SYSTEMATIC DESIGN OF A TRIPLET PHOTOGRAPHIC OBJECTIVE

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ABSTRACT

IN THIS paper is given an account of how the primary aberrations are controlled in the design of a triplet system. A differential method of adjusting the spherical aberration, when the other three shape-dependent aberrations are corrected by solution of proper shapes, is proposed. An example illustrating the numerical computation with the aid of the above methods, is also given.

INTRODUCTION

In the design of a triplet of three separated lenses, the designer has at his disposal for a given choice of glasses, eight variables, namely, the three powers, the two air separations and the three shapes for controlling the primary aberrations. The three powers and the two separations can be adjusted to satisfy the total power condition and to give the required correction of the two chromatic aberrations and the Petzval curvature. The three shapes can then be determined to obtain the desired correction of the three shape-variant aberrations, namely, coma, astigmatism and distortion. The primary spherical aberration which remains uncorrected in the above procedure, is then adjusted by determining its variation with a free parameter.

Stephens[1] has described in detail the general method by which the shape-invariant conditions could be satisfied and has suggested the procedure to be adopted for the control of shape-dependent aberrations.

Conrady[2] has worked out a method for the control of only the three aberrations namely the spherical aberration, coma and the astigmatism.

In a recent paper, Wynne[3] has discussed the thin lens primary aberrations and suggested in broad outline how the primary aberrations could be controlled in the design of a triplet system, with the help of the thin lens primary aberration equations.

The necessary equations which are not given in published literature in a form suitable for routine design, are presented in this paper, for general information and reference.

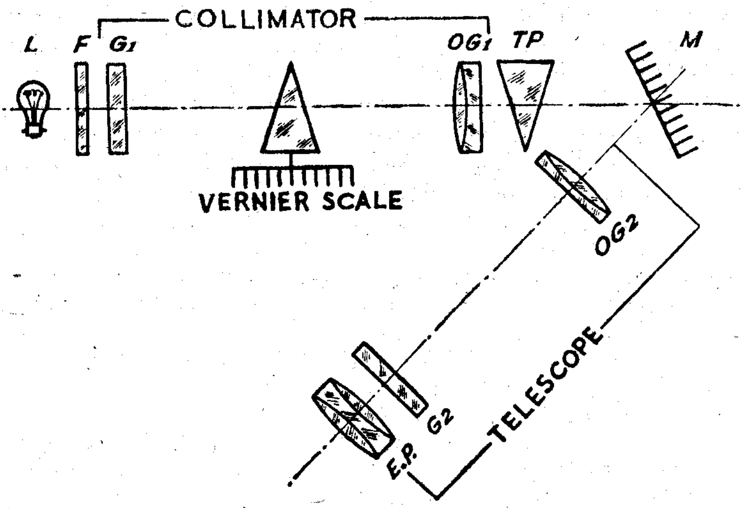

Fig.

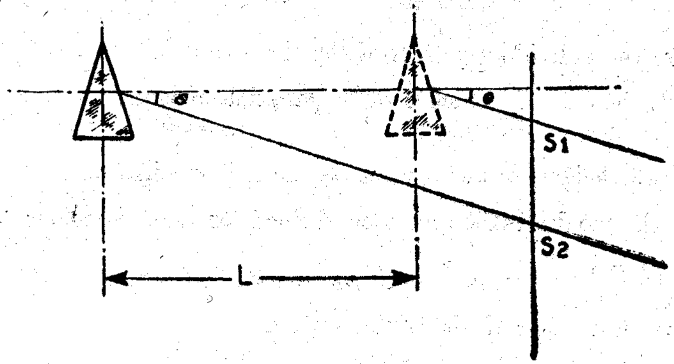

Fig.

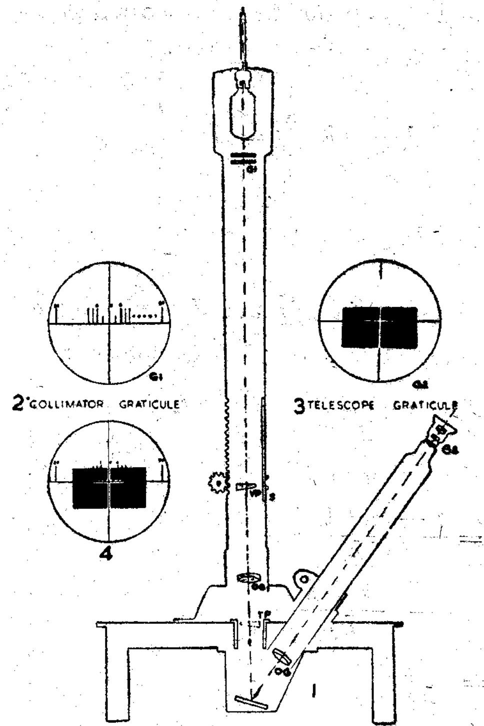

PLATE-1

EQUATIONS FOR ROUTINE DESIGN

Here Conrady's[2] method is followed with a slight modification.

