FORM FUNCTION AND VARIATION OF BURNING SURFACE AREA FOR THE PENTA-TUBULAR CHARGE

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The problem of combustion of a penta-tubular charge has been considered in this paper. This is a cylindrical charge with five holes of equal diameters, one at the centre and the other four symmetrically situated about it. The burning of the charge proceeds in three distinct phases for each of which the form function and variation of surface area has been investigated. Equivalent form-factor has also been found.

One of the important problems in internal ballistics is the consideration of the form-function for charges of different shapes. Some of the important shapes were discussed in the H.M.S.O. publication[1] and by Corner [2]. Later Tavernier [3,4] discussed the internal ballistics of heptatubular charges in much greater detail. Recently Kothari [5], Jain[6], Jain and Kapur [7] have discussed the cases of bitubular, tritubular and quadratubular charges respectively. In the present paper the authors have investigated the case of a penta-tubular charge. (The case of a hexatubular charge is also being investigated and it is intended later to compare the results of all these papers to find the optimum shapes under various conditions.)

In the earlier cases there were also three stages of burning but fortunately for mathematical purposes the second and third stages can be combined and we talk of only two stages of burning. In the present case, however, this is not possible since the shape of the sliver changes from curvilinear quadrilateral to a curvilinear triangle at the end of the second stage.

NOMENCLATURE

δ Propellant density

C Mass of the grain

V_0 Initial volume of the grain

V Volume of the grain at any instant of time t

d The diameter of each of the five holes of the grain

D The distance between two holes as indicated in fig. (1) or the websize of the grain

L The length of the grain at any instant

m The ratio of the exterior diameter of the grain to the diameter of the hole

ρ The ratio of the length of the grain to the exterior diameter of the grain

S_0 Initial surface of the grain exposed to combustion

S The surface of combustion at any instant t

z Fraction of the charge burnt at any instant t

f The fraction of the initial thickness (web-size) remaining at any instant t, for the first phase of combustion; while for the second and third phases of combustion, f is defined as the ratio of the distance receded (from the beginning of the second phase of combustion upto the instant considered) to the initial thickenss D.

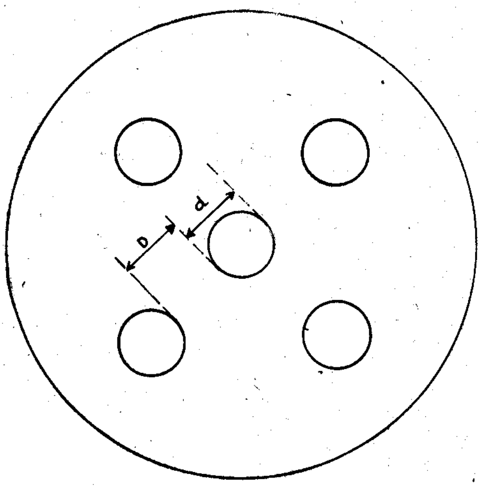

FIG. 1

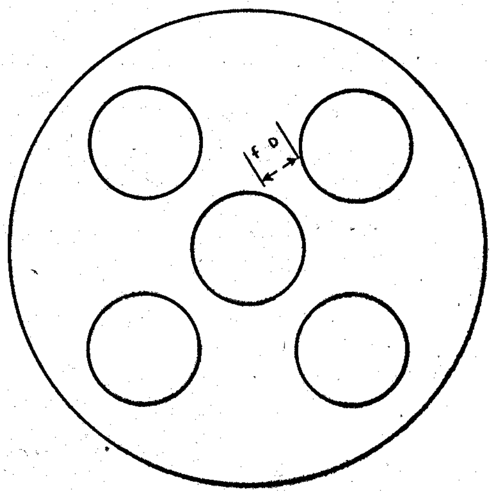

FIG. 2

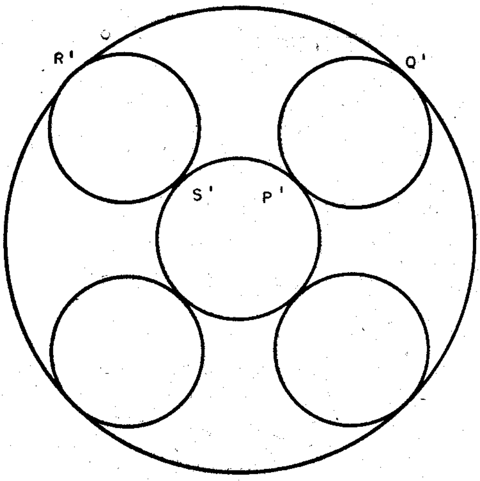

FIG. 3

Figures 1, 2 & 3 show the three stages of burning.

FIRST PHASE OF COMBUSTION

Initial mass of the grain is given by

$$C = V_o\delta = \left\{ \pi\left(\frac{md}{2}\right)^2 - 5\,\pi\left(\frac{d}{2}\right)^2\right\} m\,\rho\, d.\,\delta, \qquad (1)$$
$$= \tfrac{1}{4}\,\pi\, m\rho\delta d^3\,(m^2-5)$$

Mass of the grain when a fraction f of D of the grain remains is given by

$$C\,(1-z) = V\delta = [\pi\,\{\tfrac{1}{2}\,md - \tfrac{1}{2}\,D\,(1-f)\,\}^2 - 5\pi\,\{\tfrac{1}{2}\,d + \tfrac{1}{2}\,D\,(1-f)\}^2]\;L.\delta, \qquad (2)$$

where the length L is given by

$$L = m\rho d - (1-f)\,D. \qquad (3)$$

Also md and $4D+3d$ are the two values of the exterior diameter of the grain. Hence

$$\frac{D}{d} = \frac{m-3}{4}\,. \qquad (4)$$

Equation (2) simplifies to

$$C\,(1-z) = \tfrac{1}{4}\,\pi\delta\,\{m\rho d-(1-f)\,D\}\,[d^2\,(m^2-5)-2Dd\,(1-f)\,(m+5)\,-4D^2\,(1-f\,)^2] \qquad (5)$$

From equations (1), (4) and (5) we get,

$$z = (1-f)\left[\frac{(m-3)\,(12m^2\rho + 28\,m\rho + m^2 + 2m+1)}{16m\rho\,(m^2-5)} - \frac{(m-3)^2\,(m\rho - m - 1)}{4m\rho\,(m^2-5)}\,f - \frac{(m-3)^3}{16m\rho\,(\,m^2-f\,)}\,f^2\right]$$

which is of the form

$$z = (1-f)\,(a - bf - cf^2) \qquad (6)$$

where

$$a = \frac{(m-3)\,(12m^2\rho + 28m\rho + m^2 + 2m + 1)}{16m\rho\,(m^2-5)} \qquad (7)$$

$$b = \frac{(m-3)^2\,(m\rho - m - 1)}{4m\rho\,(m^2-5)} \qquad (8)$$

and

$$c = \frac{(m-3)^3}{16m\rho\,(m^2-5)} \qquad (9)$$

By putting $f = 0$, z becomes a which gives the fraction of the grain which is burnt at the end of the first phase of combustion.

