.D. Ghildyal and D.D. Bhattacharya

Defence Science Laboratory, Delhi

The effect of the introduction of a gauze in a field of weak axisymmetric turbulence has been studied. Results have been compared with the experimental values obtained by Townsend and the theoretical results obtained by Taylor and Batchelor. The results obtained here give a better fit to Townsend's experimental values.

It is well known that the introduction of a wire gauze in a wind tunnel reduces the turbulent velocity fluctuations and also reduces the spatial variations of mean velocity It was Prandtl[1], who first gave some approximate mathematical model to explain this behaviour. Collar[2] also gave some other form for the same type of flow pattern. Later, Taylor and Batchelor[3] investigated this problem and gave a general theory to explain this behaviour. In addition to the longitudinal effect, they have considered the side effect of the variations of the velocity due to the guaze. The equations thus obtained contain both the resistance and deflection coefficients. They also observed that if the turbulence in the upstream is isotropic, it becomes axisymmetric in the downstream region. But the experimental evidence shows that there is initially a great tendency towards isotropy near the gauze just after crossing it. In the final stage the turbulence tends slowly to isotropy. Townsend[4] pointed out that the results obtained by Schubauer *et. al.* [5] keep a wide deviation from the predictions made by Taylor and Batchelor[3]. In his work, he also remarked that the theory proposed by Taylor and Batchelor gives a quite good approximate description of the flow in the upstream region of the gauze, but fails to do so in the downstream region.

In the present paper, to explain the difference between the experiment and the theory in the downstream region of the gauze, we have contrary to Taylor and Batchelor's assumption of isotropy, assumed the turbulence to be homogeneous and weakly axisymmetric. By comparing graphically, we observe that the present model of turbulent flow gives a better fit to the experimental findings of Townsend in both the regions.*

THEORY

Suppose in a field of homogeneous turbulence, a wire gauze having the aerodynamical properties be placed such that the normal to its plane is at a small inclination to the mean flow. Obviously, the flow pattern will change in the downstream. For simplicity, it is assumed that the pressure-drop across the gauze is the same for all stream-lines and that the longitudinal velocity is continuous across the gauze. It is also assumed that there

*Recently Michio Ohji [6—9] has tried to explain the difference between the theoretical results obtained by Taylor & Batchelor and the experimental observations made by Townsend. However, he assumes the turbulance to be disotropic in the far upstream region of the gauze and axisymmetric in the downstream region. In this paper, we have taken the whole field of turbulence to be axisymmetric.

is no dissipation of energy in the turbulent flow due to the presence of the gauze, but there will be some sharing of energy between the lateral and longitudinal motion near the gauze in the downstream region. Therefore, to study the statistical characteristic of the turbulence, Batchelor[10] has obtained the relation between the spectral tensors for the homogeneous and steady turbulence for upstream and for downstream from the gauze. They are in the longitudinal and transverse directions respectively given as

$$\overset{\prime\prime}{\phi}_{11}\,(\underline{\chi}) = J\,J^*\,\overset{\prime}{\phi}_{11}\,(\underline{\chi}) \tag{1}$$

and $$\overset{\prime\prime}{\phi}_{22}\,(\underline{\chi}) + \overset{\prime\prime}{\phi}_{33}\,(\underline{\chi}) = \alpha^2\,[\overset{\prime}{\phi}_{22}\,(\underline{\chi}) + \overset{\prime}{\phi}_{33}\,(\underline{\chi})] + \beta^2\,(J\,J^* - \alpha^2)\,\overset{\prime}{\phi}_{11}\,(\underline{\chi}) \tag{2}$$

where

$\overset{\prime}{\phi}_{ii}\,(\underline{\chi})$ = the upstream spectral function at the wave number $\underline{\chi}$,

$\overset{\prime\prime}{\phi}_{ii}\,(\underline{\chi})$ = the downstream spectral function at the wave number $\underline{\chi}$,

such that when $i = 1$, they give the longitudinal components and when $i = 2, 3$, they give the lateral components.

α = the deflection coefficient,

$$\beta = \frac{\lambda_1}{\sqrt{\lambda_2^2 + \lambda_3^2}} \quad \text{where} \quad \lambda_1^2 \cdot \lambda_2^2 + \lambda_3^2 = \chi^2$$

and J = a function of β.