The general basic equations involving the two chromatic aberration coefficients and the total power of a triplet, are:

$$\left.\begin{aligned} h_1\,\varphi_1 + h_2\,\varphi_2 + h_3\,\varphi_3 &= \Phi \\ h^2_1\,\frac{\varphi_1}{\nu_1} + \frac{{h_2}^2\varphi_2}{\nu_2} + {h_3}^2\,\frac{\varphi_3}{\nu_3} &= L \\ p_1 h_1\frac{\varphi_1}{\nu_1} + p_2 h_2\frac{\varphi_2}{\nu_2} + p_3\,h_3\,\frac{\varphi_3}{\nu_3} &= T \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Where

$\varphi_1, \varphi_2, \varphi_3$ are the individual powers of the three thin lenses,

h_1, h_2, h_3 are the heights of incidence of a paraxial aperture ray from an infinite object point, at the successive lenses,

p_1, p_2, p_3 are the heights of incidence of a paraxial principal ray,

ν_1, ν_2, ν_3 are the modified Abbe numbers defined for the two colours D and G^1, as

$\nu_{D-G^1} = \frac{\mu_{G^1} - 1}{\mu_{G^1} - \mu_D}$ where μ_{G^1} and μ_D are the refractive indices for the above colours. Φ is the total power of the triplet system.

When the entire system is scaled such that $\Phi = 1$, the eqs (1) become with the assumption that $h_1 = 1$, $T = 0$ and $p_2 = 0$ (for the stop located at the middle lens),

$$\left.\begin{aligned} \varphi_1 + h_2\varphi_2 + h_3\varphi_3 &= 1 \\ \frac{\varphi_1}{\nu_1} + h_2^2\frac{\varphi_2}{\nu_2} + h_3^2\frac{\varphi_3}{\nu_3} &= L \\ p_1\frac{\varphi_1}{\nu_1} + p_3h_3\frac{\varphi_3}{\nu_3} &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Letting $J = -\frac{p_1}{p_3}$ and choosing ϕ_1 and J arbitrarily, eqs (2) may be solved for h_2, φ_2, h_3 and φ_3, in terms of φ_1, J and L as follows:

$$\left.\begin{aligned} h_2 &= \frac{1 - \varphi_1(1+J) + L\nu_1}{1 + \frac{\nu_1}{\nu_2}\left[1 - \varphi_1(1+J)\right]} \\ \varphi_2 &= \frac{1 - \varphi_1(1+J)}{h_2} \\ h_3 &= 1 - \frac{1 - h_2}{J\varphi_1} \\ \varphi_3 &= \frac{J\varphi_1}{h_3} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (3)$$

In general, if A, B, C and E denote respectively the coefficients of the primary spherical aberration, coma, astigmatism and distortion of a single thin lens of power ϕ in air, and if the stop is located at the lens, then the above coefficients may be evaluated as follows:

$$\left.\begin{aligned} \bar{A} &= \frac{\varphi^3 h^4}{4}(a + b\bar{P}^2 + c\bar{P}S + dS^2) \\ \bar{B} &= \frac{\varphi^2 h^2}{2}(e\bar{P} + fS) \\ C &= \varphi \\ E &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where a, b, c, d, e, f are functions of the refractive index μ, defined by

$$\left.\begin{aligned} a &= \frac{\mu^2}{(\mu - 1)^2} \\ b &= \frac{3\mu + 2}{\mu} \\ c &= \frac{4(\mu + 1)}{\mu(\mu - 1)} \\ d &= \frac{\mu + 2}{\mu(\mu - 1)^2} \\ e &= \frac{2\mu + 1}{\mu} \\ f &= \frac{\mu + 1}{\mu(\mu - 1)} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (4a)$$

$\overline{P}$ is Coddington's position factor for the lens, defined by $\frac{u + u^1}{u - u^1}$ where u and u^1 are the angles of obliquity of the paraxial ray with the axis in the object and image spaces respectively, S is Coddington's shape factor for the lens, defined by $S = \frac{\rho_1 + \rho_2}{\rho_1 - \rho_2}$ where ρ_1 and ρ_2 are the curvatures of the two spherical surfaces of the lens, h is the height of incidence of the paraxial ray at the lens.

When the stop is not located at the lens the coefficients are modified as follows, taking into considertion the height p of the paraxial principal ray,

$$\left.\begin{aligned} \overline{A} = A &= \frac{\varphi^3 h^4}{4}(a + b\,\overline{P}^2 + c\overline{P}S + dS^2) \\ B &= \overline{B} + sA \\ C &= \varphi + s(B + \overline{B}) \\ E &= s\,[(3\varphi + P) + s(B + 2\overline{B})] \end{aligned}\right\} \quad \ldots \quad \ldots \quad (5)$$

Where $s = \frac{p}{h}$ at the lens.

After the necessary powers of the lenses and heights of incidence of the paraxial ray have been determined from (3), the values of the coefficients for the three lenses of the triplet system may be expressed, with the aid of eqs (4) and (5), as functions of the shapes S_1, S_2 and S_3 of the lenses, as