From (3) and (4) we find that the burning will be over before rupture if *

$$\rho \leqslant \frac{m-3}{4m}$$

*The above result is only mathematically true.

and the second stage of burning will begin if

$$\rho \geqslant \frac{m-3}{4m}$$

From (7)

$$a = a_o(m) + \frac{a_1(m)}{\rho}$$

where

$$a_o(m) = \frac{(m-3)(3m+7)}{4(m^2-5)}, \text{ and } a_1(m) = \frac{(m-3)(2m^2+2m+1)}{16m(m^2-5)}.$$

Differentiating this we find that when $m > 3$, a is an increasing function of m and decreasing function of ρ, but

$$\rho_{min} = \frac{m-3}{4m}.$$

Hence for any given value of m,

$$a_{max} = \frac{(m-3)\,4m}{16m\,(m^2-5)\,(m-3)}\left[4m\,(3m+7)\frac{m-3}{4m} + m^2+2m+1\right] = 1,$$

a value independent of m.

Also for a given value of m, a_{min} occurs at $\rho = \infty$, so that

$$a_{min} = a_o(m)$$

At $m = 3$, $a_o(m) = o$ and at $m = \infty$, $a_o(m) = 0{\cdot}75$ so that $a_o(m)$ varies from 0 to $0{\cdot}75$ as m varies from 3 to ∞.

From (6), we get

$$\frac{S}{S_o} = \frac{(dz/df)}{(dz/df)_{f=1o}} = \frac{a+b}{a-b-c} - \frac{2(b-c)}{(a-b-c)} f - \frac{3c}{a-b-c} f^2,$$

which is of the form

$$\frac{S}{S_o} = \alpha - \beta f - \gamma f^2 \qquad (10)$$

where α, β, γ are given by

$$\alpha = \frac{(m+1)(16\,m\rho - 3\,m + 13)}{4\,(2m^2\,\rho + 10m\rho + m^2 - 5)}, \qquad (11)$$

$$\beta = \frac{(m-3)\;(4m\rho - 5m - 1)}{2\,(2m^2\rho + 10m\rho + m^2 - 5)}, \qquad (12)$$

and

$$\gamma = \frac{3\,(m-3)^2}{4\,(2m^2\rho + 10m\rho + m^2 - 5)} \qquad (13)$$

At the end of the first stage, $S/S_o = \alpha$ which gives the ratio of the surface at rupture to the initial surface. Equation (6) and (10) determine the (S, z) relation, f as the parameter. The results are given in Table 1.

TABLE 1

VALUE OF Z AND $S/S\hat{o}$ FOR THE FIRST PHASE OF COMBUSTION, FOR SOME SETS OF VALUES OF m AND ρ

m	ρ	$f \to$	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0·0
4	1/2	Z	0·0	0·0531	0·1056	0·1575	0·2088	0·2594	0·3095	0·3589	0·4076	0·4562	0·5028
		S/S_o	1·0	0·9982	0·9772	0·9658	0·9541	0·9521	0·9297	0·9173	0·9040	0·8906	0·8780
4	1	Z	0·0	0·0475	0·0942	0·1411	0·1880	0·2348	0·2816	0·3282	0·3746	0·4211	0·4673
		S/S_o	1·0	0·9987	0·9973	0·9957	0·9938	0·9917	0·9896	0·9876	0·9846	0·9819	0·9760
4	9/4	Z	0·0	0·0438	0·0878	0·132	0·1765	0·2212	0·2661	0·3112	0·3561	0·4019	0·4476
		S/S_o	1·0	1·0053	1·0102	1·0152	1·0201	1·0249	1·0298	1·0343	1·0388	1·0433	1·0476
7	9/4	Z	0·0	0·0615	0·1240	0·1876	0·2522	0·3178	0·3844	0·4521	0·5202	0·5893	0·6594
		S/S_o	1·0	1·0178	1·0354	1·0545	1·0690	1·0849	1·1013	1·1131	1·1293	1·1429	1·1560
10	9/4	Z	0·0	0·0611	0·1246	0·1902	0·2580	0·3278	0·3997	0·4735	0·5493	0·6269	0·7064
		S/S_o	1·0	1·0527	1·0716	1·0915	1·1245	1·1245	1·1475	1·1676	1·1877	1·2068	1·2250
∞	9/4	Z	0·0	0·0631	0·1298	0·2002	0·2739	0·3508	0·4309	0·5137	0·5993	0·6874	0·7780
		S/S_o	1·0	1·0620	1·1216	1·1788	1·2335	1·2840	1·3326	1·3785	1·4217	1·4622	1·5000
∞	∞	Z	0·0	0·0525	0·11	0·1725	0·240	0·3125	0·3900	0·4725	0·5600	0·6525	0·7500
		S/S_o	1·0	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0

Progressive and Degressive nature of Burning

From (10) we have

$$\frac{d}{df}\left(\frac{S}{S_\circ}\right) = -\beta - 2\gamma f \text{, and } \frac{d^2}{df^2}\left(\frac{S}{S_\circ}\right) = -2\gamma. \tag{14}$$

Since γ is always + ve, $\frac{d^2}{df^2}\left(\frac{S}{S_\circ}\right) < 0$ so that $\frac{d}{df}\left(\frac{S}{S_\circ}\right)$ is a decreasing function of f.

At the end of the first phase $f=0$

i.e.

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=0} = -\beta \tag{15}$$

and at the beginning of the combustion

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=1} = -\beta - 2\gamma \tag{16}$$

From (11), (12) and (13), we get

$$\left.\begin{aligned} \left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=0} &= \frac{(m-3)(5m-4m\rho+1)}{2(2m^2\rho+10m\rho+m^2-5)} \\ \text{and} \quad \left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=1} &= \frac{(m-3)(m+5-2m\rho)}{(2m^2\rho+10m\rho+m^2-5)} \end{aligned}\right\} \tag{17}$$

$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]$ has opposite signs at $f=0$ and at $f=1$.

