ON THE FORM FUNCTION AND PROPERTIES OF BITUBULAR POWDERS

L. S. KOTHARI

Department of Mathematics Rajasthan College, Jaipur

ABSTRACT

IN PART A of the present paper the form function for a bitubular charge i.e., a twin perforated cylindrical grain has been derived for both the phases of combustion and its general properties have been discussed. In part B the charge has been modified by cutting out in the very beginning, the slivers remaining at the end of first phase of combustion and the surface inhibited from burning, thus eliminating the complicated second stage of burning. The form function for such a modified bitubular charge also has been obtained and its properties have been discussed.

INTRODUCTION

It is known that multitubular powders have become popular and almost standard in United States and many other countries. Tavernier[1,2] and Gupta[3] have discussed the theory for heptatubular powders, while Kapur and Jain[4] have discussed the form function for modified heptatubular and tritubular charges. Corner[5] has mentioned about a long twin perforated cylindrical grain which had been used for experimental purposes in England but the form function and its properties have not been discussed so far.

In part A of this paper the form function for a bitubular charge for both the phases of combustion has been derived and its general properties discussed. In part B the form function for a modified bitubular charge (i.e., a bitubular charge in which the complicated second stage of burning has been eliminated by cutting out in the very beginning the slivers remaining at the end of first phase of combustion and inhibiting the surface of the slivers from burning) has been obtained and its properties discussed. Values of different parameters occurring in our treatment have also been tabulated for different sets of values of m and ρ in case of the modified charge. As is clear the internal ballistics of a bitubular charge will remain the same as that of a heptatubular charge discussed by Tavernier[2] and Gupta[3] except changes in the values of the constants, the same has therefore not been discussed.

NOTATIONS

D = The exterior diameter of the charge grain.

d = The diameter of the holes of the grain.

L = The length of the grain.

e = The distance between the two holes or between any hole and the curved surface of the grain i.e., the web size of the grain.

m = The ratio of the exterior diameter of the grain to the diameter of any hole $= \dfrac{D}{d}$

ρ = The ratio of the length of the grain to the exterior diameter of the grain $= \dfrac{L}{D}$

S_o = Initial surface of the grain.

S = Surface of the grain at any instant t.

Z = Fraction of the charge burnt at any instant t.

f = Fraction of the initial thickness (web size e) remaining at any instant t, for the first phase of combustion; while for the second phase of combustion defined as the ratio of the distance receded (from the beginning of the second phase upto the instant considered) to the initial thickness e.

δ = Propellant density.

V_o = Initial volume of the grain.

V = Volume of the grain at any instant t.

Part A

We shall consider the two stages of combustion:

(*i*) before the rupture of the grain.

(*ii*) after the rupture of the grain.

Form function for the first phase of combustion

The cross section of a bitubular grain will appear as shown in figs. 1 and 2.

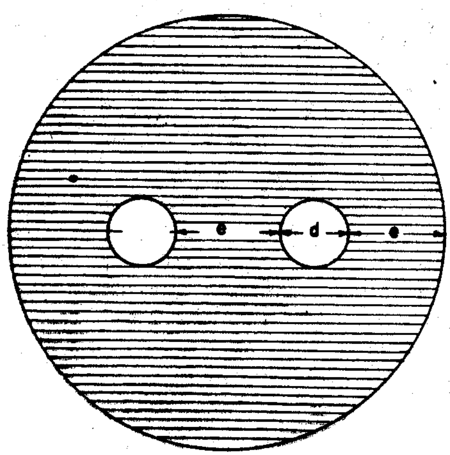

Fig. 1—Bitubular charge

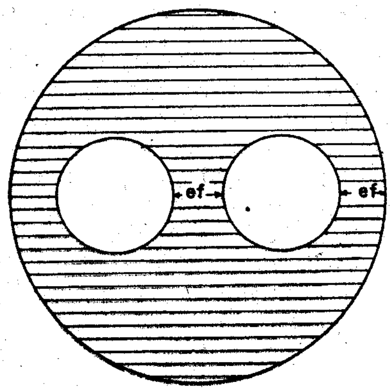

Fig. 2—Bitubular charge when a fraction f of e remains.

We have as defined earlier,

$$D = md \qquad \ldots \quad (1)$$

$$L = mpd \qquad \ldots \quad (2)$$

and from fig. 1

$$D = 2d + 3e = md \qquad \ldots \quad (3)$$

$$\therefore \quad e = \frac{m-2}{3} \, d \qquad \ldots \quad (4)$$

The initial volume $V_\circ$ of the grain is given as

$$V_\circ = \left\{ \pi \left(\frac{D}{2}\right)^2 - 2\pi \left(\frac{d}{2}\right)^2 \right\} \times L$$

$$= \frac{\pi \rho m d^3}{4} \left\{ m^2 - 2 \right\} \quad \dots \quad (5)$$

The volume V of the grain at any instant t is given as

$$V = \left[\pi \left\{ \frac{D}{2} - \frac{e(1-f)}{2} \right\}^2 - 2\pi \left\{ \frac{d}{2} + \frac{e(1-f)}{2} \right\}^2 \right] \times \left[L - e(1-f) \right] \quad \dots \quad (6)$$

$$= \frac{\pi d^3}{36} \left[9m^2 - (m-2)^2 (1-f)^2 - 18 - 6(m^2-4)(1-f) \right] \times \left[\frac{3m\rho - (m-2)(1-f)}{3} \right] \quad \dots \quad (7)$$

The fraction of the grain burnt at an instant t is

$$Z = \frac{V_\circ \delta - V\delta}{V_\circ \delta} = 1 - \frac{V}{V_\circ} \quad \dots \quad (8)$$