$$
\left.\begin{aligned}
\Sigma A &= \bar{d}_1 S_1^2 + \bar{c}_1 S_1 + (\bar{a}_1 + \bar{b}_1) + \bar{d}_2 S_2^2 + \bar{c}_2 S_2 \\
&\quad + (\bar{a}_2 + \bar{b}_2) + \bar{d}_3 S_3^2 + \bar{c}_3 S_3 + (\bar{a}_3 + \bar{b}_3) \\
\Sigma B &= s_1 \bar{d}_1 S_1^2 + (s_1 \bar{c}_1 + \bar{f}_1) S_1 + s_1 (\bar{a}_1 + \bar{b}_1) + \bar{e}_1 \\
&\quad + \bar{f}_2 S_2 + s_3 \bar{d}_3 S_3^2 + (s_3 \bar{c}_3 + \bar{f}_3) S_3 + \bar{e}_2 \\
&\quad + s_3 (\bar{a}_3 + \bar{b}_3) + \bar{e}_3 \\
\Sigma C &= s_1^2 \bar{d}_1 S_1^2 + (s_1^2 \bar{c}_1 + 2 s_1 \bar{f}_1) S_1 + s_1^2 (\bar{a}_1 + \bar{b}_1) \\
&\quad + 2 s_1 \bar{e}_1 + \varphi_1 + \varphi_2 + s_3^2 \bar{d}_3 S_3^2 \\
&\quad + (s_3^2 \bar{c}_3 + 2 s_3 \bar{f}_3) S_3 + s_3^2 (\bar{a}_3 + b_3) \\
&\quad + 2 s_3 \bar{e}_3 + \varphi_3 \\
\Sigma E &= s_1^3 \bar{d}_1 S_1^2 + (s_1^3 \bar{c}_1 + 3 s_1^2 \bar{f}_1) S_1 + s_1^3 (\bar{a}_1 + \bar{b}_1) \\
&\quad + 3 s_1^2 \bar{e}_1 + s_1 (3\varphi_1 + P_1) + s_3^3 d_3 S_3^2 \\
&\quad + (s_3^3 c_3 + 3 s_3^2 \bar{f}_3) S_3 + s_3^3 (\bar{a}_3 + \bar{b}_3) \\
&\quad + 3 s_3^2 \bar{e}_3 + s_3 (3\varphi_3 + P_3)
\end{aligned}\right\} \quad ..
$$

Where

$$
\left.\begin{aligned}
s_1 &= \frac{p_1}{h_1} = \frac{h_2 - 1}{h_2 \varphi_1} \\
s_3 &= \frac{p_3}{h_3} = \frac{h_2 - h_3}{h_2 h_3} \cdot \frac{1}{(1 - J\varphi_1)}
\end{aligned}\right\} \quad ..
$$

The quantities $\bar{a}$ to $\bar{f}$ and $\bar{P}$ are calculated for each lens from the following :

$$
\left.\begin{aligned}
\bar{a} &= \frac{\varphi^3 h^4}{4} a \\
\bar{b} &= \frac{\varphi^3 h^4}{4} b \bar{P}^2 \\
\bar{c} &= \frac{\varphi^3 h^4}{4} c \bar{P} \\
\bar{d} &= \frac{\varphi^3 h^4}{4} d \\
\bar{e} &= \frac{\varphi^2 h^2}{2} e \bar{P} \\
\bar{f} &= \frac{\varphi^2 h^2}{2} f \\
\text{and } \bar{P}_1 &= -1 \quad \text{for an infinite object point.} \\
\bar{P}_2 &= -\left\{ \frac{2\varphi_1}{1 - \varphi_1 (1 + J)} + 1 \right\} \\
\bar{P}_3 &= \left\{ 1 - \frac{2}{J \varphi_1} \right\}
\end{aligned}\right\}
$$

It may be seen from (6) that ΣA is a quadratic function in each of the three shapes; ΣB is a quadratic function of S_1 and S_3 and linear in S_2; ΣC and ΣE are both quadratic in S_1 and S_3.

According to Conrady the eqs (6) may be solved for S_1, S_2 and S_3 to give the desired correction of ΣA, ΣB and ΣC. For a chosen value of S_1, the equation for ΣC may be solved for S_3 by completing the squares for S_1 and S_3. The values of S_1 and S_3 thus determined are substituted in the equation for ΣB and the value of S_2 obtained. ΣA can now be evaluated with the known values of S_1, S_2 and S_3. The process is repeated for different values of S_1, the proper S_1 for the required ΣA being obtained by interpolation. While this method offers a quick means for obtaining correction in ΣA, ΣB and ΣC, the correction of ΣE is completely left out of consideration.

As an alternative to the above method of Conrady, the following method can also be adopted. However, in view of the time and labour involved in the use of this method, it is preferable to adopt it after a satisfactory correction for ΣA, ΣB and ΣC has been achieved by the more simple method of Conrady.

Since ΣC and ΣE are functions of S_1 and S_3 alone, the equations may be solved by treating them as two simultaneous equations in the variables S_1 and S_3. Since both S_1 and S_3 occur in quadratic form, the eliminant of either S_1 or S_3 from the equation, is a quartic equation in S_3 or S_1. The solution of the two equations for ΣC and ΣE thus provide the two values for S_1 and S_3, which when substituted in the equation for ΣB provides the solution for S_2. ΣA can now be evaluated. ΣA will not generally be corrected by this method in the first attempt. The whole procedure has to be repeated with a different choice of the arbitrarily chosen free parameter. By successive interpolation or extrapolation, the desired correction for ΣA may be achieved.

While the above methods are adequate and satisfactory for controlling ΣA, the same result could be achieved with relatively lesser effort by the adoption of the differential approach.

DIFFERENTIAL METHOD

Differential method for simultaneous control of primary aberrations of the triplet system is given here.

φ_1 and J having been arbitrarily chosen as free parameters, the differentials of ΣA, ΣB, ΣC and ΣE with respect to these two parameters can be ob'ained.

