FORM FUNCTION AND VARIATION OF BURNING SURFACE AREA FOR THE ECCENTRIC CYLINDRICAL CHARGE

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The combustion problem of an eccentric cylindrical charge i.e. a cylindrical charge with an eccentric hole has been considered in this paper. The burning of the charge proceeds in two distinct phases for each of which the form function and variation of burning surface area have been investigated. We also find the equivalent form factor. Finally the English and French Co-efficients of progressivity have been evaluated.

The tubular charge bounded by two co-axial cylindrical surfaces is the most common charge shape, used in guns. But due to manufacturing defects, the section does not always give concentric circles. The object of the present paper is to examine the effect of eccentricity on the degressive nature of the burning. Kleider investigated one such shape (Corner)[1]. The general problem has been investigated by the present authors in detail.

NOMENCLATURE

δ	Propellant density
C	Mass of the grain
V	Volume of the grain at any instant
a	Radius of the outer cylinder
b	Radius of the hole
c	Distance between their centres
D	Web-size of the grain
λD	Initial length of the grain
S	Surface of combustion at any instant
z	Fraction of the charge burnt
f	The fraction of the initial thickness (web-size) remaining at any instant.

Subscript

0 refers to the initial value.

THEORY

Let the cross-section consist of two circles of radius a and b ($a > b$) with their centres at a distance c apart, the web-size being

$$D = a - b - c. \qquad \text{(See figure 1)}$$

The burning takes place in two phases. In the first phase the radius of the inner surface increases while that of the outer surface decreases till the two surfaces touch at a point. In the second phase the shape is that of the crescent moon whose thickness goes on decreasing till it vanishes.

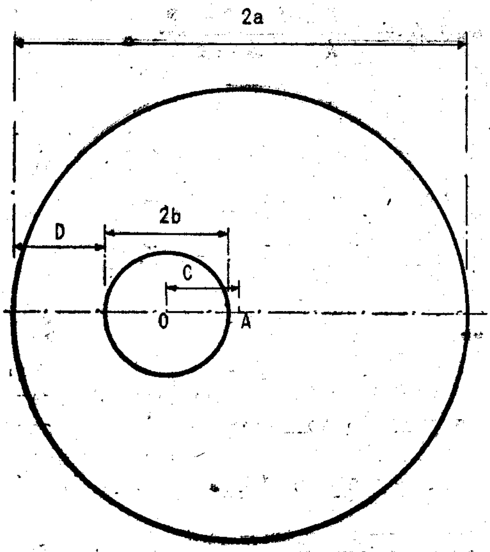

FIG. 1

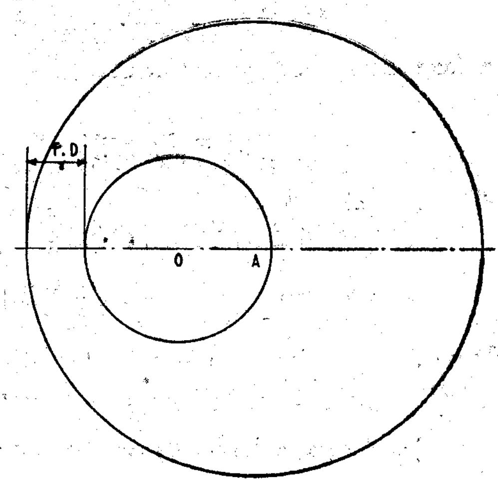

FIG. 2

FIRST PHASE OF COMBUSTION

At time t, let $f\,D$ be the fraction of the web-size remaining (See figure 2) so that the radii r_1, r_2 of the inner and outer surfaces respectively are given by

$$r_1 = b + \tfrac{1}{2} D (1 - f) \tag{1}$$

$$r_2 = a - \tfrac{1}{2} D (1 - f) \tag{2}$$

and the area of the cross-section remaining is

$$\pi (r_2^2 - r_1^2) = \pi (a + b) [c + (a - b - c) f] \tag{3}$$

Since λD is the initial length of the charge, the initial charge mass C is given by -