Using (5) and (7) in (8), we have on simplification

$$Z = (1-f)\Bigg[\left\{ \frac{2(m^2-4)}{3(m^2-2)} + \frac{(m-2)}{3m\rho} + \frac{(m-2)^2}{9(m^2-2)} - \frac{2(m^2-4)(m-2)}{9m\rho(m^2-2)} - \frac{(m-2)^3}{27m\rho(m^2-2)} \right\}$$
$$- \left\{ \frac{(m-2)^2}{9(m^2-2)} - \frac{2(m^2-4)(m-2)}{9m\rho(m^2-2)} - \frac{2(m-2)^3}{27m\rho(m^2-2)} \right\} f$$
$$- \left\{ \frac{(m-2)^3}{27m\rho(m^2-2)} \right\} f^2 \Bigg] \quad \dots \quad (9)$$

or

$$Z = (1-f)(a - bf - cf^2) \quad \dots \quad (10)$$

where

$$a = \frac{(m-2)}{27m\rho(m^2-2)} \left[21m^2\rho + 30m\rho + 2(m+1)^2 \right] \quad \dots \quad (11)$$

$$b = \frac{(m-2)^2}{27m\rho(m^2-2)} \left[3m\rho - 8m - 8 \right] \quad \dots \quad (12)$$

and

$$c = \frac{(m-2)^3}{27m\rho(m^2-2)} \quad \dots \quad (13)$$

(10) is the relation between Z and f for the first period of combustion. If we put $f = 0$, in (10), we get $Z=a$, which gives the fraction of the grain burnt at the end of the first phase of combustion, that is at the rupture of the grain.

From (4), we see that $m > 2$,

and from (2) and (4), we have

$$L = \frac{3\,e}{m-2}\, m\,\rho \qquad \text{(14)}$$

from which we see that if

$$\rho \leqslant \frac{m-2}{3\,m},$$

L is less than or equal to e, which means that total combustion of the grain precedes the rupture or coincides at the point of rupture of the grain. Therefore as explained by Tavernier[1] we should have

$$\rho > \frac{m-2}{3\,m} \qquad \text{(15)}$$

which means that there is definitely a rupture at $f = 0$.

Writing (11) as

$$a = a_{\circ}(m) + \frac{a_1(m)}{\rho} \qquad \text{(16)}$$

where

$$a_{\circ} = \frac{(m-2)(21\,m+30)}{27\,(m^2-2)} \qquad \text{(17)}$$

and

$$a_1 = \frac{2\,(m-2)\,(m+1)^2}{27\,m\,(m^2-2)} \qquad \text{(18)}$$

Differentiating (17) and (18) we get

$$\frac{da_{\circ}}{dm} = \frac{4\,(m+1)\,(m+2)}{9\,(m^2-2)^2} \qquad \text{(19)}$$

and

$$\frac{da_1}{dm} = \frac{4\,(m+1)\,(m^2+2\,m-2)}{27\,m^2\,(m^2-2)^2} \qquad \text{(20)}$$

From (19) and (20) it can be seen that $\frac{da_{\circ}}{dm}$ and $\frac{da_1}{dm}$ are always positive for possible values of m,

Therefore a is an increasing function of m and a decreasing functin of ρ.

From (15) we have

$$\rho_{min} = \frac{m-2}{3m} \quad \ldots \quad (21)$$

Therefore for a given value of m

$$a_{max} = \frac{(m-2)}{27(m^2-2)}\left[21m + 30 + \frac{6(m+1)^2}{m-2}\right] = 1, \quad \ldots \quad (22)$$

a value independent of m.

Also for a given value of m, a is minimum when ρ is maximum i.e., $\rho = +\infty$

Therefore

$$a_{min} = \frac{(m-2)(21m+30)}{27(m^2-2)} = a_{\circ}(m) \quad \ldots \quad (23)$$

When $m = 2$, $a_{\circ}(m) = 0$, and when $m = \infty$, $a_{\circ}(m) = \frac{21}{27} = \cdot 777$

$\therefore$ $a_{\circ}(m)$ varies from 0 to $\cdot 777$, when m varies from 2 to ∞.

We know that S is proportional to $\frac{dz}{df}$ and therefore

$$\frac{S}{S_{\circ}} = \frac{\frac{dz}{df}}{\left(\frac{dz}{df}\right)_{f=1}} \quad \ldots \quad (24)$$

Differentiating (10) we have

$$\frac{dz}{df} = -(a+b) + 2(b-c)f + 3cf^2 \quad \ldots \quad (25)$$

and

$$\left(\frac{dz}{df}\right)_{f=1} = -a + b + c \quad \ldots \quad (26)$$

$$\therefore \quad \frac{S}{S_{\circ}} = \frac{a+b}{a-b-c} - \frac{2(b-c)}{a-b-c}f - \frac{3c}{a-b-c}f^2 \quad \ldots \quad (27)$$

or

$$\frac{S}{S_{\circ}} = \alpha - \beta f - \gamma f^2 \quad \ldots \quad (28)$$

Where

$$\alpha = \frac{a+b}{a-b-c} = \frac{2(m+1)\ \ 4m\rho - m + 3\}}{3\ 2m^2\rho + 4m\rho + m^2 - 2\}} \quad \ldots \quad (29)$$

$$\beta = \frac{2(b-c)}{a-b-c} = \frac{2(m-2)\ \ m\rho - 3m - 2\}}{3\ \ 2m^2\rho + 4m\rho + m^2 - 2\}} \quad \ldots \quad (30)$$

and

$$\gamma = \frac{3\,c}{a - b - c} = \frac{(m-2)^2}{3\ \{2\,m^2\rho + 4\,m\,\rho + m^2 - 2\}} \quad \ldots \quad (31)$$

(28) gives the relation between $\frac{S}{S_o}$ and f for the first phase of combustion, and if we put $f=0$, in (28) we get $\frac{S}{S_o} = \alpha$, which is the ratio of the surface at rupture to the initial surface.

From (28) we have

$$\frac{d}{df}\left(\frac{S}{S_o}\right) = -\beta - 2\,\gamma f \quad \ldots \quad (32)$$

and

$$\frac{d^2}{df^2}\left(\frac{S}{S_o}\right) = -2\,\gamma \quad \ldots \quad (33)$$

But from (31) we see that γ is always positive, therefore $\frac{d^2}{df^2}\left(\frac{S}{S_o}\right)$ is always negative and $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is a decreasing function of f.