φ_1 *differentials:*

Differentiating 6 (a) and 6 (b) with respect to φ_1 (treating J constant) and substituting in (6), the following can be obtained after some simplification.

$$\sum \frac{dA}{d\varphi_1} = \bar{l}_1 \bar{d}_1 S_1^2 + \bar{l}_1 \bar{c}_1 S_1 + \bar{l}_1 (\bar{a}_1 + \bar{b}_1) + \bar{l}_2 \bar{d}_2 S_2^2$$

$$+ (\bar{l}_2 + \bar{n}_2) \bar{c}_2 S_2 + \bar{l}_2 (\bar{a}_2 + \bar{b}_2)\, 2 \bar{b}_2 \bar{n}_2$$

$$+ \bar{l}_3 \bar{d}_3 S_3^2 + (\bar{l}_3 + \bar{n}_3) \bar{c}_3 S_3 + \bar{l}_3 (\bar{a}_3 + \bar{b}_3)$$

$$+ 2 \bar{b}_3 \bar{n}_3$$

$$\sum \frac{dB}{d\varphi_1} = \left(\bar{l}_1 s_1 \bar{d}_1 + \bar{d}_1 \frac{d s_1}{d \phi_1}\right) S_1^2 + \left(s_1 \bar{l}_1 \bar{c}_1 + \bar{c}_1 \frac{d s_1}{d \phi_1} + \bar{m}_1 \bar{f}_1\right) S_1$$

$$+ s_1 \bar{l}_1 (\bar{a}_1 + \bar{b}_1) + (\bar{a}_1 + \bar{b}_1) \frac{d s_1}{d_1} + \bar{m}_1 \bar{e}_1$$

$$+ \bar{m}_2 \bar{f}_2 S_2 + \bar{e}_2 (\bar{m}_2 + \bar{n}_2)$$

$$+ \left(\bar{l}_3 s_3 \bar{d}_3 + \bar{d}_3 \frac{d s_3}{d\varphi_1}\right) S_3^2$$

$$+ \left[s_3 \bar{c}_3 (l_3 + \bar{n}_3) + \bar{c}_3 \frac{d s_3}{d\varphi_1} + \bar{m}_3 \bar{f}_3\right] S_3 + s_3 \left[\bar{l}_3 (\bar{a}_3 + \bar{b}_3) + 2 \bar{b}_3 \bar{n}_3\right]$$

$$+ \left(\bar{a}_3 + \bar{b}_3\right) \frac{d s_3}{d\varphi_1} + \bar{e}_3 \left(\bar{m}_3 + \bar{n}_3\right)$$

$$\sum \frac{dC}{d\varphi_1} = \left(\bar{l}_1 s_1^2 \bar{d}_1 + 2 \bar{d}_1 s_1 \frac{d s_1}{d\varphi_1}\right) S_1^2$$

$$+ \left[\bar{l}_1 s_1^2 \bar{c}_1 + 2 \bar{c}_1 s_1 \frac{d s_1}{d\varphi_1} + 2 \bar{f}_1 \left(\bar{m}_1 s_1 + \frac{d s_1}{d\varphi_1}\right)\right] S_1$$

$$+ \left[\bar{l}_1 s_1^2 (\bar{a}_1 + \bar{b}_1) + 2 (\bar{a}_1 + \bar{b}_1) s_1 \frac{d s_1}{d\varphi_1} + 2 \bar{e}_1 \left(\bar{m}_1 s_1 + \frac{d s_1}{d\varphi_1}\right) + 1\right]$$

$$+ \frac{d\varphi_2}{d\varphi_1} + \left(\bar{l}_3 s_3^2 \bar{d}_3 + 2 \bar{d}_3 s_3 \frac{d s_3}{d\varphi_1}\right) S_3^2$$

$$+ \left[s_3^2 \bar{c}_3 (\bar{l}_3 + \bar{n}_3) + 2 \bar{c}_3 s_3 \frac{d s_3}{d\varphi_1} + 2 \bar{f}_3 \left(\bar{m}_3 s_3 + \frac{d s_3}{d\varphi_1}\right)\right] S_3$$

$$+ s_3^2 [\bar{l}_3 (\bar{a}_3 + \bar{b}_3) + 2 \bar{b}_3 \bar{n}_3] + 2 (\bar{a}_3 + \bar{b}_3) s_3 \frac{d s_3}{d\varphi_1}$$

$$+ 2 \bar{e}_3 \left[(\bar{m}_3 + \bar{n}_3) s_3 + \frac{d s_3}{d\varphi_1}\right] + \frac{d\varphi_3}{d\varphi_1}$$

$$\sum \frac{dE}{d\varphi_1} = \left(\bar{l}_1 s_1^3 \bar{d}_1 + 3 \bar{d}_1 s_1^2 \frac{d s_1}{d\varphi_1}\right) S_1^2$$

$$+ \left[\bar{l}_1 s_1^3 \bar{c}_1 + 3 \bar{c}_1 s_1^2 \frac{d s_1}{d\varphi_1} + 3 \bar{f}_1 \left(\bar{m}_1 s_1^2 + 2 s_1 \frac{d s_1}{d\varphi_1}\right)\right] S_1$$

$$+ \bar{l}_1 s_1^3 (\bar{a}_1 + \bar{b}_1) + 3 (\bar{a} + \bar{b}_1) s_1^2 \frac{d s_1}{d\varphi_1}$$

$$+ 3\ \bar{e}_1 \left(\bar{m}_1\ s_1^2 + 2\ s_1 \frac{d\ s_1}{d\varphi_1} \right) + s_1 \left(3 + \frac{1}{\mu_1} \right) + (3\ \varphi_1 + P_1)\ \frac{d\ s_1}{d\varphi_1}$$

$$+ \left(\bar{l}_3\ s_3^3\ \bar{d}_3 + 3\ \bar{d}_3\ s_3^2\ \frac{d\ s_3}{d\varphi_1} \right) S_3^2$$

$$+ \left[s_3^3\ \bar{c}_3\ (\bar{l}_3 + \bar{n}_3) + 3\ \bar{c}_3\ s_3^2 \frac{d\ s_3}{d\ \varphi_1} + 3\ \bar{f}_3 \left(\bar{m}_3\ s_3^2 + 2\ s_3 \frac{d\ s_3}{d\varphi_1} \right) \right] S_3$$