$$C = V_\circ \, \delta = \pi (a^2 - b^2) \lambda D \delta \tag{4}$$

and the charge mass remaining at time t is,

$$C (1 - z) = \pi (a + b) [c + (a - b - c) f] [\lambda D - D (1 - f)] \delta,$$

$$z = (1 - f) \left[\frac{a - b - c}{a - b} + \frac{c}{\lambda} \; \frac{1}{a - b} + \frac{a - b - c}{a - b} \; \frac{f}{\lambda} \right]. \tag{5}$$

which is of the form

$$z = (1 - f)(A + Bf), \tag{6}$$

where

$$A = \frac{a-b-c}{a-b} + \frac{c}{\lambda}\,\frac{1}{a-b}\,, \text{ and } B = \frac{1}{\lambda}\,\frac{a-b-c}{a-b}.$$

For long tubes, λ is large and we have

$$z = \left(1 - \frac{c}{a-b}\right)(1-f). \tag{7}$$

Also

$$S_o = 2\pi(a^2 - b^2) + \lambda D(2\pi a + 2\pi b),$$
$$= 2\pi(a+b)[(a-b) + \lambda D]. \tag{8}$$

and

$$S = 2\pi(r_2^2 - r_1^2) + 2\pi(r_1 + r_2)[\lambda D - D(1-f)],$$
$$= 2\pi(a+b)[c + (a-b-c)f + \lambda D - D(1-f)].$$

$$\therefore \frac{S}{S_o} = \frac{c + (a-b-c)(\lambda - 1) + 2(a-b-c)f}{(a-b) + (a-b-c)\lambda} \tag{9}$$

Equation (6) gives the relation between z & f and the equation (9) between S_o/S_o & f. These values *i.e.* z, S/S_o have been calculated for different values of f, at $\lambda = 10$ and $\lambda = \infty$ and the results are illustrated in figures 4 and 5.

SECOND PHASE OF COMBUSTION

In fig. (3), A and O are the centres of the outer and the inner holes. At the beginning of this phase the charge is like the crescent moon with radii AE and OE given by $(a+b+c)/2$ and $(a+b+c)/2$ respectively with the thickness as $A'D' = 2c$.

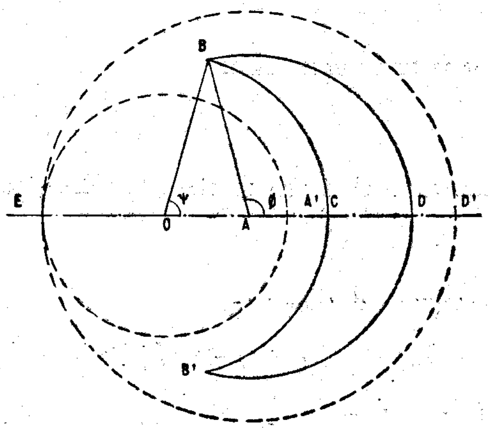

FIG. 3

As burning proceeds it shrinks into $BDB'\,CB$ and the radii change to AD and OC *i.e.*

$$\left.\begin{aligned} AD = r_2 &= \frac{a+b+c}{2} - \frac{a-b-c}{2}\,f \\ \text{and}\qquad OC = r_1 &= \frac{a+b-c}{2} + \frac{a-b-c}{2}\,f \end{aligned}\right\} \tag{10}$$

and, if ψ and ϕ are the angles as shown in the figure (3), the area of the cross-section $B\,D\,B'\,C\,B$ remaining at time is given by

$$r_2^2\,\phi - r_1^2\,\psi + r_1\,r_2 \sin(\phi - \psi) \tag{11}$$

$$\therefore\ 1 - z = \frac{[\,r_2^2\,\phi - r_1^2\,\psi + r_1\,r_2 \sin(\phi - \psi)\,]\,[\,\lambda D - D(1-f)\,]}{(\pi a^2 - \pi b^2)\,\lambda D} \tag{12}$$