Now when $f=1$, i.e., at the beginning of the combustion of the grain we have from (32)

$$\frac{d}{df}\left(\frac{S}{S_o}\right) = -\beta - 2\,\gamma = \frac{2\,(m-2)\ \{2\,m + 4 - m\,\rho\}}{3\ \{2\,m^2\rho + 4\,m\,\rho + m^2 - 2\}} \quad \ldots \quad (34)$$

and when $f = 0$, i.e., at the rupture of the grain

$$\frac{d}{df}\left(\frac{S}{S_o}\right) = -\beta = \frac{2\,(m-2)\ \{3\,m + 2 - m\,\rho\}}{3\ \{2\,m^2\rho + 4\,m\,\rho + m^2 - 2\}} \quad \ldots \quad (35)$$

Therefore for $\frac{S}{S_o}$ to have a maximum value at any point between the beginning of combustion and the rupture of the grain, we must have

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=1}\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=o} < 0 \quad \ldots \quad (36)$$

i.e., $\{2\,m + 4 - m\rho\}\ \{3\,m + 2 - m\rho\} < 0$ .. .. .. (37)

The first member of this inequality becomes zero for

$$\rho_1 = \frac{2\,m + 4}{m} \quad \ldots \quad (38)$$

and the second member for

$$\rho_2 = \frac{3\,m + 2}{m} \quad \ldots \quad (39)$$

Therefore for the existence of a maximum value of $\frac{S}{S_o}$, ρ should lie between ρ_1 and ρ_2.

From (38) and (39) it can be easily seen that ρ_1 and ρ_2 are both decreasing functions of m.

Also we have

$$\rho_2 - \rho_1 = \frac{m-2}{m} \quad \ldots \quad (40)$$

$\therefore$ In every case $\rho_2 \geqslant \rho_1$, since $m \geqslant 2$.

For $\frac{S}{S_o}$ to have a maximum value, $\frac{d}{df}\left(\frac{S}{S_o}\right) = 0$, so that from (32) we have $-\beta - 2\gamma f = o$

or

$$f = -\frac{\beta}{2\gamma} = \frac{3m + 2 - m\rho}{m-2} \quad \ldots \quad (41)$$

(41) can also be written as

$$f = 1 - \frac{\rho - \rho_1}{3\rho_{min}} \quad \ldots \quad (42)$$

We have from (34), (35), (38) and (39)

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=1} = \frac{2m(m-2)(\rho_1 - \rho)}{3\{2m^2\rho + 4m\rho + m^2 - 2\}} \quad \ldots \quad (43)$$

and

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=o} = \frac{2m(m-2)(\rho_2 - \rho)}{3\{2m^2\rho + 4m\rho + m^2 - 2\}} \quad \ldots \quad (44)$$

Hence

(*i*) For $\frac{m-2}{3m} \leqslant \rho \leqslant \rho_1$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is always positive, accordingly the powder is degressive throughout, as in the cases when $m = 4$, $\rho = \frac{1}{2}$; or $m = 4$, $\rho = 1$.

(*ii*) For $\rho_1 < \rho < \rho_2$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is negative in the beginning and then positive, accordingly the powder is first progressive and then degressive, as in the cases when $m = 4$, $\rho = \frac{13}{4}$; or $m = 7$, $\rho = 3$.

(*iii*) For $\rho \geqslant \rho_2$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is always negative, accordingly the powder is always progressive, as in the cases when $m = 4$, $\rho = \frac{7}{2}$; or $m = 7$, $\rho = \frac{7}{2}$; or $m = 10$, $\rho = \frac{7}{2}$.

The results obtained above are given in Table 1:—

TABLE 1

	Case (i)	Case (ii)	Case (iii)
ρ	$\rho_{min} \leqslant \rho \leqslant \rho_1$	$\rho_1 < \rho < \rho_2$	$\rho \geqslant \rho_2$
$\frac{S}{S_0}$	Decreasing function of Z. The powder is degressive.	Increasing function of Z in the beginning, then decreasing function of Z. The powder is first progressive and then degressive.	Increasing function of Z. The powder is progressive.

FORM FUNCTION FOR THE SECOND PHASE OF COMBUSTION

In the beginning of the present period of combustion the powder grain consists of two prisms of base identical to the curvilinear triangle $C\,D\,E$, as shown in fig 3, and of common length

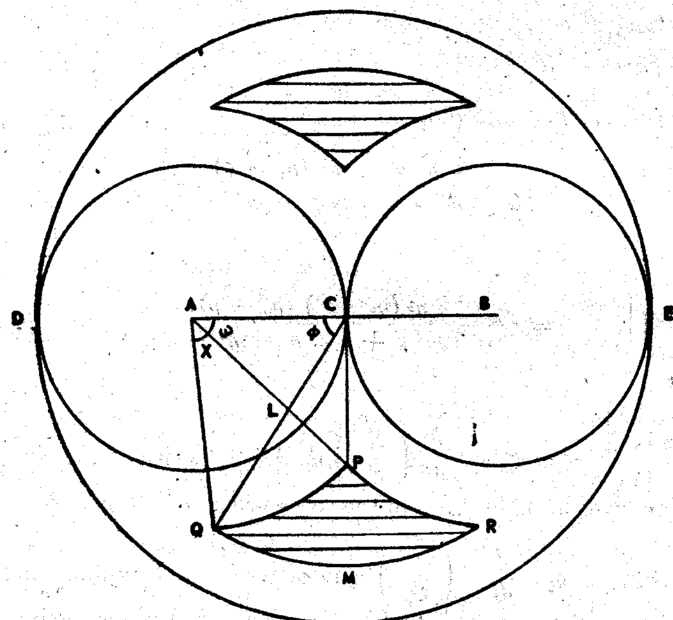

Fig 3—Bitubular charge during the second phase of combustion

$$L - e = \left\{\frac{3\,m\rho - m + 2}{3}\right\}d \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

The radius of the arcs like $D\,E$ is

$$C\,D = \frac{D - e}{2} = \frac{(m+1)}{3}\,d \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (46)$$

and that of arcs like $C\,D$ or $C\,E$ is

$$A\,C = \frac{d+e}{2} = \frac{(m+1)}{6}\,d \quad \ldots \quad (47)$$

As burning proceeds the triangle $C\,D\,E$ shrinks into the triangle $P\,Q\,R$.

Let the radius of the arcs like $P\,Q$ and $P\,R$ be equal to r, then that of $Q\,R$ will be

$$\frac{(m+1)}{3}\,d - \left(r - \frac{e+d}{2}\right) = \frac{(m+1)}{2}\,d - r \quad \ldots \quad (48)$$

Let us now denote the angles CAP, ACQ and PAQ by w, ϕ and χ respectively.