$$+ s_3^3\ [\bar{l}_3\ (\bar{a}_3 + \bar{b}_3) + 2\ \bar{b}_3\ \bar{n}_3] + 3\ (\bar{a}_3 + \bar{b}_3)\ s_3^2\ \frac{d\ s_3}{d\ \varphi_1}$$

$$+ 3\ \bar{e}_3 \left[s_3^2\ (\bar{m}_3 + \bar{n}_3) + 2\ s_3 \frac{d\ s_3}{d\varphi_1} \right] + s_3 \frac{d\varphi_3}{d\ \varphi_1} \left(3 + \frac{1}{\mu_3} \right)$$

$$+ (3\ \varphi_3 + p_3)\ \frac{d\ s_3}{d\varphi_1} \qquad \ldots \ldots \qquad (7)$$

Where

$$\bar{l}_1 = \frac{3}{\varphi_1}$$

$$\bar{m}_1 = \frac{2}{\varphi_1}$$

$$\bar{l}_2 = \frac{3}{\varphi_2} \frac{d\varphi_2}{d\varphi_1} + \frac{4}{h_2} \frac{dh_2}{d\varphi_1}$$

$$\bar{m}_2 = \frac{2}{\varphi_2} \frac{d\varphi_2}{d\varphi_1} + \frac{2}{h_2} \frac{dh_2}{d\varphi_1}$$

$$\bar{n}_2 = \frac{1}{\bar{P}_2} \frac{d\bar{P}_2}{d\varphi_1}$$

$$\bar{l}_3 = \frac{3}{\varphi_3} \frac{d\varphi_3}{d\varphi_1} + \frac{4}{h_3} \frac{dh_3}{d_1}$$

$$\bar{n}_3 = \frac{1}{\bar{P}_3} \frac{d\bar{P}_3}{d\varphi_1}$$

$$\bar{m}_3 = \frac{2}{\phi_3} \frac{d\phi_3}{d\phi_1} + \frac{2}{h_3} \cdot \frac{dh_3}{d\phi_1}$$

and

$$\frac{d\varphi_2}{d\varphi_1} = -\left[\frac{\frac{dh_2}{d\varphi_1}\left\{1 - \varphi_1(1+J) + h_2(1+J)\right\}}{h_2^2}\right]$$

$$\frac{dh_2}{d\varphi_1} = \frac{(1+J)\left(\frac{\nu_1}{\nu_2}\ L\,\nu_1 - 1\right)}{\left[1 + \frac{\nu_1}{\nu_2}\left\{1 - \varphi_1(1+J)\right\}\right]^2}$$

$$\frac{d\varphi_3}{d\varphi_1} = \frac{J\left(h_3 - \varphi_1 \frac{dh_3}{d\ _1}\right)}{h_3^2}$$

$$\frac{dh_3}{d\varphi_1} = \frac{J\ \varphi_1 \frac{dh_2}{d\varphi_1} + J(1 - h_2)}{(J\ \ _1)^2}$$

$$\frac{d\bar{P}_2}{d\varphi_1} = -\frac{2}{[1 - \varphi_1(1+J)]^2}$$

$$\frac{d\bar{P}_3}{d\varphi_1} = \frac{2\,J}{(J\ \varphi_1)^2}$$

$$\frac{ds_1}{d\varphi_1} = \frac{\varphi_1 \frac{dh_2}{d\varphi_1} + h_2\ (1 - h_2)}{(h_2\ \varphi_1)^2}$$

$$\frac{ds_3}{d\varphi_1} = \frac{(1 - J\ \varphi_1)\left(h_3^2\ \frac{dh_2}{d\varphi_1} - h_2^2\frac{dh_3}{d\varphi_1}\right) + J\,h_2\,h_3\,(h_2 - h_3,}{[h_2\,h_3\,(1 - J\ \varphi_1)]^2} \qquad 7(a$$

J differentials:

Similar expressions as given for φ_1 differentials, may be obtained for J changes also. These may be written, since $\frac{dA_1}{dJ} = 0$, (φ_1 being chosen constant), as follows:

$$\sum\frac{dA}{dJ} = l_2\bar{d}_2 S_2^2 + (l_2 + n_2)\,\bar{c}_2 S_2 + l_2(\bar{a}_2 + \bar{b}_2) + 2\bar{b}_2 n_2$$
$$+ l_3\bar{d}_3 S_3^2 + (l_3 + n_3)\bar{c}_3 S_3 + l_3(\bar{a}_3 + \bar{b}_3) + 2\bar{b}_3 n_3$$

$$\sum\frac{dB}{dJ} = \bar{d}_1\ \frac{ds_1}{dJ} S_1^2 + \bar{c}_1\ \frac{ds_1}{dJ}\ S_1 + (\bar{a}_1 + \bar{b}_1)\frac{ds_1}{dJ} + m_2\bar{f}_2 S_2$$
$$+ \bar{e}_2(m_2 + n_2) + \left(l_3\bar{d}_3 s_3 + \bar{d}_3\frac{ds_3}{dJ}\right)S_3^2$$
$$+ \left[s_3\bar{c}_3(l_3 + n_3) + \bar{c}_3\ \frac{ds_3}{dJ} + m_3\bar{f}_3\right]S_3$$
$$+ s_3[l_3(\bar{a}_3 + \bar{b}_3) + 2\bar{b}_3 n_3] + (\bar{a}_3 + \bar{b}_3)\ \frac{ds_3}{dJ} + \bar{e}_3(m_3 + n_3)$$

