AN AMPLIFIER FOR FOURTH ORDER SYSTEMS

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ABSTRACT

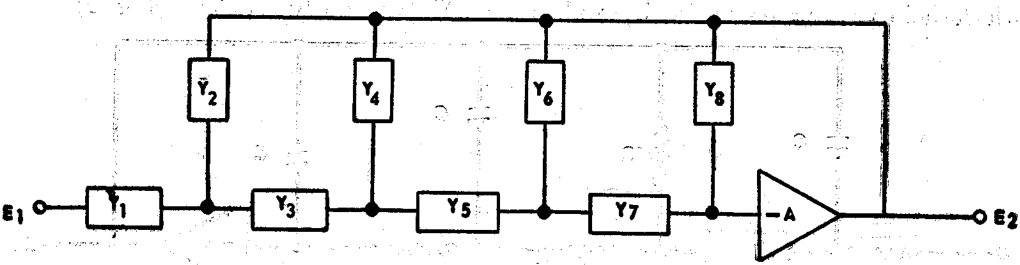
This work presents a circuit for simulating fourth order linear systems using a single operational amplifier. Every one of the three forms discussed is illustrated by one arbitrary simple choice of circuit elements. Simulation is possible for only limited range of values of the systems parameters. The circuit can be extended, by adding more laterally inverted L Sections, to simulate higher order linear systems.

INTRODUCTION

In his earlier work[7], the author has discussed a circuit which simulates general third order linear systems using a single operational amplifier in conjunction with a *minimum* number of resistive and capacitive circuit elements. General analysis of the proposed circuit was presented; workable design equations to calculate the values of the circuit elements were set forth. It was pointed out that simulation was possible only for a restricted range of values of the systems parameters, because of the fact that the values of resistors and capacitors have got to be real positive individually. The circuit offered various alternative combinations of the circuit elements. There were certain restrictions on the values of the systems parameters which were common to all the various alternative combinations and these constituted the basic limitations of the circuit; other restrictions, which were not common to all the alternatives, only meant that if a given set of values of the systems parameters cannot be simulated by one choice of alternative, it can be simulated by another.

It can be shown that by adding more laterally inverted L Sections (comprised by resistor-capacitor combinations) to the circuit proposed for simulating third order linear systems, we can represent higher orders of linear systems with one operational amplifier.

Thus, to the circuit proposed for simulating third order linear systems is added one more laterally inverted L section comprised by resistor-capacitor combinations. The resulting circuit is shown in Fig. I.



$$\frac{E_2}{E_1} = \frac{-b_0\,(b_3 S^3 + b_2 S^2 + b_1 S + 1)}{a_4 S^4 + a_3 S^3 + a_2 S^2 + a_1 S + 1}$$

FIG. 1

That the resulting circuit is capable of simulating fourth order linear systems is illustrated by offering one arbitrary, and comparatively simple, choice of resistor-capacitor combination for each of the three forms of linear fourth order systems. Seven other forms are illustrated elsewhere [8,9].

It will be seen that simulation is possible only when the systems parameters have certain constraints put on them. In this text these constraints appear mostly as equalities but it is so only because, for the sake of simplicity, we have taken too few variables to represent the resistor and the capacitor values. By taking at least as many variables as the systems-constants in the transfer function of the given system, it can be seen that the constraints on these systems-constants will appear mostly as a set of inequalities. Some of these inequalities appearing in a particular form of the fourth order systems will be common to all the alternative choices of resistor-capacitor combinations available for simulating that form, while others are not. The former alone constitute the limitations to the applicability of the circuit as a whole. These constraints, in the form of inequalities, arise because of the fact that all the variables chosen to represent the resistor-capacitor values are required to be real and positive.

It is intended to present, in subsequent contributions, general analyses of the various alternatives available for simulating each of these three forms (and also of many others) individually with the help of the proposed circuit using one operational amplifier and, as far as possible, a *minimum* number of resistive and capacitive elements.