Therefore $\left(\frac{S}{S_\circ}\right)$ will have its maximum value in the interval of variation of f,

if

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=0} \left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=1} < 0$$

i.e. if

$$(5m - 4m\rho + 1)(m + 5 - 2m\rho) < 0$$

or if

$$(\rho_2 - \rho)(\rho_1 - \rho) < 0$$

where

$$\rho_1 = (5 + m)/2m \tag{18}$$

and

$$\rho_2 = (5m + 1)/4m \tag{19}$$

ρ_1 and ρ_2 are decreasing functions of m and vary from (4/3 to 1/2) and (4/3 to 5/4) where m takes up the values from 3 to ∞.

(S/S_o) is maximum when

$$f = -\frac{\beta}{2\gamma} = -\frac{4m\rho - 5m - 1}{3(m-3)}$$

which can also be written in the form,

$$f = 1 - \frac{\rho - \rho_1}{3\rho_{min}}$$

The values of ρ_1, ρ_2 and ρ_{min} are tabulated for different values of m viz. 3, 4,...... 9, 10 in Table 2.

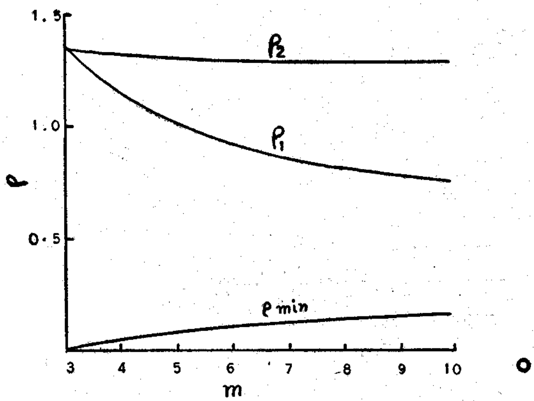

FIG. 4— Relation between ρ_1, ρ_2 and ρ_{min}

TABLE 2

VALUES OF ρ_1, ρ_2 & ρ_{min} FOR DIFFERENT VALUES OF m

m	3	4	5	6	7	8	9	10	∞
ρ_1	1·333	1·3125	1·3	1·292	1·285	1·2812	1·2778	1·275	1·25
ρ_2	1·333	1·125	1·0	0·9167	0·8571	0·8025	0·7778	0·75	0·5
ρ_{min}	0·0	0·0625	0·1	0·125	0·1429	0·1562	0·1667	0·175	0·25

An alternative way of deriving (10), following Jain[8], is as follows:

$$S_o = \tfrac{1}{2}\,(2m^2\rho + 10m\rho + m^2 - 5)\;\pi d^2$$

$$S = [2\{\pi\tfrac{1}{2}md - \tfrac{1}{2}D\,(1-f\,)\} - 10\pi\;\{\tfrac{1}{2}d + \tfrac{1}{2}D\,(1-f\,\}]\;\{m\rho d - D\,(1-f\,)\}$$
$$+\;2[\pi\,\{\tfrac{1}{2}md - \tfrac{1}{2}D\,(1-f\,)\}^2 - 5\pi\;\{\tfrac{1}{2}.d + \tfrac{1}{2}.D\,(\,1-f\,)^2\}],$$
$$= \tfrac{1}{2}\pi d^2\left[\left\{(2m^2\rho + 10m\rho + m^2 - 5) + \frac{D}{d}\;(2m\rho - m - 5) - 12\left(\frac{D}{d}\right)^2\right\}\right.$$
$$\left. +\;4\left(\frac{D}{d}\right)\left\{m + 5 - 2m\rho + 6\;\left(\frac{D}{d}\right)\right\}f - 12\left(\frac{D}{d}\right)^2 f^2\right],$$

using equation (4)

$S = \frac{1}{2}\pi d^2 \left[\frac{1}{4}(16m\rho - 3m + 13)(m+1) - \frac{1}{2}(m-3)(4m\rho - 5m - 1) f - \frac{3}{4}(m-3)^2 f^2\right]$

Hence the expression for $\left(\frac{S}{S_\circ}\right)$ is given as

$$\frac{S}{S_\circ} = \frac{1}{(2m^2\rho + 10m\rho + m^2 - 5)} \left[\frac{1}{4}(m+1)(10m\rho - 3m + 13) - \frac{1}{2}(m-3)(4m\rho - 5m - 1) f - \frac{3}{4}(m-3)^2 f^2 \right] \quad (20)$$

which is of the form

$$\frac{S}{S_\circ} = \alpha - \beta f - \gamma f^2$$

where α, β, γ are the constants and are same as that of (15) *i.e.* the same expression which has been derived from the value of z.

The value of z can also be deduced from the equation (20) and is as follows:—

$$\left(\frac{S}{S_\circ}\right) = \frac{\left(\frac{dz}{df}\right)}{\left(\frac{dz}{df}\right)_{f=1}} = \alpha - \beta f - \gamma f^2. \quad (21)$$

so that

$$\frac{dz}{df} = K(\alpha - \beta f - \gamma f^2), \text{ where } K = \left(\frac{dz}{df}\right)_{f=1}$$

Integrating we get

$$z = - K(1-f)\{(\alpha - \beta/2 - \gamma/3) - (\beta/2 + \gamma/3) f - (\gamma/3) f^2\},$$

which is the required (z, f) relation and is of the form (6) with

$$a = - K(\alpha - \beta/2 - \gamma/3), \quad b = - K(\beta/2 + \gamma/3), \quad c = - K(\gamma/3) \quad (22)$$

For finding the value of K, we note that

$$\left(\frac{dz}{df}\right)_{f=1} = - \frac{DS_\circ\delta}{2c} = - \frac{DS_\circ}{2V_\circ}$$

so that

$$K = - \frac{DS_\circ}{2V_\circ}$$

Substituting the values of D, $S_\circ$ and $V_\circ$ this gives

$$K = - \frac{(m-3)(2m^2\rho + 10m\rho + m^2 - 5)}{4m\rho(m^2 - 5)}.$$

Substituting the values of α, β, γ and K to the equation (20), we get the same values for a, b, c as given in equations (7), (8) and (9).

SECOND PHASE OF COMBUSTION

In figure 5, O, A, B are the centres of the central and two other holes. OAB is an isosceles triangle right angled at O, of side $2a=D+d$. At the beginning of this phase there are four curvilinear prisms like P′Q′R′S′ with radii a while the length of the side OP′ is $3a$.