We have from fig. 3

$$\cos\,\omega = \frac{AC}{AP} = \frac{(m+1)}{6r}\,a \quad \ldots \quad (49)$$

$$\cos\,\varphi = \frac{CQ^2 + AC^2 - AQ^2}{2AC.CQ} = \frac{5(m+1)d - 18r}{3(m+1)d - 6r} = \frac{5\cos\,\omega - 3}{3\cos\,w - 1} \quad \ldots \quad (50)$$

and

$$\cos\,(\omega + \chi) = \frac{AQ^2 + AC^2 - CQ^2}{2AQ.AC} = 3 - 4\,\cos w \quad \ldots \quad (51)$$

The complete combustion of the grain corresponds to $\phi = 90°$,

i. e., $\frac{5\,\cos\omega - 3}{3\,\cos\omega - 1} = O$, which gives $\omega = 53°8'$.

During the second period of combustion w varies from $0°$ to $53°\,8'$ and accordingly χ varies from $180°$ to $0°$.

When the section of the prisms become like $P\,Q\,R$ their common length is given as

$$\lambda = L - 2\left(r - \frac{e+d}{2}\right) - e = \left\{\rho m + 1 - \frac{m+1}{3\cos\,\omega}\right\} d \quad \ldots \quad (52)$$

The complete combustion of the grain occurs when the triangle $P\,Q\,R$ vanishes, with the condition that λ, in the range of variation of w ($0°$ to $53°\,8'$) is a decreasing function of w.

For this we have

$$\rho m + 1 \geqslant \frac{m+1}{3\cos\,53°8'} = \frac{5}{9}\,(m+1) \quad \ldots \quad (53)$$

or

$$\rho \geqslant \frac{5}{9} - \frac{4}{9m}$$

Let us write

$$\rho_3 = \frac{5}{9} - \frac{4}{9m} \quad \ldots \quad (54)$$

so that $\rho \geqslant \rho_3$.

Also ρ_3 is always greater than ρ_{min} and less than ρ_2.

Let us now consider the triangle PQR:

Area of $\triangle PQR$ = 2 Area PQM

$=2$ [sector $CQM - \triangle CLP$ — sector $APQ + \triangle ALQ$]

$$= \frac{(m+1)^2 d^2}{36} \left[\left\{ \left(\frac{\pi}{2} - \Phi \right) \left(3 - \frac{1}{\cos \omega} \right)^2 \right\} - \left\{ \frac{\sin \omega \tan \omega \sin \left(\frac{\pi}{2} - \phi \right)}{\sin(\pi - \omega - \phi)} \right\} \right.$$

$$\left. - \left\{ \frac{\chi}{\cos^2 \omega} \right\} + \left\{ \frac{\sin\phi \sin \chi}{\sin(\pi - \omega - \phi) \cos\omega} \right\} \right]$$

$$= \frac{(m+1)^2 d^2}{36} \, G(\omega) \quad \ldots \quad (55)$$

where

$$G(\omega) = \left[\left(\frac{\pi}{2} - \phi \right) \left(3 - \frac{1}{\cos\omega} \right)^2 - \frac{\sin \omega \tan \omega \sin \left(\frac{\pi}{2} - \phi \right)}{\cos(\pi - \omega - \varphi)} - \frac{\chi}{\cos^2\omega} \right.$$

$$\left. + \frac{\sin \phi \sin \chi}{\sin (\pi - \omega - \phi) \cos w} \right] \quad \ldots \quad (56)$$

Therefore the volume of the two prisms of base identical to PQR, i.e., the volume at any instant during the second period of combustion is given as

$$2\, V(PQR) = \frac{2(m+1)^2 d^2}{36} G(w). \lambda$$

$$= \left\{ \frac{(m+1)^2 d^2}{18} G(\omega) \right\} \left\{ \rho\, m + 1 - \frac{m+1}{3 \cos\omega} \right\} \quad \ldots \quad (57)$$

The fraction of the charge burnt at any instant during the second period of combustion is given as

$$Z = \frac{V_\circ \delta - 2V(PQR)\, \delta}{V_\circ \delta} \quad \ldots \quad (58)$$

or

$$Z = 1 - \frac{2(m+1)^2}{9\pi(m^2-2)} \left\{ 1 + \frac{1}{\rho m} - \frac{m+1}{3\rho m \cos\omega} \right\} G(\omega) \quad \ldots \quad (59)$$

When $\omega = 0°$, $\phi = 0°$, $\chi = 180°$ ond $G(\omega) = \pi$, accordingly

when $\omega = 0°$, Z is given as

$$Z = 1 - \frac{2(m+1)^2}{9(m^2-2)} \left\{ 1 + \frac{1}{\rho m} - \frac{m+1}{3\rho m} \right\}$$

$$= \frac{(m-2)\{21m^2\rho + 30m\rho + 2(m+1)^2\}}{27m\rho(m^2-2)} \quad .. \quad .. \quad (60)$$

which is same as the value of a at the end of first phase of combustion.

Now when $\omega=53° \; 8'$, $\phi= 90°$, $\chi=0°$, and $G(\omega)=0$, accordingly $Z=1$, the value which it should attain.

Let us now define f again as the ratio of the distance receded (from the beginning of the second phase of combustion to the instant considered) to the initial thickness e, we have

$$f = \frac{2\left(\frac{e+d-r}{2}\right)}{e} = \frac{2}{e}\left[\frac{m+1}{6}d - \frac{m+1}{6\cos\omega}d\right]$$

$$= \frac{(m+1)}{(m-2)}\left[1 - \frac{1}{\cos\omega}\right] \quad .. \quad .. \quad .. \quad .. \quad (61)$$

For $\omega = 0°, f = 0$ and for $\omega = 53° \; 8'$

$$f = -\frac{2}{5}\left(\frac{m+1}{m-2}\right)$$

which is the minimum value of f and a function of m only.