The transfer function of the circuit, (Fig. I), can be shown to be[10]

$$(1)\quad \frac{Y_1 Y_3 Y_5 Y_7}{Y_1 Y_5 (Y_3 Y_8 + Y_4 Y_7 + Y_4 Y_8) + Y_5 (Y_7 + Y_8)(Y_2 Y_3 + Y_3 Y_4 + Y_2 Y_4) + (Y_6 Y_7 + Y_6 Y_8 + Y_7 Y_8) \times (Y_1 Y_3 + Y_1 Y_4 + Y_1 Y_5 + Y_2 Y_3 + Y_2 Y_4 + Y_2 Y_5 + Y_3 Y_4 + Y_3 Y_5)}$$

Further, a general fourth order linear system may be represented by an expression of the type.

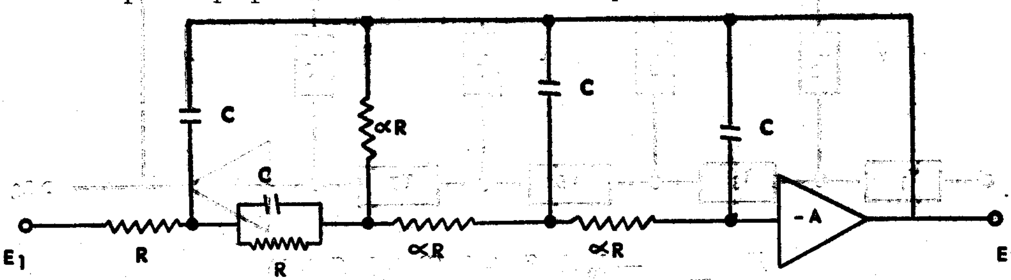
$$(2)\qquad F(s) = \frac{-b_0 (b_3 s^3 + b_2 s^2 + b_1 s + 1)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + 1}$$

where the a's and the b's are real, non-zero, positive constants. In particular cases, some of the b's may be zero.

(*i*) Consider a fourth order linear system of the form

$$(3)\qquad F(s) = \frac{-b_0 (b_1 s + 1)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + 1}$$

where the a's and b's are real, non-zero, positive constants. In order to simulate this system with the help of the proposed circuit, we choose the Y's arbitrarily as follows (Fig. 2).



$$\frac{E_2}{E_1} = \frac{-b_0 (b_1 S + 1)}{a_4 S^4 + a_3 S^3 + a_2 S^2 + a_1 S + 1}$$

FIG. 2

$$
\text{(4)}\qquad
\begin{aligned}
Y_1 &= \frac{1}{R} \\
Y_2 &= Y_6 = Y_8 = SC \\
Y_3 &= SC + \frac{1}{R} \\
Y_4 &= Y_5 = Y_7 = \frac{1}{\alpha R}
\end{aligned}
$$

and we denote $T = RC$

Substituting (4) into (1) and comparing the coefficients of like powers of S in the resulting expression and in (3) we get,

(5) $$bo = \frac{\alpha}{2}$$

(6) $$b_1 = T$$

(7) $$a_4 = \tfrac{1}{2}\, T^4\, \alpha^3$$

(8) $$a_3 = \tfrac{1}{2}\,(2\alpha + 7)\, T^3\, \alpha^2$$

(9) $$a_2 = \tfrac{1}{2}\,(\alpha^2 + 10\alpha + 11)\, T^2\, \alpha$$

10) $$a_1 = \tfrac{1}{2}\,(3\alpha^2 + 11\alpha + 2)\, T$$

(5) and (6) give the values of α and T

(5′) $$\alpha = 2b_0$$

(6′) $$T = b_1$$

Equations (7), (8), (9), (10) show that the following constraints need be put on the a's and the b's in order that simulation of this form of fourth order systems is possible by means of this choice of the Y's;