As burning proceeds the prism $P'Q'R'S'$ shrinks into $PQRS$ of circular arcs, where the radius of the arcs PQ, RS and S R is r while that of the arc $Q\ R$ is $(4a-r)$, since the burning is assumed to be by parallel layers. If χ, ω and φ are the angles as shown in figure (5), from $\triangle OPV$,

$$r = a \sec \omega \tag{23}$$

and

$$\cos \varphi = \frac{OP^2+OV^2-PV^2}{2OP.OV}.$$

Using equation (23), $\cos \varphi$ becomes

$$\cos \varphi = \frac{5-2 \sec \omega}{4-\sec \omega} \tag{24}$$

End of the second phase will occur at $\omega=\pi/4$, i.e. $\varphi = 32°54'$; from equation (24).

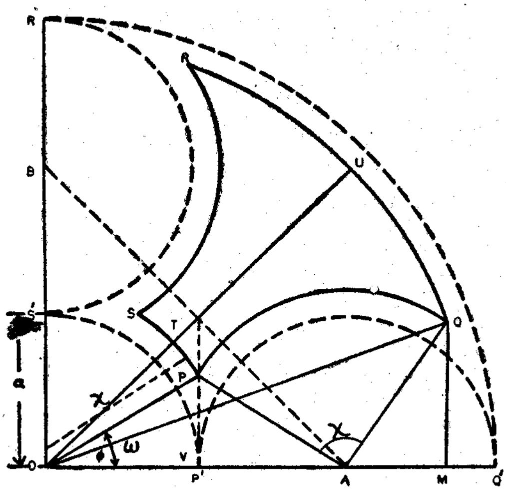

FIG. 5— Second Phase of Combustion.

From $\triangle OQM$

$$QM=OQ \sin \varphi,$$

and $\triangle\ QAM$ gives

$$QM=QA \sin (\omega+\chi),$$

by equating the two values of Q M

$$\sin (\omega+\chi) = (4-\sec \omega) \sin \varphi \cos \omega. \tag{25}$$

With the help of equation (24) and (25) χ and φ are tabulated in the parameter ω at the interval of 5° and is given in Table 3.

TABLE 3

VALUES OF ϕ AND χ IN PARAMETER ω

ω deg	ω (Radians)	ϕ deg	ϕ radians	χ deg	χ radians
0°	0·0000	0°·00′	0·0000	180°	3·1416
5°	0·0873	2°·55′	0·0509	166°16′	3·0774
10°	0·1745	5°·51′	0·1021	152°37′	2·6637
15°	0·2618	8° 51′	0·1545	138°59′	2·3751
20°	0·3491	12° 1′	0·2097	125°00′	2·1817
25°	0·4363	15°22′	0·2682	111°00′	1·9373
30°	0·5236	18°59′	0·3314	96° 52′	1·6906
35°	0·6109	22°54′	0·4012	82° 22′	1·4376
40°	0·6981	27°32′	0·4806	67°32′	1·1787
45°	0·7854	32°54′	0·5762	51° 57′	0·9067

As r and ω increases, the length of the prism at the instant is given by

$$L = mpd - D - 2\,(r - a) = mpd - D - 2a\,(\sec\omega - 1) \qquad (26)$$

Since $L = mpd - D$ at the end of the first phase, and the burning proceeds for a distance $(r - a)$ at each end.

Hence the fraction f of D is given by

$$f = \frac{2(a - r)}{D} = \frac{2a}{D}(1 - \sec\omega).$$

Area of the sector $PQRS$

$$= 2\,[\text{Sector } OUQ + \triangle OQA - \text{Sector } PAQ - \text{Sector } TPO - \triangle OPA]. \qquad (27)$$

Area of Sector OUQ

$$= \tfrac{1}{2}OQ^2\left(\frac{\pi}{4} - \varphi\right) = \tfrac{1}{2}\,a^2\left(\frac{\pi}{4} - \varphi\right)\left(4 - \sec\omega\right)^2 .$$

Similarly

$$\triangle\ OQA = \tfrac{1}{2}OA\cdot OQ\,\sin\varphi = a^2\,(4 - \sec\omega)\,\sin\varphi.$$

$$\text{Sector}\quad PAQ = \tfrac{1}{2}AP^2.\ \chi = \tfrac{1}{2}a^2\,\chi\,\sec^2\omega\ .$$

$$\text{Sector}\quad TPO = \tfrac{1}{2}OP^2\left(\frac{\pi}{4} - \omega\right) = \tfrac{1}{2}a^2\left(\frac{\pi}{4} - \omega\right)\sec^2\omega.$$

$$\triangle\quad OPA = \tfrac{1}{2}OP.OA\ \sin\omega = a^2\tan\omega\ .$$

$$\text{Sector}\quad PQRS = a^2H(\omega) \qquad (28)$$

where,

$$H(\omega) = 4\pi + 8\sin\varphi - 16\phi - 2\tan\omega - 2(\pi - 4\varphi + \sin\varphi)\sec\omega - (\chi + \varphi - \omega)\sec^2\omega \quad (29)$$

Hence the total volume of the prism. Also,

$$V = 4a^2 H(\omega)\, L$$

and

$$z = 1 - 4a^2 H(\omega) \frac{\{(4m\rho d - D) - 2a(\sec\omega - 1)\}}{\pi/4\rho m\, d^3 (m^2 - 5)},$$

Substituting the values of a, d, D in terms of m and ρ then

$$z = 1 - (m+1)^2 \frac{\{4m\rho - (m+1)\sec\omega + 4\}}{16\pi m\rho\,(m^2 - 5)} H(\omega) \quad (30)$$

The length of the circumference of PQRS

$$= 2r\left(\frac{\pi}{4} - \omega\right) + 2(4a - r)\left(\frac{\pi}{4} - \varphi\right) + 2r\chi,$$

$$= 2a\left[(\varphi + \chi - \omega)\sec\omega + \pi - 4\varphi\right] \quad (31)$$

and the surface of combustion is,

S = four times the length of the circumference $PQRS$ + twice × four times the area $PQRS$

$$= 8a\,L[(\phi + \chi - \omega)\sec\omega + \pi - 4\varphi] + 4.2a^2\, H(\omega).$$

for $0 \leqslant \omega \leqslant 45°$

Hence the function of progressivity is given by

$$\frac{S}{S_0} = \frac{(m+1)[2(\varphi + \chi - \omega)\sec\omega + \pi - 4\phi\}\{4 + 4m\rho - (m+1)\sec\omega\} + (m+1)H(\omega)]}{4\pi(2m^2 + 10m\rho + m^2 - 5)} \quad (32)$$

Equations (26) & (30) give the relation between z & f and (26) & (32) give the relation between $\frac{S}{S_0}$ & f but both will determine in the parameter ω. Their tabulated values are given in Table 4.

