SOLAR RADIATION IN RELATION TO ALTITUDE AND HUMIDITY

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ABSTRACT

A simplified theory of attenuation of incoming direct total solar radiation as a function of humidity of the atmosphere and local altitude under clear sky condition with minimal dust and smoke content has been presented. For this purpose the atmosphere has been idealised and broadly divided into two components—one responsible for absorption and the other, for scattering and reflection. The equation has been developed with the object of predicting local solar radiation from a knowledge of local humidity and altitude, with the help of simple charts.

INTRODUCTION

Extensive scientific efforts are being directed towards utilisation of solar energy and new developments in this field are being frequently reported. However, for utilisation of solar energy it is necessary to know the amounts of depletion of incoming solar radiation by the atmosphere. It has been reported that for clear sky conditions the fractions of direct solar radiation which is depleted due to various causes are[1]: —

Atmospheric Scattering	9 %
Surface reflection	6 %
Other gases, smoke, dust etc.	3 %

The values are obviously for zenith—sun (unit air mass). Thus overall transmission coefficient of dry atmosphere at sea level is about 82 %. Besides the above depletions, water vapour present in the atmosphere plays a predominant role in the attenuation of solar radiation. Under extreme humid conditions for cloudless sky, the transmission coefficient may be as low as 55 %[2].

Lack of precise information regarding the attenuation of solar radiation by water vapour has been a major handicap in meteorological research. Much work has been done regarding absorption by water vapour of terrestial radiation which is exclusively long-infrared in the range 4 to 80μ. On the other hand, attenuation by water vapour of incoming solar radiation, 99 % of which is below 4μ, remains comparatively unexplored, at least for the purpose of quick estimation of local insolation. Van Vleck-Weisskoff[3] developed an expression for the attenuation of solar radiation by water vapour in relation to frequency. From a knowledge of attenuation at various frequencies, the total depletion by water vapour over the whole solar spectrum can be computed. But the actual procedure is highly complicated and tedious.

In the present paper an attempt has been made to develop a simplified theory of atmospheric attenuation of direct solar radiation with reference to humidity and altitude, for a cloudless sky and minimal smoke and dust content. As such the effect of varying dust and smoke has not been taken into account.

A simplified theory of atmospheric attenuation of solar radiation.

For the purpose of simplification, we introduce the concept of an idealised atmosphere under clear sky conditions as consisting of two broad components:—

(1) Molecules responsible for absorption

(2) Molecules responsible for scattering and reflection.

Then change in the intensity of solar radiation S for zenith—sun, at an altitude h after traversing a thickness dh of the atmosphere may be given by :

$$dS = (k_1 \rho_1 + k_2 \rho_2) S dh$$

Since h is measured upward, positive sign has been taken because S and h increase in the same direction. Here k_1 is the integrated attenuation constant for the whole solar spectrum and ρ_1 , the effective density, of the absorbing molecules. Similarly k_2 and ρ_2 are the corresponding quantities for the scattering and reflecting molecules. Density being practically exponentially related to altitude, we can write :

$$\rho_1 = \rho_{10} e^{-\alpha_1 h}$$

$$\rho_2 = \rho_{20} e^{-\alpha_2 h} ,$$

where ρ_{10} and ρ_{20} are the effective densities of the respective components at sea level and α_1 and α_2 are the respective constants in the exponents. On integration we have :

$$ln\ S/S_c = - (Ae^{-\alpha_1 h} + Be^{-\alpha_2 h}) \quad \ldots \quad (1)$$

where S_c = Solar Constant i.e. Intensity of incoming solar radiation outside the atmosphere, or mathematically

$$S \longrightarrow S_c \quad as \quad h \longrightarrow \infty$$

A and B are constants given by :

$$A = \frac{k_1 \rho_{10}}{\alpha_1}$$

$$B = \frac{k_2 \rho_{20}}{\alpha_2}$$

and at sea level, we have, $ln\ S_o/S_c = - (A + B)$

The absorbing component consists of water vapour, carbon dioxide, minor gases, smoke and dust etc. For minimal dust and smoke content of the atmosphere, particularly far from industrial cities, and in the absence of dust storms etc, water vapour is practically the only variable absorbing component. Therefore the constant A should vary linearly with density of water vapour, and can thus be written as:—

$$A = a + b' \rho_{w_o} ,$$

where 'a' is the integrated attenuation constant for all absorbing components except water vapour, and b' is the effective absorption constant per unit density of water vapour at sea level, ρ_{w_o} being the density at sea level.

$\rho_{w_\circ}$, being equal to $\frac{p_{w_\circ}}{R_w T}$, depends both on pressure and temperature at sea level. Ignoring seasonal variations in temperature, if we take $\overline{T}$, the mean annual temperature at sea level on the absolute scale, we can regard $\rho_{w_\circ}$ practically proportional to $p_{w_\circ}$, the water vapour pressure at sea level. In which case, the constant A can be put in the form : $A = a + b\, p_{w_\circ}$,

where b is the effective absorption constant per unit vapour pressure corresponding to a fixed value of $\overline{T}$, the mean annual temperature at sea level.