(11) $$a_4 = 4\, b^3{}_0\, b^4{}_1$$

(12) $$a_3 = 2\, b^2{}_0\, b^3{}_1\, (7 + 4\, b_0)$$

(13) $$a_2 = b_0 b^2{}_1\, (4\, b^2{}_0 + 20\, b_0 + 1)$$

(14) $$a_1 = b_1\, (6b^2{}_0 + 11\, b_0 + 1)$$

(*ii*) Let the form of a fourth order linear system be

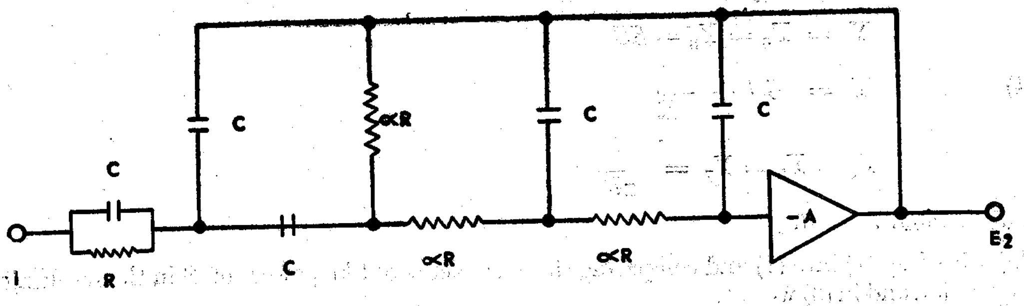
(15) $$F(s) = \frac{-b_1\, s\, (b_2\, s + 1)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + 1}$$

where the a's and b's are all real, positive, non-zero constants.

We hit upon the following arbitrary choice of Y's to simulate such a system by means of the proposed circuit;

$$
\text{(16)}\qquad
\begin{aligned}
Y_1 &= S\,C + \frac{1}{R} \\
Y_2 &= Y_3 = Y_6 = Y_8 = SC \\
Y_4 &= Y_5 = Y_7 = \frac{1}{\alpha R}
\end{aligned}
$$

Fig. 3 shows the resulting circuit configuration.



$$\frac{E_2}{E_1} = \frac{-b_1 S\,(b_2 S + 1)}{a_4 S^4 + a_3 S^3 + a_2 S^2 + a_1 S + 1}$$

FIG. 3

Substituting (16) into (1) and comparing the coefficients of like powers of S in the resulting expression and in (15) we get,

(17) $b_1 = T\alpha$

(18) $b_2 = T$

(19) $a_4 = 2\,T^4\,\alpha^3$

(20) $a_3 = T^3\,\alpha^2\,(3\,\alpha + 10)$

(21) $a_2 = T^2\,\alpha\;(2\,\alpha^2 + 3\alpha + 14)$

(22) $a_1 = T\,(5\alpha + 3)$

From (17) and (18) we get the values of T and α to be

(23) $\alpha = \dfrac{b_1}{b_2}$

(24) $T = b_2$

Equations (19), (20), (21), (22), then, lay down the following restrictions on a's and b's in order that simulation of this form of fourth order linear systems is possible by means of this choice of the Y's in the proposed circuit;

(25) $a_4 = 2\,b^3{}_1\,b_2$

(26) $a_3 = b^2{}_1\,(3\,b_1 + 10\,b_2)$

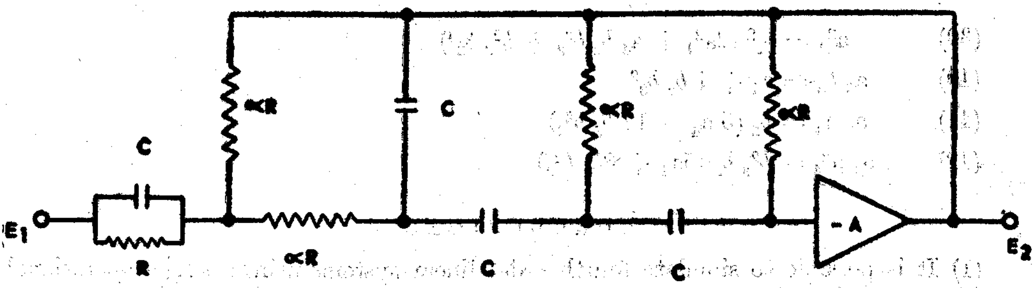
(27) $a_2 = b_1\left(3\,b_1 + 14\,b_2 + \dfrac{2b^2{}_1}{b_2}\right)$

(28) $a_1 = 5\,b_1 + 3\,b_2$

(*iii*) Fourth order linear systems of the form

(29) $$F(s) = \frac{-b_2\,s^2\,(b_3\,s + 1)}{\alpha^4\,s^4 + a_3\,s^3 + a_2\,s^2 + a_1\,s + 1}$$

where the a's and the b's are all real, positive non-zero constants, can be simulated by means of the proposed circuit. This is illustrated by the following arbitrary choice of Y's (Fig.-4)



$$\frac{E_2}{E_1} = \frac{-b_2 S^2 (b_3 S + 1)}{a_4 S^4 + a_3 S^3 + a_2 S^2 + a_1 S + 1}$$