THIRD PHASE OF COMBUSTION

The third phase of combustion begins, when $PQRS$ shrinks to HIJ with the arc $HA = BH = A = \sqrt{2}.\, a$ *i. e.* during this phase the powder grain consists of four curvilinear triangular prisms of bases like HIJ as shown in figure (6) and of common length L

where $L = m\rho d - D - (2A - \sqrt{2}A)$

TABLE 4

VALUE OF z, S/S_o AND f FOR DIFFERENT VALUES OF m, & ρ FOR THE SECOND PHASE OF COMBUSTION

m	ρ		0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
4	1/2	f	0·0000	—0·0191	—0·0771	—0·1763	—0·3209	—0·5169	—0·7735	—1·1038	—1·5270	—2·0709
		z	0·5028	0·5380	0·5477	0·5749	0·6239	0·6719	0·7510	0·8211	0·8927	0·9538
		S/So	0·8780	0·8242	0·7511	0·6490	0·6115	0·5226	0·4487	0·3592	0·2660	0·1697
4	1	f	0·0000	—0·0191	—0·0771	—0·1763	—0·3209	—0·5169	—0·7735	—1·1038	—1·5270	—2·0709
		z	0·4673	0·5043	0·5125	0·5383	0·5867	0·6335	0·7156	0·7891	0·8677	0·9403
		S/So	0·9790	0·9306	0·8368	0·7771	0·6895	0·6047	0·5261	0·4357	0·3404	0·2359
4	9/4	f	0·0000	—0·0191	—0·0771	—0·1763	—0·3209	—0·5169	0·7735	—1·1038	—1·5270	—2·0071
		z	0·4476	0·4856	0·4929	0·5179	0·5660	0·6122	0·6959	0·7714	0·8539	0·9315
		S/So	1·0476	0·9976	0·8950	0·8912	0·7425	0·6598	0·5786	0·4876	0·3909	0·2869
7	9/4	f	0·0000	—0·0076	—0·0308	—0·0705	—0·1284	—0·2068	—0·3094	—0·4415	—0·6108	—0·8284
		z	0·6594	0·6831	0·6876	0·7032	0·7326	0·7609	0·8124	0·8603	0·9067	0·9576
		S/So	1·1560	1·1012	0·9878	0·9167	0·8361	0·7597	0·6394	0·4326	0·3113	0·3113
10	9/4	f	0·0000	—0·0060	—0·0242	—0·0554	—0·1008	—0·1624	—0·2431	—0·3469	—0·4800	—0·6508
		z	0·7064	0·7265	0·7304	0·7438	0·7691	0·7936	0·8380	0·8782	0·9221	0·9639
		S/So	1·2250	1·1665	1·0464	0·9710	0·8685	0·7764	0·6776	0·5715	0·4588	0·3304
∞	9/4	f	0·0000	—0·0038	—0·0154	—0·0353	—0·0642	—0·1034	—0·1547	—0·2208	—0·3054	—0·4142
		z	0·7780	0·7930	0·7960	0·8061	0·8252	0·8437	0·8773	0·9077	0·9409	0·9722
		S/So	1·5000	1·4285	1·2813	1·1887	1·0637	0·9512	0·8306	0·7012	0·5634	0·4062
∞	∞	f	0·0000	—0·0038	—0·0154	—0·0352	—0·0642	—0·1034	—0·1557	—0·2208	—0·3054	—0·4142
		z	0·7500	0·7671	0·7700	0·7809	0·8028	0·8218	0·8592	0·8932	0·9309	0·9670
		S/So	2·0000	1·9068	1·7075	1·5040	1·4245	1·2799	1·1288	0·9640	0·7874	0·5803

and ξ, η, ζ are the angles in same sense as ω, φ, χ as in fig. (5) and (6) respectively.
From $\triangle AEH$

$$R = A \sec \zeta \tag{33}$$

and

$$\cos \xi = \frac{OF^2 + OA^2 - AF^2}{2OF.OA} = \frac{5 - 2\sqrt{2} \sec \zeta}{4 - \sqrt{2} \sec \zeta} \tag{34}$$

And there will be a complete combustion of the grain at $\xi = \frac{\pi}{4}$ so that $\zeta = 32° \ 39'$.

Also from $\triangle\ OFA$

$$\cos \left(\frac{\pi}{4} + \eta + \zeta \right) = \frac{A\,F^2 + O\,A^2 - O\,F^2}{2\,A\,F \cdot O\,A} = 2 - \frac{3}{\sqrt{2}} \cos \zeta \tag{35}$$

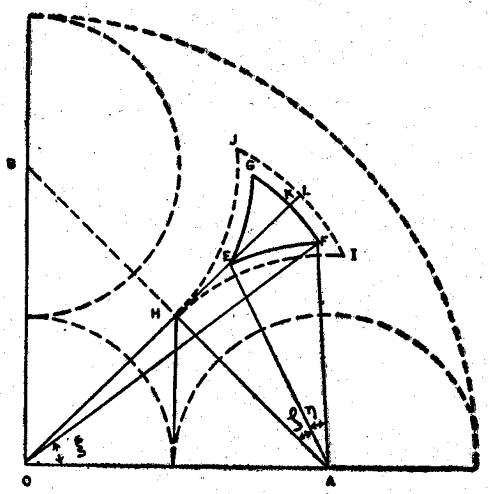

FIG. 6 —Third phase of combustion.

With the help of equations (34) and (35), (η, ξ) are tabulated in the parameter ζ at the interval of five minutes and is given in Table 5.

TABLE 5

VALUES OF η & ξ IN PARAMETER ζ.