when $\lambda \to \infty$ *i.e.* for long tubes, we get

$$1 - z = \frac{r_2^2\,\phi - r_1^2\,\psi + r_1\,r_2 \sin(\phi - \psi)}{\pi a^2 - \pi b^2} \tag{13}$$

where ψ and ϕ are given by

$$\cos\psi = \frac{r_1^2 + c^2 - r_2^2}{2\,r_2\,c} = \frac{c^2 - (a+b)\,c - (a+b)\,(a-b-c)\,f}{c\,[\,(a+b-c) - (a-b-c)\,f\,]} \tag{14}$$

and

$$\cos\phi = -\frac{r_2^2 + c^2 - r_1^2}{2\,r_1\,c} = -\frac{c^2 + (a+b)\,c + (a+b)\,(a-b-c)\,f}{c\,[\,(a+b-c) + (a-b-c)\,f\,]} \tag{15}$$

Also S for the second phase of combustion is given by

$$\begin{aligned} S = {} & 2\,[\,r_2^2\,\phi - r_1^2\,\psi + r_1\,r_2 \sin(\phi - \psi)] \\ & + 2\,(r_1\,\psi + r_2\,\phi)\,[\,\lambda D - D(1-f)\,] \end{aligned} \tag{16}$$

Hence with the help of equation (8) and (16), we get

$$\frac{S}{S_o} = \frac{r_2^2\,\phi - r_1^2\,\psi + r_1\,r_2 \sin(\phi - \psi) + (r_1\,\psi + r_2\,\phi)\,[\,\lambda D - D(1-f)\,]}{\pi\,(a+b)\,(a-b+\lambda D)} \tag{17}$$

The equations (13) and (17) determine the values of z and S/S_o respectively, in terms of ψ and ϕ. ψ and ϕ are determined in terms of f from (14) and (15). Tables 1 and 2 give the values of z, S/S_o in terms of the parameter f for $\lambda = 10, \infty$ respectively.

The figures 4 and 5 show the variation of z with S/S_o. From figure 5 we find that burning surface area is constant for the first phase of combustion and steadily decreases to zero during the second phase of combustion.

EQUIVALENT FORM FACTOR θ.

For finding the value of θ we have the following system of equations [Tavernier],[23] for the period before and after the rupture of the grain:

$$\left.\begin{aligned} (A+B) &= (1+\theta)\,(1-f) \\ \text{and}\qquad A &= (1-f)\,(1+\theta f) \end{aligned}\right\} \tag{18}$$