With (61), (60) can be written as

$$Z = 1 - \frac{2(m+1)^2}{27\pi m\rho(m^2-2)}\left[3m\rho - (m-2)(1-f)\right]G(\omega) \quad (62)$$

Now for the second period of combustion we can find a relation between w and $\frac{S}{S_o}$, we have

$$\frac{dz}{df} = \frac{\frac{dz}{d\omega}}{\frac{df}{d\omega}}$$

$$= \frac{2(m+1)(m-2)}{27\pi m\rho(m^2-2)}\left[\left\{3m\rho + 3 - \frac{(m+1)}{\cos\omega}\right\}G'(\omega)\frac{\cos^2\omega}{\sin\omega} - (m+1)G(\omega)\right] \quad .. \quad .. \quad .. \quad .. \quad (63)$$

and accordingly

$$\frac{S}{S_o} = \frac{\frac{dz}{df}}{\left(\frac{dz}{df}\right)_{f=1}}$$

$$= -\frac{2(m+1)\left[\left\{3m\rho + 3 - \frac{m+1}{\cos\omega}\right\}G'(\omega)\frac{\cos^2\omega}{\sin\omega} - (m+1)G(\omega)\right]}{9\pi\{2m^2\rho + 4m\rho + m^2 - 2\}} \quad .. \quad (64)$$

For simplification let us write

$$G'(\omega)\frac{\cos^2\omega}{\sin\omega} = K(\omega) \quad .. \quad .. \quad .. \quad (65)$$

and

$$G'(\omega) \cot \omega + G(\omega) = I(\omega) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (66)$$

then

$$\frac{S}{S_o} = -\frac{2(m+1)\{3(m\rho+1)k(\omega)+(m+1)I(\omega)\}}{9\pi\{2m^2\rho+4m\rho+m^2-2\}} \quad \ldots \quad \ldots (67)$$

For $\omega = 0°$, $K(\omega) = -4\pi$ and $I(\omega) = -3\pi$, and accordingly

$$\frac{S}{S_o} = \frac{2(m+1)\{4m\rho-m+3\}}{3\{2m^2\rho+4m\rho+m^2-2\}} = a,$$

the value which $\frac{S}{S_o}$, must have at the end of first phase of combustion.

For $\omega = 53° 8'$, $G(\omega)=O$, $G'(\omega)=O$, and therefore $\frac{S}{S_o} = O$.

Part B

FORM FUNCTION FOR THE MODIFIED BITUBULAR POWDER

Let us now suppose that we have a modified bitubular charge i.e., a bitubular charge in which the slivers remaining at the end of the first phase of combustion are cut out in the very beginning and their surfaces are inhibited from burning. The cross section of such a modified bitubular grain will appear as shown in figs. 4 and 5.

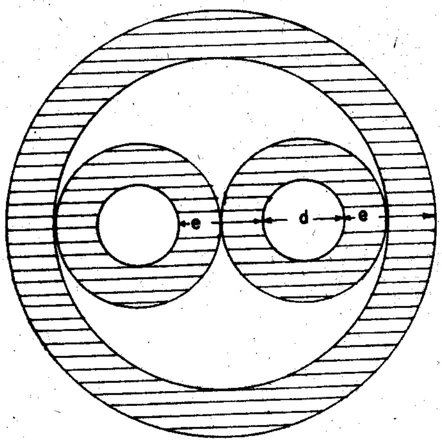

Fig. 4—Modified bitubular charge

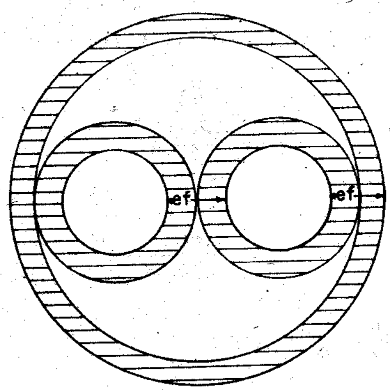

Fig 5—Modified bitubular charge when a fraction f of e. remains

With the same notations, the exterior diameter of the grain when the inner and outer circles touch each other as shown in fig. 3 is

$$D - e = \tfrac{2}{3}(m+1)\,d \quad \ldots \quad \ldots \quad \ldots (68)$$

The area of the base of the two curvilinear triangular prisms i.e., of the slivers is

$$\pi\left(\frac{D-e}{2}\right)^2 - 2\pi\left(\frac{d+e}{2}\right)^2 = \frac{\pi(m+1)^2}{18}\,d^2 \quad \ldots \quad \ldots (69)$$

If these slivers are cut out in the very beginning of combustion and the surface inhibited from burning as shown in fig. 4, then the initial volume Vo of the grain is given as

$$V_{o} = \left[\pi \left(\frac{D}{2}\right)^2 - \pi \frac{(m+1)^2}{18} d^2 - 2\pi\left(\frac{d}{2}\right)^2 \right] \times L$$

$$= \frac{\pi \rho m d^{3\prime}}{36} \left[7m^2 - 4m - 20 \right] \quad \ldots \quad \ldots \quad \ldots (70)$$

The volume V of the grain at any instant t is then given as

$$V = \left[\pi \left\{ \frac{D}{2} - \frac{e(1-f)}{2} \right\}^2 - 2\pi \left\{ \frac{d}{2} + \frac{e(1-f)}{2} \right\}^2 - \pi \frac{(m+1)^2}{18} d^2 \right] \times \left[\rho m d - e(1-f) \right]$$

$$= \frac{\pi d^3}{36} \left[(7m^2 - 4m - 20) - 6(m^2 - 4)(1-f) - (m-2)^2 (1-f)^2 \right] \times \left[\rho m - \frac{(m-2)(1-f)}{3} \right] \quad \ldots \quad \ldots (71)$$

The fraction of the grain burnt at any instant t is

$$Z = \frac{V_{o}\delta - V\delta}{V_{o}\delta} = 1 - \frac{V}{V_{o}} \quad \ldots \quad \ldots (72)$$

Using (70) and (71) in (72), we have on simplification

$$Z = (1-f)\left[1 - \left\{ \frac{3m\rho(m-2)^2 - 6(m^2-4)\,m - 2\,(m-2)^3}{3m\rho\,(7m^2 - 4m - 20)} \right\} f - \left\{ \frac{(m-2)^3}{3m\rho\,(7m^2 - 4m - 20)} \right\} f^2 \right] \quad \ldots \quad \ldots \quad \ldots (73)$$

or

$$Z = (1-f)(1 - Bf - Cf^2) \quad \ldots \quad \ldots \quad \ldots (74)$$

where

$$B = \frac{(m-2)(3m\rho - 8m - 8)}{3m\rho\,(7m + 10)} \quad \ldots \quad \ldots \quad \ldots (75)$$

and

$$C = \frac{(m-2)^2}{3m\rho\,(7m + 10)} \quad \ldots \quad \ldots \quad \ldots (76)$$

Thus (74) gives the form function for the modified bitubular powders.