ζ deg	ζ radians	ξ deg	ξ radians	η deg	η radians
0°	0·0000	32° 54′	0·5576	51° 57′	0·9067
5°	0·0873	33° 00	0·5783	46° 30′	0·8116
10°	0·1746	33° 54′	0·5917	39° 39′	0·6938
15°	0·2618	35° 13′	0·6147	32° 51′	0·5734
20°	0·3491	37° 13′	0·6496	24° 33′	0·4285
25°	0·4363	39° 36′	0·6912	15° 32′	0·2711
30°	0·5236	42° 54′	0·7487	5° 38′	0·0986
32° 39′	0·5690	45° 00′	0·7854	0° 00′	0·0000

As R and ζ increases, the length of the prism is given by

$$L = m\,\rho\,d - D + \sqrt{2}\,(1 - \sqrt{2}\sec\zeta) \tag{36}$$

Hence the fraction f of D is given by

$$f = \frac{2a - 2R}{D} = \frac{2a}{D}\,(1 - \sqrt{2}\sec\zeta) \tag{37}$$

Area of the curvilinear triangle LEF is given by

$$\triangle LEF = 2\,[\,\text{Sector } LOF + \triangle OFA - \text{Sector } EFA - \triangle OAE\,] \tag{38}$$

Area of the sector $L\,O\,F$

i. e. $\text{Sector } LOF = \tfrac{1}{2}\,OF^2\left(\frac{\pi}{4} - \xi\right) = \tfrac{1}{2}\,A^2\,(2\sqrt{2} - \sec\zeta)^2\left(\frac{\pi}{4} - \xi\right)$

Similarly

$$\triangle OFA = \tfrac{1}{2}\,OA \cdot OF\sin\xi = \tfrac{1}{2}\sqrt{2}\,A^2\,(2\sqrt{2} - \sec\zeta)\sin\xi$$

$$\text{Sector } EFA = \tfrac{1}{2}\,(EA)^2\,\beta = \tfrac{1}{2}\,\beta\,A^2\sec^2\zeta$$

$$\triangle OAE = \tfrac{1}{2}\,OA \cdot AE\sin\left(\frac{\pi}{4} + \zeta\right) = \frac{A^2}{\sqrt{2}}\sec\zeta\sin\left(\frac{\pi}{4} + \zeta\right)$$

with the help of these values the area $G\,E\,F$.

$\triangle GEF = A^2\,K(\zeta)$

where

$$K(\zeta) = \left[\,2(\pi - 4\xi + 2\sin\xi) - \sqrt{2}\left\{(\pi - 4\xi) + \sin\xi + \sin\left(\frac{\pi}{4} + \zeta\right)\sec\zeta\right\} - \left(\xi + \eta - \frac{\pi}{4}\right)\sec^2\zeta\,\right]$$

And the volume of this prism is

$$V = 4\,A^2\,K(\zeta)\,L$$

But

$$z = 1 - \frac{V}{V_o}$$

$$= 1 - \frac{4\,A^2\,K(\zeta)\left\{m\,\rho\,d - D + \sqrt{2}\,A\,(1 - \sqrt{2}\sec\zeta)\right\}}{\tfrac{1}{4}\,\pi\,\rho\,m\,d^3\,(m^2 - 5)}$$

$$= 1 - \frac{(m+1)^2\left\{4\,m\,\rho - \sqrt{2}\,(m+1)\sec\zeta + 4\right\}K(\zeta)}{8\,\pi\,m\,\rho\,(m^2 - 5)} \tag{39}$$

Now the length of the circumference $E\ F\ G$

$$= 2 R \eta + (2\sqrt{2}\, A - R)\left(\frac{\pi}{2} - 2\xi\right)$$

$$= A\,[2\,(\eta + \xi - \pi/4)\sec\zeta + \sqrt{2}\,(\pi - 4\,\xi)\,].$$

Surface of combustion at the instant t is given by

S = four times the length of the circumference of $E\ F\ G$ × length of the grain at the instant t + twice × four times the area $E\ F\ G$

$$= 4\,A\left\{2\left(\xi + \eta - \frac{\pi}{4}\right)\sec\zeta + \sqrt{2}\,(\pi - 4\,\xi)\right\}$$

$$\left\{(m\,\rho\,d - D + \sqrt{2}\,A)\,(1 - \sqrt{2}\sec\zeta\right\} - 8\,A^2\,K\,(\zeta)$$

$$\therefore \frac{S}{S_0} = \frac{(m+1)\left[\,2\left\{4\,(m\,\rho + 1) - \sqrt{2}\sec\zeta\,(m+1)\right\}\left\{\sqrt{2}(\xi + \eta - \pi/4)\sec\zeta + (\pi - 4\,\xi)\right\} + 2\,(m+1)\,K\,(\zeta)\,\right]}{4\,\pi\,(2\,m^2\,\rho + 10\,m\,\rho + m^2 - 5)} \qquad (40)$$

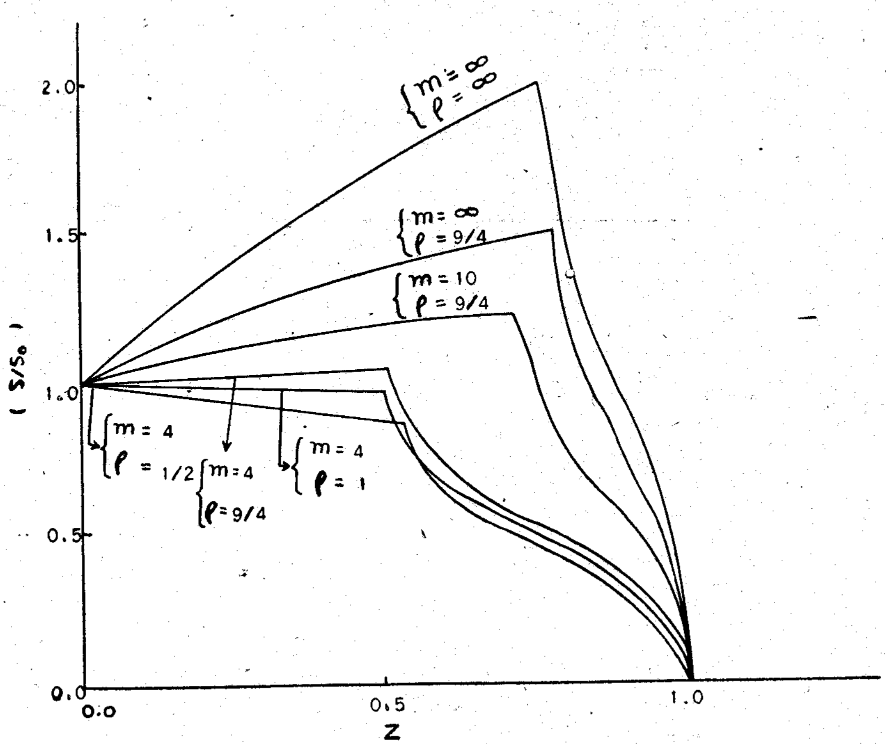

FIG. 7—Relation between S/S_0 and z for some set of values of m and ρ

Equations (37), (39), (37) & (40) give the relation between (z, f) and $\left(\frac{S}{S_o}, f\right)$ respectively but both will determine in the parameter ζ. Their tabulated values are given in Table 6.

VARIATION OF (S/S_o) MAX.