The reduction factors in the total energy of the longitudinal and lateral fluctuations are given by

$$\mu = \frac{\overline{u_1''^2}}{\overline{u_1'^2}} = \frac{\int J\,J^*\,\overset{\prime}{\phi}_{11}\,(\underline{\chi})\,\partial\chi}{\int \overset{\prime}{\phi}_{11}\,(\underline{\chi})\,\partial\chi} \tag{3}$$

and $$\nu = \frac{\overline{u_2''^2} + \overline{u_3''^2}}{\overline{u_2'^2} + \overline{u_3'^2}} = \alpha^2 + \frac{\int \beta^2\,(J\,J^* - \alpha^2)\,\overset{\prime}{\phi}_{11}\,(\underline{\chi})\,d\chi}{\int [\,\overset{\prime}{\phi}_{22}\,(\underline{\chi}) + \overset{\prime}{\phi}_{33}\,(\underline{\chi})\,]\,d\chi} \tag{4}$$

ν can be expressed as a function of μ and the directional energy distribution in the turbulence far upstream, *viz.*

$$\nu = \alpha^2 + \tfrac{1}{4}\,[\,(1 + \alpha - \alpha K)^2 - (1 + \alpha + K)^2\,\mu\,]\,\frac{\overline{u_1'^2}}{\overline{u_2'^2} + \overline{u_3'^2}} \tag{5}$$

where

K = the resistance coefficient

$\overline{u_i'^2}$ = the mean square fluctuating velocity in the i_{th} direction.

It is, therefore, just sufficient to know the value of μ. But in order to determine the value of μ, one must know the spectrum of the turbulence in the upstream region.

Kampé dFé'riet[11] has given the general spectral tensor in the case of an incompressible fluid. It is

$$\phi_{\alpha\beta}(\underline{\chi}) = C_{\alpha}(\underline{\chi})\, {}^{*}C_{\beta}(\underline{\chi}) + b^2\left(\delta_{\alpha\beta} - \frac{\lambda_\alpha \lambda_\beta}{\chi^2}\right) \tag{6}$$

$$\text{with } \sum C_{\alpha}(\underline{\chi})\,\lambda_{\alpha} = 0, \tag{7}$$

$$\text{where } C_{\alpha}(\underline{\chi}) = \sqrt{1 - \frac{b^2}{a^2}}\, a_{\alpha}(\underline{\chi}), \tag{8}$$

and $\underline{\chi}$, $\underline{a}$ and $\underline{b}$ are orthogonal complex vectors. $\underline{a}$ and $\underline{b}$ being functions of $\underline{\chi}$ such that $\underline{\chi}$, $\underline{a}$ and $\underline{b}$ form a trirectangular trihedral and $a_{\alpha}(\underline{\chi})$ is a vector component of $a(\underline{\chi})$ in α direction.

The spectral tensor $\varphi_{\alpha\beta}(\underline{\chi})$ can be split up into two tensors, one can be attributed to isotropy and the other to axisymmetry. Thus, we have

$$\phi_{\alpha\beta}(\underline{\chi}) = \overset{\mathrm{I}}{\phi}_{\alpha\beta}(\underline{\chi}) + \overset{\mathrm{A}}{\phi}_{\alpha\beta}(\underline{\chi}) \tag{9}$$

where

$$\overset{\mathrm{I}}{\phi}_{\alpha\beta}(\underline{\chi}) = \text{the isotropic tensor} = b^2\left(\delta_{\alpha\beta} - \frac{\lambda_\alpha \lambda_\beta}{\chi^2}\right)$$

and

$$\overset{\mathrm{A}}{\phi}_{\alpha\beta}(\underline{\chi}) = \text{the axisymmetric tensor} = C_{\alpha}(\underline{\chi})^{*}C_{\beta}(\underline{\chi}).$$

Now let us assume that the turbulence is weak axisymmetric, such that $\left(\frac{a^2}{b^2} - 1\right)$, say θ, is not zero, but is small and independent of χ.

As the total energy per unit mass of the fluid due to the axisymmetric part has to be positive, we have

$$\int \sum \overset{\mathrm{A}}{\phi}_{\alpha\alpha}(\underline{\chi})\,d\chi > 0$$

using the condition (8), we have

$$\int (a^2 - b^2)\,d\chi > 0$$

Thus, we find that θ is always positive. Again since the energy of the fluid in any direction in the transverse plane is the same, we have

$$\int \phi_{22}(\underline{\chi})\,d\chi = \int \phi_{33}(\underline{\chi})\,d\chi$$

or

$$\int \overset{\mathrm{A}}{\phi}_{22}(\underline{\chi})\,d\chi = \int \overset{\mathrm{A}}{\phi}_{33}(\underline{\chi})\,d\chi.$$