Differentiating (74), we have

$$\frac{dz}{df} = -(1 + B) + 2(B - C)f + 3Cf^2 \quad \ldots (77)$$

and

$$\left(\frac{dz}{df}\right) f = 1 = -1 + B + C \quad \ldots \quad \ldots (78)$$

$$\therefore \frac{S}{S_o} = \frac{\frac{dz}{df}}{\left(\frac{dz}{df}\right)_{f=1}} = \frac{1 + B - 2(B - C)f - 3Cf^2}{1 - B - C} \quad \ldots \quad \ldots (79)$$

or

$$\frac{S}{S_o} = \alpha' - \beta' f - \gamma' f^2 \quad \ldots \quad \ldots (80)$$

where

$$\alpha' = \frac{1 + B}{1 - B - C} = \frac{8(m+1)(3m\rho - m + 2)}{\{18m^2\rho + 36m\rho + 7m^2 - 4m - 20\}} \quad \ldots \quad \ldots (81)$$

$$\beta' = \frac{2(B - C)}{1 - B - C} = \frac{6(m-2)(m\rho - 3m - 2)}{\{18m^2\rho + 36m\rho + 7m^2 - 4m - 20\}} \quad \ldots (82)$$

and

$$\gamma' = \frac{3C}{1 - B - C} = \frac{3(m-2)^2}{\{18m^2\rho + 36m\rho + 7m^2 - 4m - 20\}} \quad \ldots (83)$$

The values of B, C, α', β', and γ' for different sets of values of m and ρ are given in tables 1 to 4 while those of Z and $\frac{S}{S_o}$ in terms of f for different sets of values of m and ρ are given in Table 5. The results of Table 5 are illustrated in figs. 6, 7 and 8.

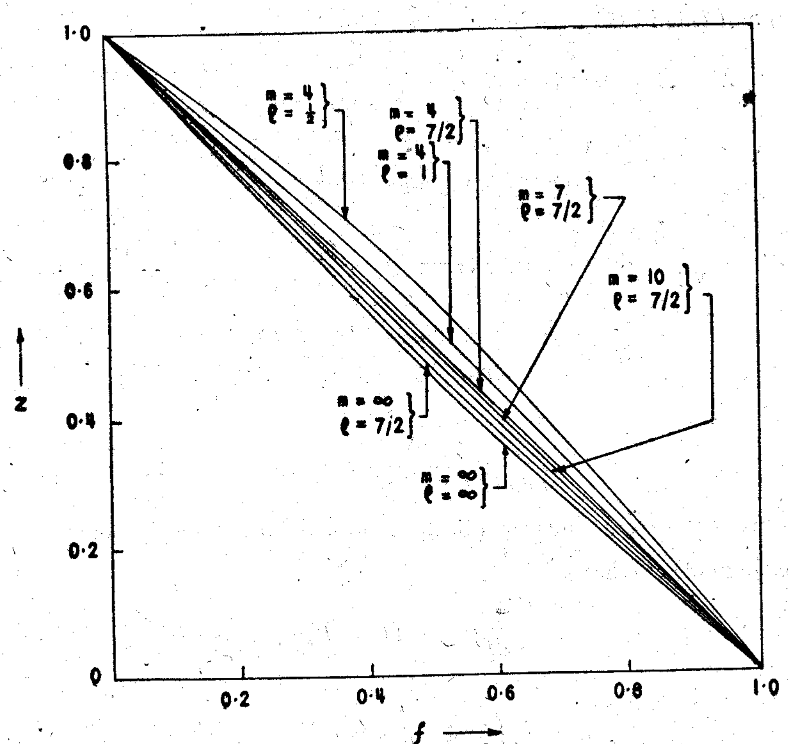

Fig. 6

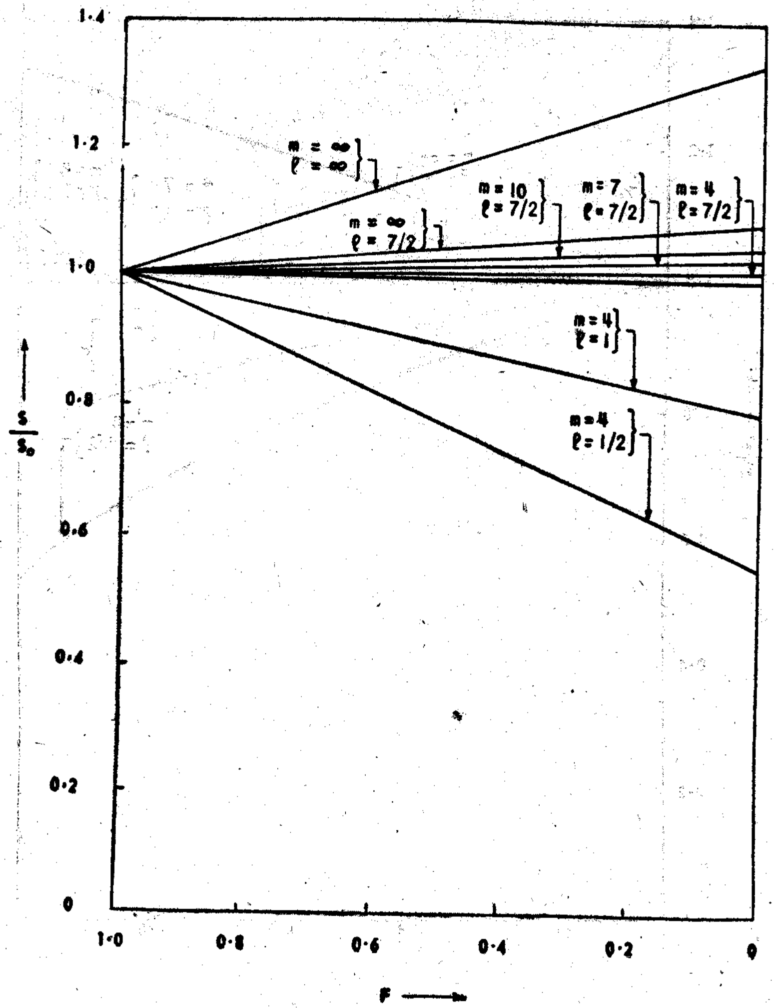

Fig. 7

TABLE 2

M=4

ρ	½	1	$\frac{7}{2}$	5	20	∞
B	—0·2982	—0·1228	0·0025	0·0175	0·0438	0·0526
C	0·0175	0·0088	0·0025	0·0017	0·0004	0·0000
α′	0·5480	0·7874	1·0075	1·0374	1·0921	1·1110
β′	—0·4930	—0·2363	0·0000	0·0322	0·0908	0·1110
γ′	0·0410	0·0237	0·0075	0·0052	0·0012	0·0000