It has been shown in (19) that $(S/So)_{max}$ for

$$f = \frac{(5\,m + 1) - m\,\rho}{3\,(m - 3)} \qquad (41)$$

with

$$\left.\begin{aligned} \rho_1 &= \frac{m + 5}{2\,m} \\ \text{and} \quad \rho_2 &= \frac{5\,m + 1}{4\,m} \end{aligned}\right\} \qquad (42)$$

also

$$\begin{aligned} \left(\frac{S}{S_o}\right)_{max} &= 1 & \rho &< \rho_1 \\ &= \alpha + \beta^2/4\,\gamma & \rho_1 \leqslant \rho &\geqslant \rho_2 \\ &= \alpha & \rho &> \rho_2 \end{aligned}$$

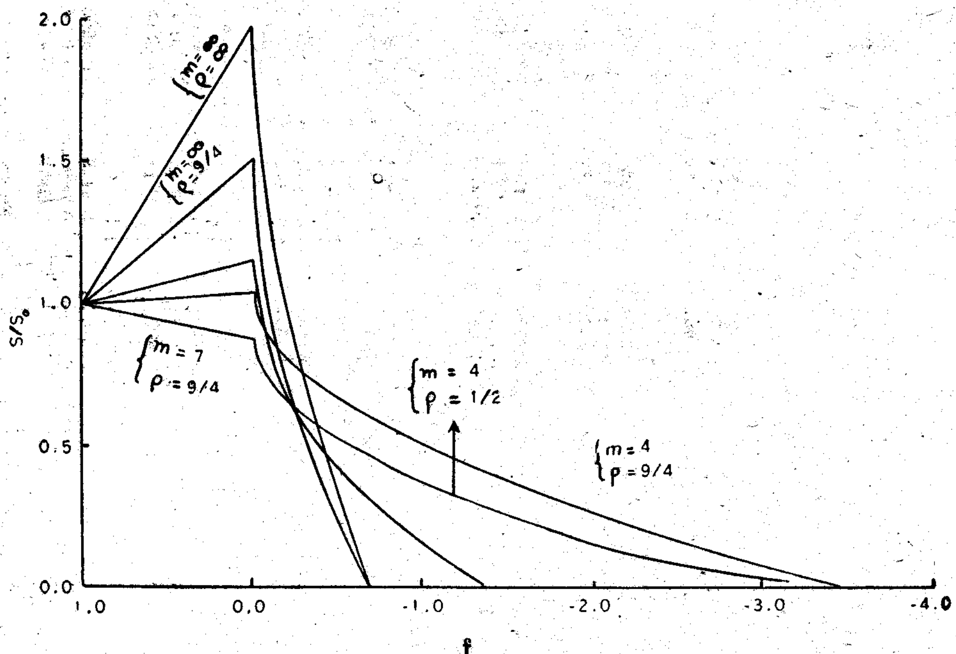

FIG. 8—Relation between S/S_o and f for some set of values of m and ρ

TABLE 6

VALUE OF z, S/S_0 AND f FOR DIFFERENT VALUES OF m & ρ FOR THE THIRD PHASE OF COMBUSTION

m	ρ		0°	5°	10°	15°	20°	25°	30°	32° 39′
4	1/2	f	—2·0709	—2·0980	—2·1801	—2·3205	—2·5248	—2·8020	—3·1649	—3·4980
		z	0·9538	0·9574	0·9608	0·9709	0·9832	0·9926	0·9991	1·0000
		S/S_o	0·1697	0·1565	0·1365	0·1124	0·0815	0·0487	0·0159	0·0000
4	1	f	—2·0709	—2·0980	—2·1801	—2·3205	—2·5248	—2·820	—3·1649	—3·4980
		z	0·9403	0·9439	0·9479	0·9606	0·9767	0·9892	0·9985	1·0000
		S/S_o	0·2359	0·2182	0·1913	0·1613	0·1216	0·0666	0·0283	0·0000
4	9/4	f	—2·0709	—2·0980	—2·1805	—2·3205	—2·5248	—2·8020	—3·1649	—3·4980
		z	0·9315	0·9363	0·9407	0·9549	0·9730	0·9873	0·9982	1·0000
		S/S_o	0·28691	0·2615	0·2285	0·1944	0·1489	0·0955	0·3634	0·0000
7	9/4	f	—0·8284	—0·8392	—0·8721	—0·9282	—1·0099	—1·1208	—1·2660	—1·3592
		z	0·9639	0·9608	0.9633	0·9712	0·9833	0·9922	0·9988	1·0000
		S/S_o	0·3113	0·2885	0·2533	0·2156	0·1651	0·1060	0·0404	0·0000
10	9/4	f	—0·6508	—0·6502	—0·6750	—0·7291	—0·7973	—0·8805	—0·9845	—1·0670
		z	0·9639	0·9660	0·9683	0·9759	0·9856	0·9932	0·9991	1·0000
		S/S_o	0·3304	0·2905	0·2552	0·2172	0·1663	0·1608	0·0406	0·0000
∞	9/4	f	—0·4142	—0·4196	—0·4360	—0·4641	—0·5050	—0·5604	—0·6329	—0·6961
		z	0·9722	0·9742	0·9760	0·9817	0·9890	0·9948	0·9993	1·0000
		S/S_o	0·4062	0·3761	0·3292	0·2814	0·2157	0·1387	0·5288	0·0000
∞	∞	f	—0·4142	—0·4196	—0·4360	—0·4641	—0·5050	—0·5604	—0·6330	—0·6800
		z	0·9670	0·9694	0·9714	0·7781	0·9868	0·9938	0·9992	1·0000
		S/S_o	0·5803	0·5378	0·4731	0·4050	0·3127	0·2034	0·0787	0·0000

Considering the different cases, for the values of m at 4, 7, 10 and ∞.

Case I

at $m = 4$, it gives

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{4\rho(16\rho + 18) + 114}{3(72\rho + 11)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with $\rho_1 = \frac{9}{8}$ and $\rho_2 = \frac{21}{16}$

Values of $(S/S_\circ)_{max}$ for some values of ρ are given in Table 7.

TABLE 7

VALUES OF $(S/S_\circ)_{max}$ FOR SOME VALUES OF ρ.