$$\sum \frac{dC}{dJ} = 2\bar{d}_1 s_1 \frac{ds_1}{dJ} S_1^2 + \left(2\bar{c}_1 s_1 \frac{ds_1}{dJ} + 2\bar{f}_1 \frac{ds_1}{dJ}\right) S_1$$

$$+ 2(\bar{a}_1 + \bar{b}_1) s_1 \frac{ds_1}{dJ} + 2\bar{e}_1 \frac{ds_1}{dJ} + \frac{d\varphi_2}{dJ}$$

$$+ \left(l_3 \bar{d}_3 s_3^2 + 2\bar{d}_3 s_3 \frac{ds_3}{dJ}\right) S_3^2$$

$$+ \left[s_3^2 \bar{c}_3 (l_3 + n_3) + 2\bar{c}_3 s_3 \frac{ds_3}{dJ} + 2\bar{f}_3 \left(m_3 s_3 + \frac{ds_3}{dJ}\right)\right] S_3$$

$$+ s_3^2 [l_3(\bar{a}_3 + \bar{b}_3) + 2\bar{b}_3 n_3] + 2(\bar{a}_3 + \bar{b}_3) s_3 \frac{ds_3}{dJ}$$

$$+ 2\bar{c}_3 \left[(m_3 + n_3) s_3 + \frac{ds_3}{dJ}\right] + \frac{d\varphi_3}{dJ}$$

$$\sum \frac{dE}{dJ} = 3\bar{d}_1 s_1^2 \frac{ds_1}{dJ} S_1^2 + \left(3\bar{c}_1 s_1^2 \frac{ds_1}{dJ} + 6\bar{f}_1 s_1 \frac{ds_1}{dJ}\right) S_1$$

$$+ 3(\bar{a}_1 + \bar{b}_1) s_1^2 \frac{ds_1}{dJ} + 6\bar{e}_1 s_1 \frac{ds_1}{dJ} + (3\varphi_1 + P_1) \frac{ds_1}{dJ}$$

$$+ \left(l_3 \bar{d}_3 s_3^3 + 3\bar{d}_3 s_3^2 \frac{ds_3}{dJ}\right) S_3^2$$

$$+ \left[s_3^3 \bar{c}_3 (l_3 + m_3) + 3\bar{e}_3 s_3^2 \frac{ds_3}{dJ} + 3\bar{f}_3 \left(m_3 s_3^2 + 2 s_3 \frac{ds_3}{dJ}\right)\right] S_3$$

$$+ s_3^3 [l_3(\bar{a}_3 + \bar{b}_3) + 2\bar{b}_3 n_3] + 3(\bar{a}_3 + \bar{b}_3) s_3^2 \frac{ds_3}{dJ}$$

$$+ 3\bar{e}_3 \left[s_3^2 (m_3 + n_3) + 2 s_3 \frac{ds_3}{dJ}\right] + s_3 \frac{d\varphi_3}{dJ}\left(3 + \frac{1}{M_3}\right)$$

$$+ (3\varphi_3 + P_3) \frac{ds_3}{dJ} \qquad \ldots \qquad (\;)$$

Where

$$l_2 = \frac{3}{\varphi_2} \frac{d\varphi_2}{dJ} + \frac{4}{h_2} \frac{dh_2}{dJ}$$

$$m_2 = \frac{2}{\varphi_2} \frac{d\varphi_2}{dJ} + \frac{2}{h_2} \frac{dh_2}{dJ}$$

$$n_2 = \frac{1}{\bar{P}_2} \frac{d\bar{P}_2}{dJ}$$

$$l_3 = \frac{3}{\varphi_3} \frac{d\varphi_3}{dJ} + \frac{4}{h_3} \frac{dh_3}{dJ}$$

$$m_3 = \frac{2}{\varphi_3} \frac{d\varphi_3}{dJ} + \frac{2}{h_3} \frac{dh_3}{dJ}$$

$$n_3 = \frac{1}{\bar{P}_3} \frac{d\bar{P}_3}{dJ}$$

and

$$\frac{d\varphi_2}{dJ} = -\left[\frac{\frac{dh_2}{dJ}\left\{1-\varphi_1(1+J)+h_2\varphi_1\right\}}{h_2^2}\right]$$

$$\frac{dh_2}{dJ} = \frac{\varphi_1\left(\frac{\nu_1}{\nu_2}L\nu_1 - 1\right)}{\left[1+\frac{\nu_1}{\nu_2}\left\{1-\varphi_1(1+J)\right\}\right]^2}$$

$$\frac{d\varphi_3}{dJ} = \frac{\varphi_1\left(h_3 - J\frac{dh_3}{dJ}\right)}{h_3^2}$$

$$\frac{dh_3}{dJ} = \frac{J\varphi_1\frac{dh_2}{dJ} + \varphi_1(1-h_2)}{(J\varphi_1)^2}$$

$$\frac{d\bar{P}_2}{dJ} = -\frac{2\varphi_1^2}{[1-\varphi_1(1+J)]^2}$$

$$\frac{d\bar{P}_3}{dJ} = \frac{2\varphi_1}{(J\varphi_1)^2}$$

$$\frac{ds_1}{dJ} = \frac{\varphi_1\frac{dh_2}{dJ}}{(h_2\varphi_1)^2}$$

$$\frac{ds_3}{dJ} = \frac{(1-J\varphi_1)\left(h_3^2\frac{dh_2}{dJ} - h_2^2\frac{dh_3}{dJ}\right) + \varphi_1 h_2 h_3(h_2-h_3)}{[h_2h_3(1-J\varphi_1)]^2} \qquad \text{8(a)}$$