Thus finally, equation (1) becomes

$$ln t = -\left[(a + b\, p_{w_\circ})\, e^{-a_1 h} + Be^{a_2 h} \right] \quad .. \quad .. \quad .. \quad (2)$$

where t, which is the ratio S/S_c , is the over-all transmission coefficient of the atmosphere at altitude h Km. Its value at sea level is given by

$$ln\ t_o = -[(a + bp_{w_\circ}) + B].$$

Evaluation of Constants

For determination of constants of the above equations, we shall assume that the absorbing molecules are confined to troposphere only. Ozone present in the stratosphere absorbs only in the short utltraviolet region. However, depletion of total solar radiation due to ozone alone being less than 1 %, the above assumption will not introduce any significant error in the evaluation of constants. The available values [5] of solar radiation reduced to optical path for zenith-sun at various altitudes are given in Table I.

TABLE I

INTENSITY OF SOLAR RADIATION AT NORMAL INCIDENCE FOR ZENITH—SUN AT VARIOUS ALTITUDES

h Km.	S Cal/cm²/min.	t × 100 percent
0·126	1·58	79·0
1·737	1·64	82·0
4·42	1·72	86·0
5·80	1·75	87·5
22·00	1·84	92·0

Transmission coefficient 't' has been calculated with[6,7] $Sc = 2\cdot00$ Cals/cm²/min.

Taking the mean height of the tropopause as 13 Km., the average transmission coefficient above the tropopause may be taken as 90·7%. This value is obtained from the graph drawn by plotting the above values of transmission coefficient against logarithm of altitude and its accuracy is within the permissible range of 1%.

From our assumption that beyond the tropopause, the absorption term in equation (1) may be neglected, we have :

$$\ln t = -Be^{-\alpha_2 h} \quad \text{(beyond tropopause)}$$

Using the values of transmission coefficient for h=13 and 22 Km. in the above equation, B and α_2 are evaluated. Similarly, A and a_1 are evaluated from equation (1) using transmission values at 0·126 and 4·42 Km. together with the calculated values of B and α_2.

The values obtained are :

$$A = 0{\cdot}1170$$
$$a_1 = 0{\cdot}2578$$
$$B = 0{\cdot}1226$$
$$\alpha_2 = 0{\cdot}0175.$$

Theoretical curve with the above constants substituted in equation (1) is compared with observed values in Fig. 1.

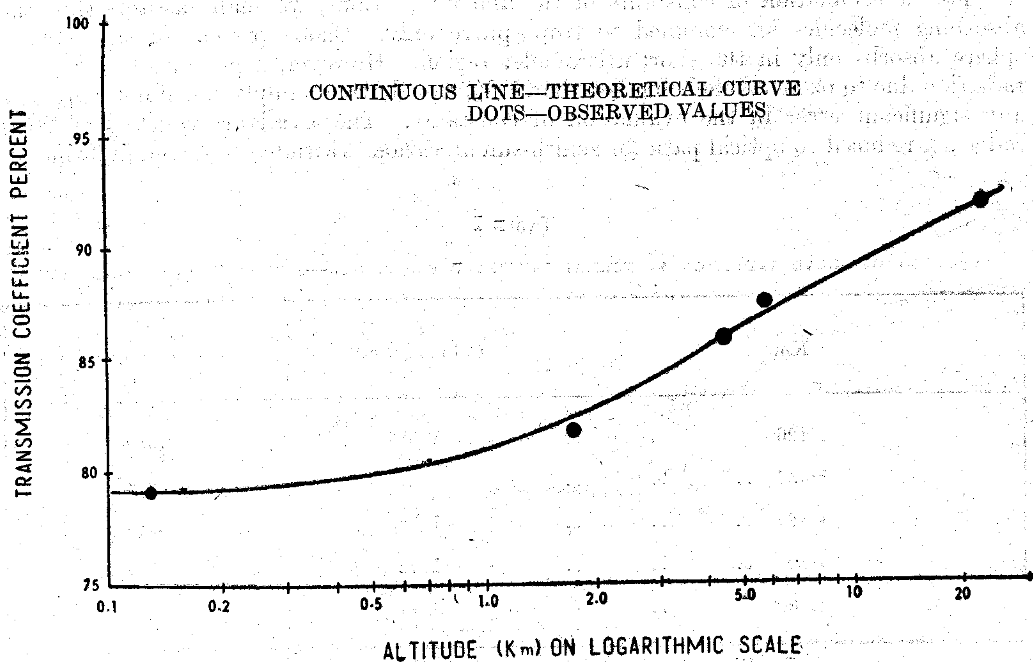

Fig I. Comparison between theoretical curve and observed values of transmission co-efficient of the atmosphere at different altitudes shown on a logarithmic scale.