TABLE 1

VALUES OF z AND S/So FOR THE SECOND PHASE OF COMBUSTION, FOR SOME SETS OF VALUES OF a/c, b/c AT $\lambda = 10$

a/c	b/c												
4	2	f	0·0	—0·2	—0·4	—0·6	—1·8	—1·0	—1·2	—1·4	—1·6	—1·8	—2·0
		z	0·5500	0·6368	0·7102	0·7720	0·8260	0·8733	0·9149	0·953	0·9726	0·9908	1·0
		S/So	0·8333	0·6533	0·5624	0·4881	0·4214	0·3597	0·3020	0·2430	0·1875	0·1239	0·0
6	3	f	0·0	—0·1	—0·2	—0·3	—0·4	—0·5	—0·6	—0·7	—0·8	—0·9	—1·0
		z	0·7000	0·7338	0·8012	0·8423	0·8784	0·9100	0·9373	0·9567	0·9788	0·9961	1·0
		S/So	0·8261	0·6551	0·5696	0·5015	0·4406	0·3834	0·3279	0·2721	0·2131	0·1611	0·
10	5	f	0·0	—0·1	—0·2	—0·3	—0·4	—0·5					
		z	0·8200	0·8794	0·9253	0·9609	0·9871	1·0					
		S/So	0·8222	0·5729	0·4508	0·3420	0·2271	0·0					
20	10	f	0·0	—0·1	—0·2	—0·22							
		z	0·9100	0·9671	0·9976	1·00							
		S/So	0·8200	0·4297	0·1624	0·00							
6	2	f	0·0	—0·1	—0·2	—0·3	—0·4	—0·5	—0·6	—0·67			
		z	0·7750	0·8333	0·8804	0·9197	0·9520	0·9770	0·9942	1·00			
		S/So	0·8235	0·6095	0·5066	0·4193	0·3370	0·2516	0·1517	0·00			
9	3	f	0·0	—0·1	—0·2	—0·3	—0·4						
		z	0·8500	0·9317	0·9534	0·9747	1·0						
		S/So	0·8214	0·5383	0·3985	0·2723	0·0						
15	5	f	0·0	—0·1	—0·2	—0·22							
		z	0·9100	0·9670	0·9978	1·00							
		S/So	0·8200	0·4297	0·1624	0·00							

TABLE 2

VALUE OF z AND S/SO FOR THE SECOND PHASE OF COMBUSTION, FOR SOME SET OF VALUE OF A/C, B/C AT $\lambda = \infty$

a/c	b/c												
4	2	f	0·0	—0·2	—0·4	—0·6	—0·8	—1·0	—1·2	—1·4	—1·6	—1·8	—2·0
		z	0·5000	0·5872	0·6630	0·7286	0·7877	0·8416	0·8909	0·9383	0·9630	0·9872	1·0
		S/SO	1·0000	0·7971	0·7964	0·6326	0·5650	0·5000	0·4367	0·3674	0·1941	0·1029	0·0
6	3	f	0·0	0·1	—0·1	—0·3	—0·4	—0·5	—0·6	—0·7	—0·8	—0·9	—1·0
		z	0·6667	0·7119	0·7741	0·8188	0·8941	0·9253	0·8478	0·9741	0·9951	0·9951	1·0
		S/SO	1·0000	0·7961	0·7057	0·6317	0·5645	0·5000	0·4355	0·3676	0·2941	0·2039	0·0
10	5	f	0·0	—0·1	—0·2	—0·3	—0·4	—0·5					
		z	0·8000	0·8645	0·8151	0·9551	0·8949	0·10					
		S/SO	1·0000	0·7052	0·5642	0·4358	0·2948	0·0					
20	10	f	0·0	—0·1	—0·2	—0·22							
		z	0·9000	0·9640	0·9973	1·00							
		S/SO	1·0000	0·5319	0·2947	0·00							
6	2	f	0·0	—0·1	—0·2	—0·3	—0·4	—0·5	—0·6	—0·67			
		z	0·7500	0·8127	0·9741	0·9077	0·9441	0·9730	0·9931	1·00			
		S/SO	1·0000	0·7841	0·6319	0·5321	0·4354	0·3323	0·2036	0·00			
9	3	f	0·0	—0·1	—0·2	—0·3	—0·4						
		z	0·8333	0·9222	0·9470	0·9710	1·0						
		S/SO	1·0000	0·6615	0·5000	0·3462	0·0						
15	5	f	0·0	—0·1	—0·2	—0·22							
		z	0·9000	0·9629	0·9975	1·00							
		S/SO	1·0000	0·5318	0·2047	0·00							

$$(A + B)(1 - f_{min}) = (1 + \theta) \tag{19}$$

But the equation (18) gives on simplification

$$\theta = \frac{[(A + B)^2 - 2B] \pm (A + B)[(A + B)^2 - 4B]^{\frac{1}{2}}}{2B} \tag{20}$$

considering that radical sign which makes $\theta < 1$. The θ has been calculated for different sets of values of a/c and b/c at $\lambda = 10$.