In the above differentials P_1 and P_3 denote the Petzval curvatures of the first and third lenses given by

$$P_1 = \frac{\varphi_1}{\mu_1}$$

$$P_3 = \frac{\varphi_3}{\mu_3}$$

The Petzval sum of the entire triplet is given by

$$\Sigma P = \frac{\varphi_1}{\mu_1} + \frac{\varphi_2}{\mu_2} + \frac{\varphi_3}{\mu_3} \qquad (9)$$

The approximate change in φ_1 or J which would give the required change in ΣA, may be evaluated from the differentials $\Sigma \frac{dA}{d\varphi_1}$ or $\Sigma \frac{dA}{dJ}$ given by (7) or (8). It should be noted, however, that any change in φ_1 or J will result in changes in the other quantities *viz*, ΣB, ΣC and ΣE, whose correction might have been already achieved by solution of the shapes. It is, therefore, important to change the shapes S_1, S_2 and S_3, after the necessary change in φ_1 or J to obtain the required correction in ΣA is incorporated. The shapes are modified such that the changes introduce in ΣB, ΣC, and ΣE due to changes in φ_1 or J, are counterbalanced and the original values of ΣB, ΣC and ΣE are restored.

The necessary changes in shapes can now be evaluated by forming the shape differentials.

Shape differentials:

For a change $\triangle\varphi_1$ in φ_1, the original values of ΣB, ΣC and ΣE will be altered. These altered values can be readily obtained by application of equations (3) and (6). Let the corresponding changes in the values of these quantities be $\triangle B, \triangle C$ and $\triangle E$.

If $\triangle s_1$, $\triangle s_2$ and $\triangle s_3$ are the required changes in S_1, S_2 and S_3 to counterbalance the effects of $\triangle\varphi_1$, these may be evaluated as follows:

$$\left[\frac{dC_1}{dS_1} + \triangle\varphi_1 \frac{d}{dS_1}\left(\frac{dC_1}{d\varphi_1}\right)\right]\triangle S_1 + \left[\frac{dC_3}{dS_3} + \triangle\varphi_1 \frac{d}{dS_3}\left(\frac{dC_3}{d\varphi_1}\right)\right] \triangle S_3 = -\triangle C$$

$$\left[\frac{dE_1}{dS_1} + \triangle\varphi_1 \frac{d}{dS_1}\left(\frac{dE_1}{d\varphi_1}\right)\right]\triangle S_1 + \left[\frac{dE_3}{dS_3} + \triangle\varphi_1 \frac{d}{dS_3}\left(\frac{dE_3}{d\varphi_1}\right)\right]\triangle S_3 = -\triangle E \quad ..(10)$$

From the above two simultaneous equations the values of $\triangle S_1$ and $\triangle S_3$ are calculated and substituted in the following equation:

$$\left[\frac{dB_1}{dS_1} + \triangle\varphi_1 \frac{d}{dS_1}\left(\frac{dB_1}{d\varphi_1}\right)\right]\triangle S_1 + \left[\frac{dB_2}{dS_2} + \triangle\varphi_1 \frac{d}{dS_2}\left(\frac{dB_2}{d\varphi_1}\right)\right]\triangle S_2$$

$$+ \left[\frac{dB_3}{dS_3} + \triangle\varphi_1 \frac{d}{dS_3}\left(\frac{dB_3}{dS_3}\right)\right]\triangle S_3 = -\triangle B \quad .. (11)$$

$\triangle S_2$ can be calculated from (11)

The values of $\frac{dB_1}{dS_1}, \frac{dC_1}{dS_1}$ etc. and $\frac{d}{dS_1}\left(\frac{dB_1}{d\varphi_1}\right), \frac{d}{dS_1}\left(\frac{dC_1}{d\varphi_1}\right)$

etc. may be written as follows, from (6) and (7).

$$\frac{dB_1}{dS_1} = 2s_1\, \bar{d}_1\, S_1 + (s_1\, \bar{c}_1 + \bar{f}_1)$$

$$\frac{dB_2}{dS_2} = \bar{f}_2$$

$$\frac{dB_3}{dS_3} = 2s_3\, \bar{d}_3\, S_3 + (s_3\, \bar{c}_3 + \bar{f}_3)$$

$$\frac{dC_1}{dS_1} = 2s_1^2\ \bar{d_1}\, S_1 + s_1^2\, \bar{c_1} + 2s_1 \bar{f_1})$$

$$\frac{dC_3}{dS_3} = 2s_3^2\ \bar{d_3}\, S_3 + (s_3^2\, \bar{c_3} + 2s_3 \bar{f_3})$$

$$\frac{dE_1}{dS_1} = 2s_1^3\ \bar{d_1}\, S_1 + (s_1^3\, \bar{C_1} + 3s_1^2 \bar{f_1})$$

$$\frac{dE_3}{dS_3} = 2s_3^3\, \bar{d_3}\, S_3 + (s_3^3\, \bar{c_3} + 3s_3^2\, \bar{f_3})$$

$$\frac{d}{dS_1}\left(\frac{dB_1}{d\varphi_1}\right) = \left(2\,\bar{l_1}\, s_1\, \bar{d_1} + 2\bar{d_1}\, \frac{ds_1}{d\varphi_1}\right) S_1 + \left(s_1\, \bar{l_1}\, \bar{c_1} + \bar{c_1}\, \frac{ds_1}{d\varphi_1} + \bar{m_1}\, \bar{f_1}\right)$$