The reported limits of transmission coefficient are 1, 2.

$$t_o = 0{\cdot}82 \quad at \quad p_{w_o} = 0 \quad mm.\ Hg.$$
$$t_o = 0{\cdot}53 \quad at \quad p_{w_o} = 40 \quad mm.\ Hg.$$

The upper limit of 't' is for complete absence of water vapour, while for the lower limit, the most probable limit of hot humid conditions is assumed. Substituting the above values of t_o in equation (2) for h=o, we get :

$$a = 0{\cdot}07010$$

$$b = 0{\cdot}01087.$$

After substituting the values of the constants in equation (2) the theoretical curves are obtained by plotting transmission coefficients against logarithm of altitude for various values of p_{w_o} , as shown in fig. 2. Each curve represents a constant water vapour-pressure line.

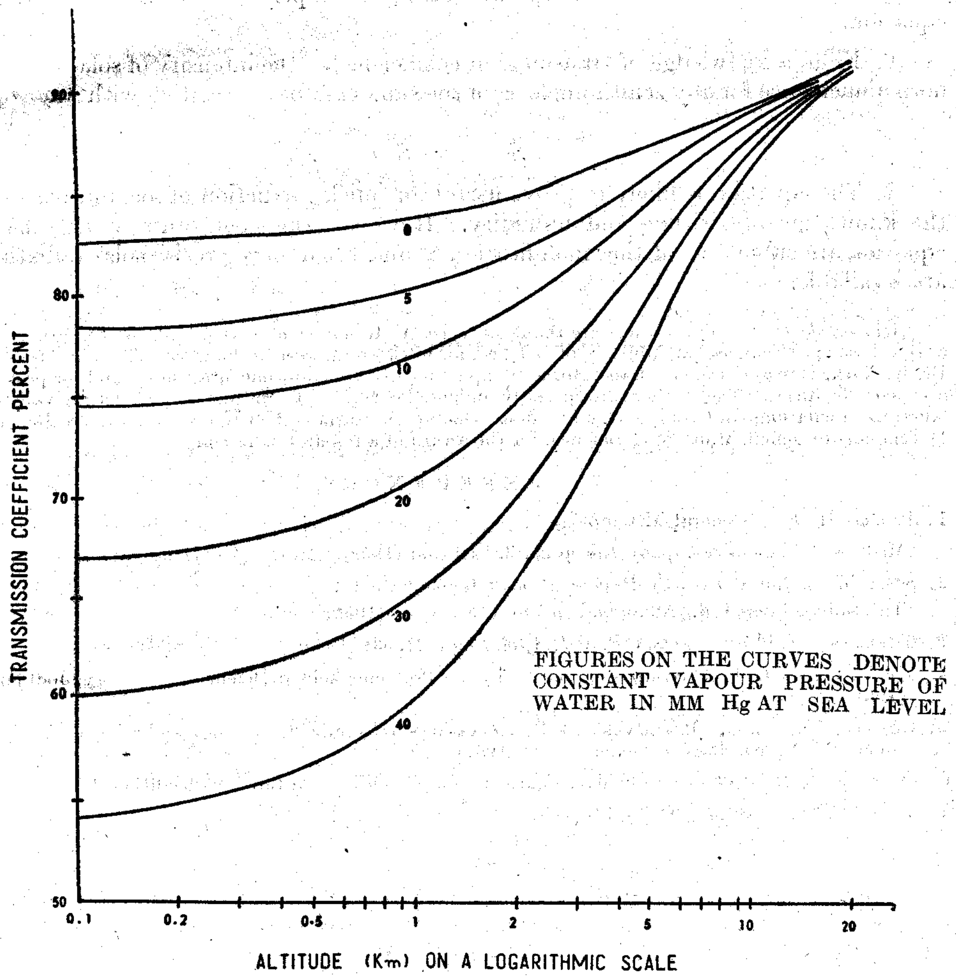

Fig 2. Theoretical transmission coefficient of the atmosphere in relation to altitude and water vapour pressure at sea level.

DISCUSSION

1. The accuracy of constants estimated is seriously limited by the lack of precise available data.

2. The assumption that the absorption term can be neglected above the tropopause is reasonably borne out by fig. 1.

3. The theoretical curve in fig. 1 shows that about 93 % of the total attenuation of solar radiation by atmosphere is confined to the atmosphere.

4. From figs. 1 and 2 it is evident that transmission coefficient is greater than 80 % above 6 Km. of altitude for all values of humidity. Thus for all practical purposes the effect of humidity is mostly confined to the lower layers of troposphere.

5. Since a constant value for the mean temperature at sea level has been assumed the effect of seasonal variations of temperature has been ignored. This effect, though small, can be taken into account by introducing a temperature factor in the theoretical equation.

6. From a knowledge of transmission coefficient 't', the intensity of solar radiation at normal incidence for any zenith-angle 'z' of the sun, can be evaluated with the equation

$$S_z = S_c \, t^{\sec z} .$$

7. The equation is likely to prove useful for quick prediction of local insolation from the knowledge of altitude and humidity. However, the constants of the theoretical equation are subject to further modification as and when more precise solar radiation data are available.

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