Therefore, by using (8), we get

$$\int \frac{a^2 - b^2}{a^2}(a_2^2 - a_3^2)d\chi = 0$$

So that

$$a_2^2 = a_3^2 \tag{10}$$

using relation (10) we have $a^2 = a_1^2 + 2a_2^2$ and equation (7) gives

$$a_1\lambda_1 + a_2(\lambda_2 + \lambda_3) = 0.$$

From the above two relations between a_1 and a_2 we get

$$a_1^2 = \frac{a^2(\lambda_2 + \lambda_3)^2}{\chi^2 + \lambda_1^2 + 2\lambda_2\lambda_3} \tag{11}$$

and

$$a_2^2 = a_3^2 = \frac{a^2\lambda_1^2}{\chi^2 + \lambda_1^2 + 2\lambda_2\lambda_3} \tag{12}$$

The expression for μ given by equation (3) can be rewritten as

$$\mu = \frac{\int JJ^*\overset{I}{\phi'}_{11}(\underline{\chi})d\chi + \int JJ^*\overset{A}{\phi'}_{11}(\underline{\chi})d\chi}{\int \overset{I}{\phi'}_{11}(\underline{\chi})dx + \int \overset{A}{\phi'}_{11}(\underline{\chi})d\chi} \tag{13}$$

Making use of the form given by Taylor and Batchelor[3]

for $\overset{I}{\phi'}_{11}(\underline{\chi})$, we have

$$\int JJ^*\overset{I}{\phi'}_{11}(\underline{\chi})d\chi = \int JJ^* \frac{C\bar{u_1}^2(\lambda_2^2 + \lambda_3^2)}{(\chi^2 + \gamma^2)^3}\, d\chi$$

$$= \frac{3}{4}\,\frac{\pi^2 C\bar{u_1}^2}{\gamma}\left[\frac{2}{3}\,\frac{(1 + a - aK)^2 + 2\alpha^2}{(1 + a + K)^2 - 4} + \frac{(1 + a - \alpha K)^2 - 4\alpha^2}{(1 + \alpha + K)^2 - 4}(1 - \eta^2)L\right]$$

where $L = 1 + \dfrac{\eta^2 - \zeta^2}{2\eta}\log\dfrac{\eta - 1}{\eta + 1}$.

Also, by using (6), (7), (10) and (13), we get

$$\int JJ^*\overset{A}{\phi^1}_{11}(\underline{\chi})d\chi = \theta\int \frac{4a^2\beta^2 + (1 + a - aK)^2}{4\beta^2 + (1 + a + K)^2}\; b^2\; \frac{(\lambda_2 + \lambda_3)^2}{\chi^2 + \lambda_1^2 + 2\lambda_2\lambda_3}\, d\chi$$

Applying the spherical polar transformation, we get

$$\int JJ^*\overset{A}{\phi'}_{11}(\underline{\chi})d\chi$$

$$= \theta\int \frac{C\bar{u_1}^2 r^2}{(r^2 + \gamma^2)^3}\,\frac{4a^2\cot^2\psi + (1 + a - aK)^2}{4\cot^2\psi + (1 + a + K)^2}\; r^2\sin^3\psi\;\frac{(\cos\theta' + \sin\theta')^2 dr d\psi d\theta'}{1 + \cos^2\psi + \sin 2\theta'\sin^2\psi}$$

$$= \frac{3}{4}\;\frac{\pi^2 C\bar{u_1}^2}{\gamma}\;\theta\,\frac{(1 + a - aK)^2 - 4a^2}{(1 + a + K)^2 - 4}\;.L$$

Similarly,

$$\int \overset{I}{\phi'}_{11}(\underline{\chi})d\chi = \int \frac{C\overline{u}^2_1(\lambda^2_2 + \lambda_3^2)}{(\chi^2 + \gamma^2)^3} \, d\chi$$

$$= \frac{\pi^2 C\overline{u}^2_1}{2\gamma}$$

and

$$\int \overset{A}{\phi'}_{11}(\chi)\underline{d}\chi = \theta \, C\overline{u}^2_1 \left[\frac{r^4}{(r^2 + \gamma^2)^3} \; \frac{\sin^3 \psi(\cos \theta' + \sin \theta')^2}{1 + \cos^2 \psi + \sin 2\theta' \sin^2 \psi} \; d^2 d\psi d\theta' \right.$$

$$= \frac{3}{4} \; \frac{\pi^2 C\overline{u}^2_1}{\gamma} \; \theta$$

$$\therefore \; \mu = \frac{2 \, \dfrac{(1 + \alpha - aK)^2 + 2a^2}{(1 + a + K)^2 - 4} + 3 \, \dfrac{(1 + a - aK)^2 - 4a^2}{(1 + a + K)^2 - 4} \, \mathrm{L} \left\{ 1 - \eta^2 + \theta \right\}}{2 + 3\theta} \qquad (14)$$

which gives the modified expression for μ.
This reduces to the expression obtained by Taylor and Batchelor[3] when $\theta = 0$.

