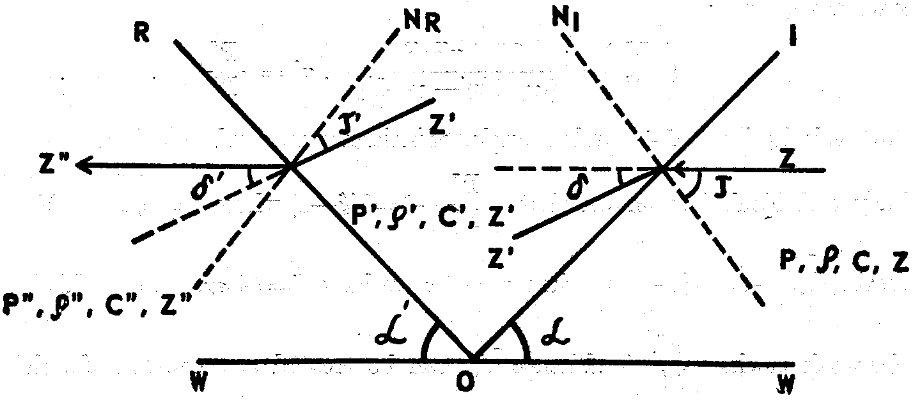
OBLIQUE REFLECTION OF SHOCK WAVE

R. S. SRIVASTAVA

Defence Science Laboratory, Delhi

Bleakney and Taub[1] state on physical grounds that the relative outflow from the reflected shock is supersonic, sonic and subsonic according as $\alpha \lesseqgtr \alpha_s$, α being the angle of incidence and α_s is that angle of incidence corresponding to which the relative outflow from the reflected shock is sonic. In this note the statement of Bleakney and Taub[1] has been given mathematical approach for moderate and weak incident shock strengths.



Figure

From the figure shown and without assigning any convention of sign as Bleakney and Taub has done we find that for the investigation of the problem we should have $\sigma = \sigma'$.

Applying this condition we obtain,

$$x' = \frac{x(1+\eta^2x^2) \pm \sqrt{x^2(1+\eta^2x^2)^2 - (1+\eta x^2)\left\{(\gamma+1)(\eta-1)+2\right\}\left\{(\gamma-1)(1+\eta x^2)+2\right\}(\eta-1)}}{(1+\eta x^2)[(\gamma+1)(\eta-1)+2]} \quad \ldots \quad \ldots \quad (1)$$

where $x = \tan\zeta,\ \alpha + \zeta = \dfrac{\pi}{2}$

$$x' = \tan\zeta',\ \alpha' + \zeta' + \delta' = \frac{\pi}{2}$$

and $\eta = \dfrac{(\gamma+1)\,\zeta + (\gamma-1)}{(\gamma-1)\,\zeta + (\gamma+1)},\ \zeta = \dfrac{P'}{P}$

Following Bleakney and Taub, neglecting the negative sign before the radical sign in (1) we have

$$x' = \frac{\frac{x^2(1+\eta^2x^2)}{(1+\eta x^2)} + \sqrt{\frac{x^2(1+\eta^2x^2)^2}{(1+\eta x^2)^2} - \left\{(\gamma+1)(\eta-1)+2\right\}(\eta-1)\left\{(\gamma-1)+\frac{2}{(1+\eta x^2)}\right\}}}{[(\gamma+1)(\eta-1)+2]} \quad \ldots \quad \ldots \quad (2)$$

From (2) we shall see that for fixed value of η, x' increases with x. Now $\frac{1+\eta^2x^2}{1+\eta\, x^2} = \eta - \frac{(\eta-1)}{1+\eta x^2}$. From this it is seen that as x increases $\frac{(\eta-1)}{1+\eta x^2}$ decreases and therefore $\eta - \frac{(\eta-1)}{1+\eta x^2}$ increases. Also $\left\{(\gamma-1)+\frac{2}{(1+\eta x^2)}\right\}$ decreases as x increases. It follows therefore from (2) that x' increases with x. Across the reflected shock we have,

$$\left(\frac{Z''}{C''}\right)_2 = \frac{2(1+\eta'^2x'^2)}{[(\gamma+1)\eta'-(\gamma-1)]}\ , \eta' = \frac{P''}{P'} \quad \ldots \quad \ldots \quad (3)$$

Now for weak shocks $\eta=1+\epsilon$ and for angles of incidence near and less than α— extreme, Bleakney and Taub has shown that $\frac{P''}{P'}-1=3(\xi-1)$ where $\xi = \frac{P''}{P'}$. From this we obtain $\frac{P''}{P'}-1=2(\xi-1)$ as when $\eta=1+\epsilon$ we have $\zeta=1+\epsilon\gamma$. From this it follows that for weak shocks $\frac{P''}{P'}$ (and hence η') can be treated as a constant for the whole range of variation in x. Also from the graphs of Polachek and Seeger (2) it can be seen that $\frac{P''}{P'}$ can be treated as a constant for weak and moderate incident shock strengths for the whole range of variation in x. It follows therefore from (3) that $\frac{Z''}{C''}$ is an increasing function of x' (*i.e.* of x) and is therefore a decreasing functions of α. This proves the statement.

The author is grateful to the Director, Defence Science Laboratory for kind permission to publish this note.

REFERENCES

1. BLEAKNEY, W. and TAUB, A. H., *Rev. Modern Phys*, **21**, 584 (1949).

2. POLACHEK, H. and SEEGER, R.J., *Proc. of Symposia in Applied Mathematics*, **I**, 119 (1949).