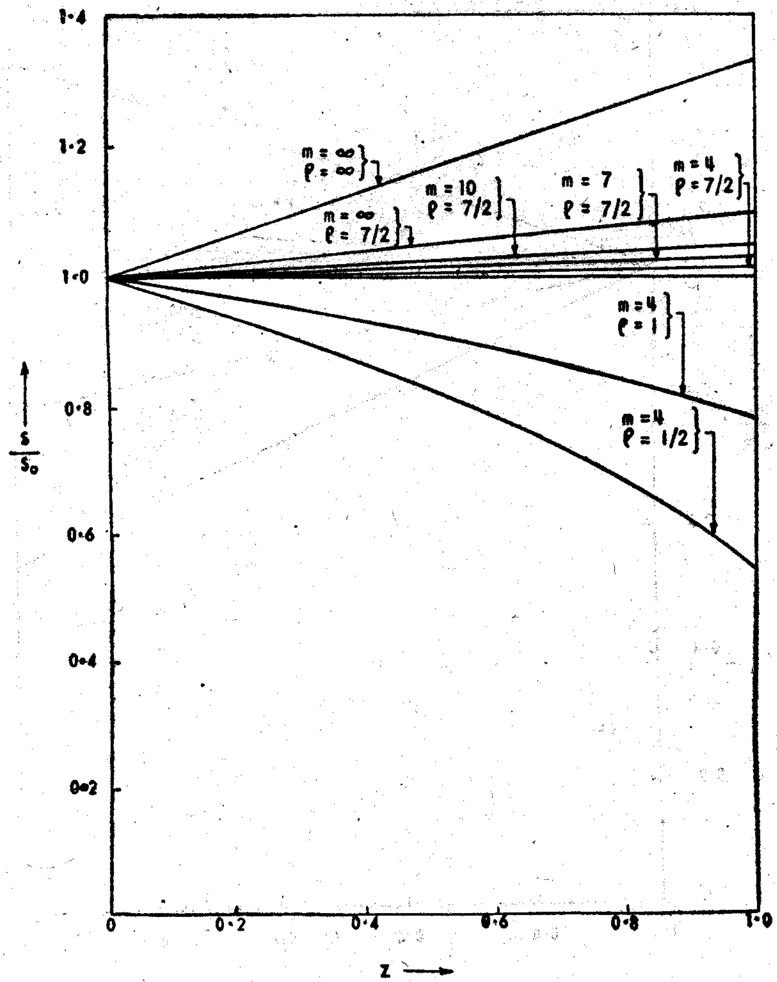

Fig. 8

TABLE 3

m=7

ρ	½	1	$\frac{7}{2}$	5	20	∞
B	—0·4280	—0·1720	0·0109	0·0331	0·0718	0·0847
C	0·0403	0·0202	0·0057	0·0040	0·0010	0·0000
α′	0·4122	0·7189	1·0280	1·0729	1·1559	1·1851
β′	—0·6749	—0·3337	0·0106	0·0604	0·1527	0·1851
γ′	0·0871	·00526	0·0174	0·0125	0·0032	0·0000

TABLE 4

m=10

ρ	½	1	7/2	5	20	∞
B	—0·4867	—0·1933	0·0162	0·0413	0·0853	0·1000
C	0·0533	0·0267	0·0076	0·0053	0·0013	0·0000
α′	0·3581	0·6915	1·0410	1·0922	1·1882	1·2222
β′	—0·7534	—0·3772	0·0176	0·0755	0·1839	0·2222
γ′	0·1115	0·0687	0·0233	0·0167	0·0043	0·0000

TABLE 5

m=∞

ρ	½	1	7/2	5	20	∞
B	—0·6190	—0·2381	0·0340	0·0666	0·1238	0·1428
C	0·0952	0·0476	0·0136	0·0095	0·0024	0·0000
α′	0·2500	0·6400	1·0857	1·1544	1·2862	1·3332
β′	—0·9374	—0·4800	0·0428	0·1236	0·2779	0·3332
γ′	0·1874	0·1199	0·0428	0·0308	0·0082	0·0000

Differentiating (80) we have

$$\frac{d}{df}\left(\frac{S}{S_\circ}\right) = -\beta' - 2\gamma' f \quad \ldots \quad \ldots \quad \ldots (84)$$

and

$$\frac{d^2}{df^2}\left(\frac{S}{S_\circ}\right) = -2\gamma' \quad \ldots \quad \ldots \quad \ldots (85)$$

From (83) we find that γ' is always positive and therefore $\frac{d^2}{df^2}\left(\frac{S}{S_\circ}\right)$ is always negative that is $\frac{d}{df}\left(\frac{S}{S_\circ}\right)$ is a decreasing function of f.