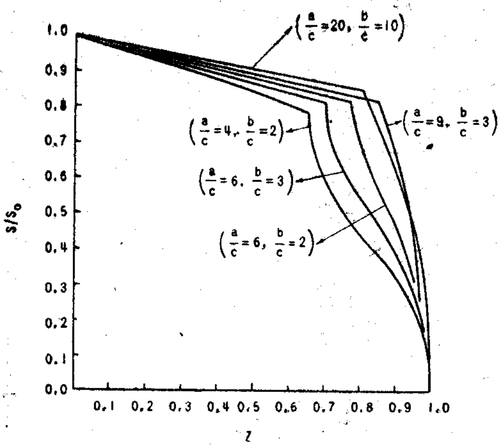

FIG. 4—Variation of z and S/S_o for some set of values of a/c, b/c at $\lambda = 10$

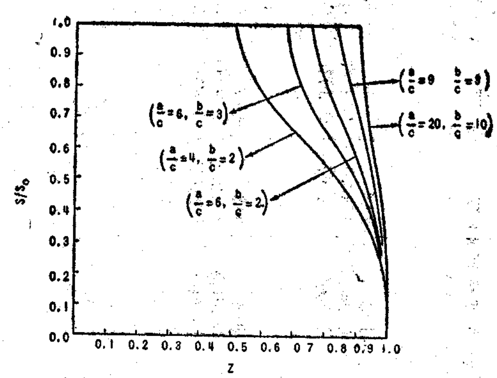

FIG. 5—Variation of z and S/S_o for some set of values of a/c, b/c at $\lambda = \infty$

TABLE 3

VALUES OF θ BEFORE THE RUPTURE OF THE GRAIN

λ	$a/c = 4$ $b/c = 2$	$a/c = 6$ $b/c = 3$	$a/c = 10$ $b/c = 5$	$a/c = 20$ $b/c = 10$	$a/c = 6$ $b/c = 2$	$a/c = 9$ $b/c = 3$	$a/c = 15$ $b/c = 5$
10	0·2	0·1446	0·125	0·1111	0·1333	0·12	0·1111

TABLE 4

VALUES OF θ AFTER THE RUPTURE OF THE GRAIN

λ	$a/c = 4$ $b/c = 2$	$a/c = 6$ $b/c = 3$	$a/c = 10$ $b/c = 5$	$a/c = 20$ $b/c = 10$	$a/c = 6$ $b/c = 2$	$a/c = 9$ $b/c = 3$	$a/c = 15$ $b/c = 5$
10	0·8	0·5333	0·35	0·2222	0·4167	0·3067	0·2222
∞	0·65	0·4000	0·2300	0·1122	0·2917	0·1900	0·1122

The relation between English and French coefficients of progressivity is

$$K = - \frac{4\,\theta}{(1+\theta)^2} \tag{21}$$

With the help of equation (20) it gives

$$K = - \frac{4\,B}{(A+B)^2}\,A \tag{22}$$

which clearly show that at $\lambda = \infty$, $K = o$; and at $\lambda = 10$ its values, for different a/c, b/c are shown in Table 5.

TABLE 5

VALUES OF K FOR DIFFERENT a/c AND b/c AT $\lambda = 10$

λ	$a/c = 4$ $b/c = 2$	$a/c = 6$ $b/c = 3$	$a/c = 10$ $b/c = 5$	$a/c = 20$ $b/c = 10$	$a/c = 6$ $b/c = 2$	$a/v = 9$ $b/c = 3$	$a/c = 15$ $b/c = 5$
10	—0·5556	—0·442	—0·3951	0·36	0·4153	0·3828	0·36

REFERENCES

1. CORNER, J., "Interior Ballistics", (John Wiley and Sons, New York), 1950
2. TAVERNIER, P., *Mem. Art. Franc*, 4°, 1015 (1956).
3. TAVERNIER, P., *Mem. Art. Franc*, 1°, 117 (1956).