ρ	ρ_1 1·125	ρ_2 1·3125	2	2·25	3	4	5	20	∞
$(S/S_\circ)_{max.}$	1	1·0071	1·0403	1·0473	1·0628	1·0744	1·0815	1·1035	1·117

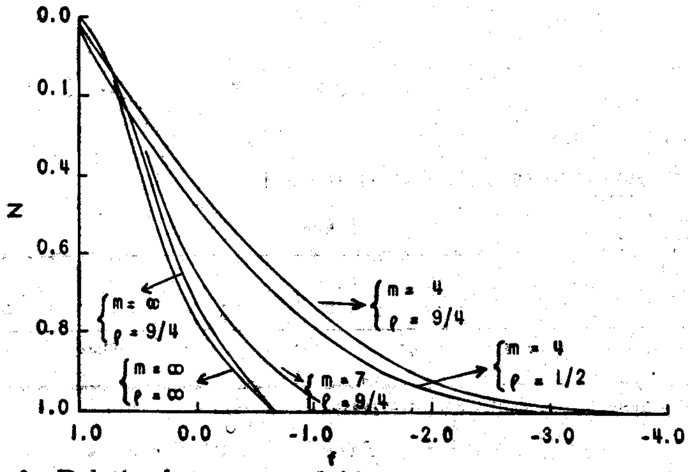

FIG. 9—Relation between z and f for some set of values of m and ρ.

Case II

at $m = 7$, it gives

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{\rho(49\rho + 42) + 69}{3(42\rho + 11)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with $\rho_1 = \frac{6}{7}$ and $\rho_2 = \frac{9}{7}$.

Values of $(S/S_\circ)_{max}$ for some values of ρ are given in Table 8.

TABLE 8

VALUES OF $(S/S_\circ)_{max}$ FOR SOME VALUES OF ρ.

ρ	ρ_1 0·8571	ρ_2 1·235	2	3	4	5	20	∞
$(S/S_\circ)_{max}$	1	1·0461	1·1368	1·1971	1·2290	1·2484	1·3114	1·3320

Case III

at $m = 10$, it gives

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{20\,\rho\,(4\,\rho + 3) + 102}{3\,(60\,\rho + 14)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with $\rho_1 = \frac{3}{4}$ and $\rho_2 = \frac{51}{40}$.

Values of $(S/S_\circ)_{max}$ for some values of ρ are given in Table 9.

TABLE 9

ρ	ρ_1 0·75	ρ_2 1·275	2	2·25	3	4	5	20	∞
$(S/S_\circ)_{max.}$	1	1·0769	1·2075	1·2251	1·2801	1·323	1·35	1·4361	1·4662

Case IV

Lastly at $m = \infty$, it gives

$$\left(\frac{S}{S_\circ}\right)_{max} = \frac{3\,\rho\,(2\,\rho + 1) + 4}{3\,(2\,\rho + 1)} \qquad \rho_1 \leqslant \rho \leqslant \rho_2$$

with $\rho_1 = 1/2$ and $\rho_2 = 5/4$.

Values $(S/S_\circ)_{max}$ for some values of ρ are given in Table 10.

TABLE 10

ρ	ρ_1 0·5	ρ_2 1·25	2	2·25	3	4	5	∞
$(S/S_\circ)_{max.}$	1	1·2143	1·45	1·5	1·607	1·6944	1·75	2·00

The results of these four cases are illustrated in fig. 10.

EQUIVALENT FORM FACTOR θ

For finding the value of θ satisfying the system of equations, (Tavernier) for the period before the rupture of the grain, the equations are

$$\left.\begin{aligned} (a - b - c) &= (1 + \theta)(1 - f \\ \text{and} \qquad a &= (1 - f)(1 + \theta f) \end{aligned}\right\} \tag{43}$$

Eliminating f, we obtain equations giving θ, viz.

$$\frac{(1 + \theta)^2}{\theta} = -\frac{(a - b - c)^2}{(b + c)} \tag{44}$$

whence

$$\theta = -\frac{\{(a - b - c)^2 + 2\,(b + c)\} \pm (a - b - c)\,\{(a - b - c)^2 + 4\,(b + c)\}^{\frac{1}{2}}}{2\,(b + c)} \tag{45}$$

Considering that radical sign which makes $\theta < 1$. θ has been tabulated for the different set of values of m and ρ and given in Table 11.

TABLE 11

θ THE EQUIVALENT FORM FACTOR BEFORE THE RUPTURE OF THE GRAIN

m/ρ	1/2	1	9/4	5	20	∞
4	0·1405	0·0125	—0·0505	—0·0751	—0·1098	—0·1255
7	0·1847	0	—0·1109	—0·1568	—0·1808	—0·1975
10	0·2244	—0·0155	—0·1924	—0·1928	—0·2306	—0·2424
∞	0·1716	—0·092	—0·2651	—0·3226	—0·386	—0·382

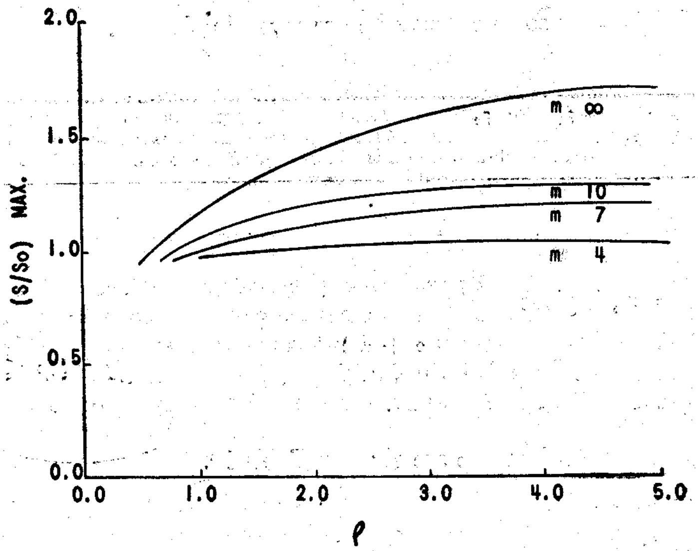

FIG. 10—Variation of $(S/S_o)_{max}$ with ρ for some values of m.

The relation between English and French coefficients of progressivity is

$$K = -\frac{4\theta}{(1+\theta)^2} \tag{46}$$

with the help of equation (44) it gives

$$K = \frac{4(b+c)}{(a-b-c)^2}$$

Its values, for the above values of m and ρ are shown in Table 12.

TABLE 12

VALUES OF COEFFICIENT OF PROGRESSIVITY FOR SOME VALUES OF m & ρ.

m/ρ	1/2	1	9/4	5	20	∞
4	—0·4302	—0·0720	0·2286	0·3917	0·5606	0·6082
7	—0·5253	0·0000	0·5449	0·8825	1·1299	1·2218
10	—0·5381	0·0733	1·1811	1·2169	1·5566	1·6877
∞	—0·5000	0·4444	1·7857	2·8099	3·6736	4·0000

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