Now for $f=1$, (beginning of combustion):

$$\left\{\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right\}_{f=1} = -\beta' - 2\gamma' = \frac{6(m-2)(2m+4-m\rho)}{\{18m^2\rho + 36m\rho + 7m^2 - 4m - 20\}} \quad \ldots (86)$$

and for $f=0$, (end of combustion):

$$\left\{\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right\}_{f=0} = -\beta' = \frac{6(m-2)(3m+2-m\rho)}{\{18m^2\rho + 36m\rho + 7m^2 - 4m - 20\}} \quad \ldots (87)$$

TABLE 6

VALUES OF Z AND S/S_o FOR SOME PARTICULAR VALUES OF m AND ρ FOR A MODIFIED BITUBULAR CHARGE

$m=4$, $\rho=1/2$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
	Z	0·0	0·1254	0·2455	0·3600	0·4690	0·5724	0·6699	0·7615	0·8471	0·9267	1·0
	S/S_o	1·0	0·9585	0·9162	0·8730	0·8290	0·7824	0·7386	0·6922	0·6450	0·5969	0·5480
$m=4$, $\rho=1$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
	Z	0·0	0·1103	0·2185	0·3245	0·4282	0·5296	0·6286	0·7252	0·8194	0·9110	1·0
	S/S_o	1·0	0·9809	0·9613	0·9412	0·9206	0·8996	0·8781	0·8561	0·8337	0·8108	0·7874
$m=4$, $\rho=7/2$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
	Z	0·0	0·0996	0·1993	0·2991	0·3990	0·4991	0·5992	0·6993	0·7995	0·8997	1·0
	S/S_o	1·0	1·0014	1·0027	1·0038	1·0048	1·0056	1·0063	1·0068	1·0072	1·0074	1·0075
$m=7$, $\rho=7/2$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
	Z	0·0	0·0985	0·1975	0·2969	0·3966	0·4966	0·5968	0·6973	0·7981	0·8990	1·0
	S/S_o	1·0	1·0044	1·0084	1·0120	1·0154	1·0183	1·0210	1·0232	1·0252	1·0268	1·0280
$m=10$, $\rho=7/2$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
	Z	0·0	0·0979	0·1964	0·2955	0·3950	0·4950	0·5954	0·6961	0·7972	0·8985	1·0
	S/S_o	1·0	1·0063	1·0120	1·0173	1·0220	1·0263	1·0302	1·0336	1·0365	1·0390	1·0410
$m=\infty$, $\rho=7/2$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
	Z	0·0	0·0958	0·1928	0·2909	0·3899	0·4898	0·5905	0·6920	0·7941	0·8968	1·0
	S/S_o	1·0	1·0125	1·0241	1·0348	1·0446	1·0536	1·0617	1·0690	1·0754	1·0810	1·0857
$m=\infty$, $\rho=\infty$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
	Z	0·0	0·0871	0·1771	0·2700	0·3657	0·4643	0·5657	0·6700	0·7771	0·8871	1·0
	S/S_o	1·0	1·0333	1·0666	1·1000	1·1333	1·1666	1·1999	1·2332	1·2666	1·3000	1·3332

Thus for $\frac{S}{S_o}$ to have a maximum value in the interval of variation of f, the values of $\frac{d}{df}\left(\frac{S}{S_o}\right)$ should have opposite signs at $f = 1$, and at $f = 0$,

$$\text{i. e.,} \left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=1} \left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=0} < o \quad \ldots \quad \ldots (88)$$

$$\text{or} \quad (2m + 4 - m\rho)\,(3m + 2 - m\rho) < o \quad \ldots \quad \ldots (89)$$

same as (37) of Part A.

Again, the first member of (89) becomes zero for

$$\rho_1 = \frac{2m + 4}{3m}$$

and the second member for

$$\rho_2 = \frac{3m + 2}{m}$$

same as (38) and (39) of Part A.

$\therefore$ when we give to ρ a value lying between ρ_1 and ρ_2 the above inequality will be satisfied. and the maximum of $\frac{S}{S_o}$ will appear if

$$\frac{d}{df}\left(\frac{S}{S_o}\right) = -\beta' - 2\gamma' f = o \quad \ldots \quad \ldots \quad \ldots (90)$$

i.e., when

$$f = -\frac{\beta'}{2\gamma'} = \frac{3m+2-m\rho}{m-2} \quad \ldots \quad \ldots (91)$$

which is same as (41) of Part A.

(91) can again be written as

$$f = 1 - \frac{\rho - \rho_1}{3\rho_{min}}$$

where $\rho_{min} = \frac{m-2}{3m}$, as explained in Part A

Taking ρ_1 and ρ_2 into consideration (86) and (87) can again be written as

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=1} = \frac{6m\,(m-2)\,(\rho_1 - \rho)}{\{18m^2\rho + 36m\rho + 7m^2 - 4m - 20\}} \quad \ldots \quad \ldots (92)$$

and

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=0} = \frac{6m\,(m-2)\,(\rho_2 - \rho)}{\{18m^2\rho + 36m\rho + 7m^2 - 4m - 20\}} \quad \ldots \quad \ldots (93)$$

Hence

(*i*) For $\frac{m-2}{3m} \leqslant \rho \leqslant \rho_1$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is always positive. Therefore the powder is degressive throughout.

(*ii*) For $\rho_1 < \rho < \rho_2$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is negative in the beginning and then positive, therefore the powder is first progressive and then degressive.

(*iii*) For $\rho \geqslant \rho_2$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is always negative. Therefore the powder is progressive throughout.

The results obtained above are given in the following Table:

	Case (i)	Case (ii)	Case (iii)
ρ	$\rho_{min} \leqslant \rho \leqslant \rho_1$	$\rho_1 < \rho < \rho_2$	$\rho \geqslant \rho_2$
$\frac{S}{S_o}$	Decreasing function of Z The powder is degressive	Increasing function of Z in the beginning, then decreasing function of Z. The powder is first progressive and then degressive.	Increasing function of Z The powder is progressive

THE EQUIVALENT FROM FACTOR FOR THE MODIFIED BITUBULAR CHARGE

Following Kapur and Jain (1961), the equivalent form factor θ for the modified bitubular charge is given as

$$\theta = -\left(B + \frac{C}{2}\right) = -\frac{(m+2)(2m\rho - 5m - 6)}{2\,m\rho\,(7m + 10)} \quad \ldots (94)$$

which will be negative if

$$\rho > \frac{5m + 6}{2m} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (95)$$

The values of Q for some sets of values of m and ρ are given in table 6.

TABLE 7

	$m = 4$, $\rho = \frac{1}{2}$	$m=4$, $\rho=1$	$m=4$, $\rho=7/2$	$m=7$, $\rho=7/2$	$m=10$, $\rho=7/2$	$m=\infty$, $\rho=7/2$	$m=\infty$ $\rho=\infty$
θ	+0·2895	+0·1184	− 0·0037	—0·0137	—0·0200	—0·0408	—0·1428

Acknowledgement—The author is highly grateful to Dr. G. C. Patni, for his interest, encouragement an guidance in the preparation of this paper.

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