Fig 4

(30) $$Y_1 = SC + \frac{1}{R}$$

$$Y_2 = Y_3 = Y_6 = Y_8 = \frac{1}{\alpha R}$$

$$Y_4 = Y_5 = Y_7 = SC$$

and we denote $T = RC$

Substituting (30) into (1) and comparing the coefficients of like powers of S in the resulting expression and in (19) we get,

(31) $$\frac{1}{b_2} = \frac{1}{\alpha T^2}\left(\frac{1}{\alpha^2} + \frac{1}{\alpha} + 2\right)$$

(32) $$b_3 = T$$

(33) $$\frac{a_4}{b_2} = \alpha T^2$$

(34) $$\frac{a_3}{b_2} = (\alpha + 7) T$$

(35) $$\frac{a_2}{b_2} = 5 + \frac{12}{\alpha}$$

(36) $$\frac{a_1}{b_2} = \frac{1}{T\alpha}\left(5 + \frac{8}{\alpha}\right.$$

(32) and (33) give the values of T and α to be

(37) $$T = b_3$$

(38) $$\alpha = \frac{a_4}{b_2 b_3^2}$$

(31), (34), (36) tell us that simulation of this form of fourth order linear systems is possible by means of this choice of Y's if the following relations between the a's and b's are satisfied;

(39) $a^3_4 = b_2^2 (2a^2_4 + a_4 b_2 b^2_3 + b^2_2 b_3^4)$

(40) $a_3 b_3 = a_4 + 7 b_2 b_3^2$

(41) $a_2 a_4 = b_2 (5 a_4 + 12 b_2 b^2_3)$

(42) $a_1 a^2_4 = b^2_2 b_3 (5a_4 + 8b_2 b^2_3)$

CONCLUSIONS

(1) It is possible to simulate fourth order linear systems using a single operational Amplifier in conjunction with the proposed network.

(2) Simulation is possible for a restricted range of values of the systems constants.

REMARKS

(1) By shorting Y_7 and opening Y_8 we get the circuit reported earlier for simulating third order sytems[5]. By shorting Y_5, Y_7 and opening Y_6, Y_8, we get a circuit, for simulating second order systems, similar to that reported by Bridgeman & Brennen[1].

(2) The relative error in the individual parameters owing to component tolerances is expected to be no more than one order of magnitude higher than the tolerances as a first approximation. The errors resulting from imperfect capacitors is sought to be minimised by keeping the number of capacitors to a minimum.

(3) The utility of this technique, as seen at present, is chiefly to the types of simulation where frequent change of parameters is not involved and an effective use of a limited amplifier capacity of a patch-board is desired, and for special purpose computers.

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REFERENCES

1. GODET, S., Synthesis of Input and Output Networks for Feed back Amplifiers; REAC Information Bull 4 Reeves Instrument Corporation, New York, (April 1950).
2. BRODLEY, F. R. & MC COY, R., *Electronics*, 25 : 144 (April 1952).
3. BRIDGEMAN, A. & BRENAN, R., *1957 WESCON Convention Record*, Part 4, pp. 273-278, San Francisco, (August 1957).
4. HUSKE & KORN, Computer Hand Book (1962), McGraw Hill, Book Co. Inc. N.Y.
5. WADHWA, L. K., : *Proc. I.R.E.*, pp. 1538 (June 1962).
6. AGGARWAL, G. K. & WADHWA, L. K., On Simulation of Third Order Systems with one Operational Amplifier. *J. Inst. Engrs., India*, Jan. 63.
7. AGGARWAL, G. K. A Single Operational Amplifier Simulates General Third Order Linear Systems. (*Proc. Ind. Acad. Scs.*, Sept. 1963).
8. AGGARWAL, G. K. A Single Operational Amplifier Simulates, Fourth Order Linear Systems: Part I, III, IV, V (To be published in distinct parts).
9. AGGARWAL, G. K. J. electronics & Control 1963 (To be published). On Fourth Order Simulation using one Amplifier.
10. AGGARWAL, G. K. Analysing the L's of One Amplifier using a Z=Y Table (To be published).