Also, α^2 the modified relationship between μ and ν is obtained from (5) and (14) as

$$\nu = \alpha^2 + \tfrac{1}{4}[(1 + \alpha - \alpha K)^2 - (1 + \alpha + K)^2\mu][\tfrac{1}{2} + \tfrac{3}{4}\theta] \qquad (15)$$

We have plotted the graphs for μ and ν for different values of K with $M = 2\cdot54$ cm., (Fig 1). The dotted lines show the theoretical results obtained by Taylor and Batchelor. The points denote the experimental values obtained by Townsend and our results are depicted by the smooth curves :

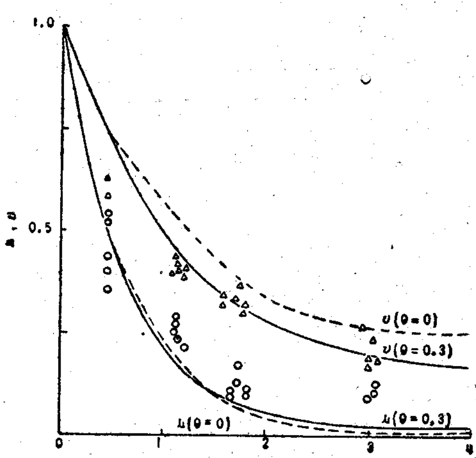

FIG. 1—Graphs for μ and ν for different values of K with M = 2·45 cm.

From Fig. 1, we find that for higher values of K, our theoretical curve (for $\theta = 0\cdot3$) for μ gives a better fit to the experimental valuesobtained by Townsend and for lower values of K, the result obtained here is practically the same as given by Taylor and Batchelor. Similarly, it is clear from the graph that our theoretical curve for ν gives a better prediction for all values of K.

From Fig. 2, we observe that the total attenuation of the turbulent intensity (taking $\theta = 0 \cdot 3$) when compared with our theoretical result gives a better approximation for the lower values of K. But for higher values of K it keeps hardly any distinction from the result of Taylor & Batchelor.

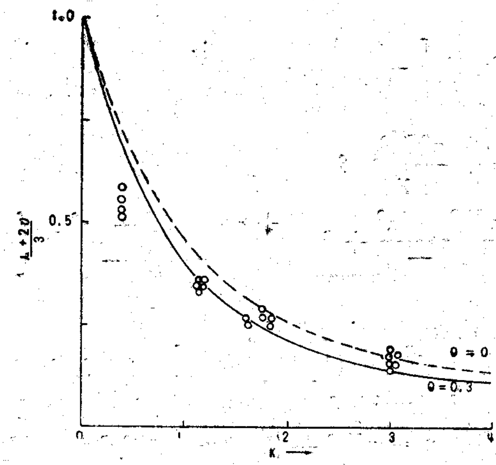

FIG. 2 — Graphs for μ and ν for different values of K with M = $2 \cdot 45$ cm.

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ACKNOWLEDGEMENTS

We are indebted to Dr. V. R. Thiruvenkatachar, for his guidance. We are also grateful to Dr. V. Ranganathan, Director, Defence Science Laboratory, Delhi, for the encouragement given to us and for his keen interest in the work.

REFERENCES

1. PRANDTL, L., *N.A.C.A. (U.S.A.), Tech. Mem.*, 726, (1933).
2. COLLAR, A. R., *Brit. Aero. Res. Coun. Rep. and Mem.*, 1867, (1939).
3. TAYLOR, G. I. & BATCHELOR, G. K., *Quart. J. Mech. Appl. Math.*, **2**, 1 (1949).
4. TOWNSEND, A. A. , *Ibid.*, **4**, 308 (1951).
5. SCHUBAUER, G. B., SPANGENBERG, W.G., & KLEBANOFF, P.S, *N.A.C.A. (U.S.A.) Tech. Note*, 2001 (1950)
6. OHJI, MICHIO, *Rep. Res. Inst. Appl. Mech.*, Kyushu Univ. **6**, 63 (1958).
7. — *Ibid.*, **6**, 153 (1958).
8. — *Ibid.*, **7**, 259 (1959).
9. — *Ibid.*, **8**, 19 (1960).
10. BATCHELOR, G. K., "The theory of Homogeneous Turbulence", (Cambridge University Press), 1953.
11. KAMPE' DE FER'IET., *Proc. Seventh. Intrnat Cong. Appl. Mech.*, **1**, 6 (1948).