$$\frac{d}{dS_2}\left(\frac{dB_2}{d\varphi_1}\right) = \bar{m_2}\, \bar{f_2}$$

$$\frac{d}{dS_3}\left(\frac{dB_3}{d\varphi_1}\right) = \left(2\,\bar{l_3}\, s_3\, \bar{d_3} + 2\bar{d_3}\, \frac{ds_3}{d\varphi_1}\right) S_3 + s_3 \bar{c_3}\, (\bar{l_3} + \bar{n_3}) + \bar{c_3}\, \frac{ds_3}{d\varphi_1} + \bar{m_3}\, \bar{f_3}$$

$$\frac{d}{dS_1}\left(\frac{dC_1}{d\varphi_1}\right) = \left(2\,\bar{l_1}\, s_1^2 \bar{d_1} + 4\,\bar{d_1}\, s_1\, \frac{ds_1}{d\varphi_1}\right) S_1 + \left[\bar{l_1}\, s_1^2\, \bar{c_1} + 2\,\bar{c_1} s_1\, \frac{ds_1}{d\varphi_1} + 2\bar{f_1}\left(\bar{m_1} s_1 + \frac{ds_1}{d\varphi_1}\right)\right]$$

$$\frac{d}{dS_1}\left(\frac{dC_3}{d\varphi_1}\right) = \left(2\,\bar{l_3}\, s_3^2\, \bar{d_3} + 4\,\bar{d_3}\, s_3\, \frac{ds_3}{d\varphi_1}\right) S_3$$
$$+ \left[s_3^2\, \bar{c_3}\, (\bar{l_3} + \bar{n_3}) + 2\,\bar{c_3}\, s_3\, \frac{ds_3}{d\varphi_1} + 2\bar{f_3}\left(\bar{m_3}\, s_3 + \frac{ds_3}{d\varphi_1}\right)\right]$$

$$\frac{d}{dS_1}\left(\frac{dE_1}{d\varphi_1}\right) = \left(2\,\bar{l_1}\, s_1^3 \bar{d_1} + 6\bar{d_1}\, s_1^2\, \frac{ds_1}{d\varphi_1}\right) S_1$$
$$+ \left[\bar{l_1}\, s_1^3\, \bar{c_1} + 3\,\bar{c_1}\, s_1^2\, \frac{ds_1}{d\varphi_1} + 3\bar{f_1}\left(\bar{m_1}\, s_1^2 + 2s_1\, \frac{ds_1}{d\varphi_1}\right)\right]$$

$$\frac{d}{dS_3}\left(\frac{dE_3}{d\varphi_1}\right) = \left(2\,\bar{l_3}\, s_3^3\, \bar{d_3} + 6\bar{d_3}\, s_3^2\, \frac{ds_3}{d\varphi_1}\right) S_3$$
$$+ \left[s_3^3\, \bar{c_3}\left(\bar{l_3} + \bar{n_3}\right) + 3\, c_3\, s_3^2\, \frac{ds_3}{d\varphi_1} + 3 f_3\left(\bar{m_3}\, s_3^2 + 2s_3\, \frac{ds_3}{d\varphi_1}\right)\right] \quad \ldots (12)$$

With the changes in shapes thus determined, the new shapes are calculated and ΣA is evaluated from eqs (6). The process is repeated, if necessary, for a different $\triangle \varphi_1$, followed by shape changes until ΣA attains the desired value.

A similar procedure as given above for φ_1 change, may be adopted for a J change.

It is important to note that the values of ΣA, ΣB, ΣC, and ΣE are always evaluated with the help of eqs (6) and not from the differentials, as the differentials indicate only the approximate changes.

When φ_1, or J is changed, the powers of the component lenses as well as the heights of incidence of the concerned rays on the individual lenses are also changed. These altered values of the powers and the heights of incidence can be calculated with the aid of equations (3) and further used in the re-evaluation of equations (6).

NUMERICAL COMPUTATION AND PRIMARY DESIGN OF A TRIPLET

The method outlined above has been adopted by the author for the design of a triplet with a severely restricted glass choice. This restriction arose from the need to design a triplet from the few types of glasses so far produced in India.

The glasses chosen were dense Barium Crown and dense flint.

Starting with the initial choice of $\varphi_1=2\cdot0$; $J=0\cdot8$, $\varphi=1$ and $L=0\cdot002$ and with the glass characteristics $\mu_1=\mu_3=1\cdot62689$ and $\mu_2=1\cdot64528$ both μ_1 and μ_2 being specified for G^1, and $\nu_1=\nu_3=45\cdot46$ and $\nu_2=28\cdot20$ both ν_1, ν_2 specified for $D\text{-}G^1$ (as earlier defined), eqs (3) give the other required data. Eqs (6) are written down numerically as

$$\Sigma A = 11\cdot3454\, S_1^2 - 20\cdot6054\, S_1 - 18\cdot3867\, S_2^2 - 18\cdot5455\, S_2 + 5\cdot0328\, S_3^2 - 2\cdot2851\, S_3 + 1\cdot4495$$

$$\Sigma B = -1\cdot5425\, S_1^2 + 7\cdot9529\, S_1 + 8\cdot4231\, S_2 + 0\cdot9872\, S_3^2 + 2\cdot8487\, S_3 - 3\cdot0824$$

$$\Sigma C = 0\cdot2097\, S_1^2 - 1\cdot7817 S_1 + 0\cdot1937\, S_3^2 + 1\cdot2035\, S_3 + 2\cdot2778$$

$$\Sigma E = -0\cdot0285\, S_1^2 + 0\cdot3375\, S_1 + 0\cdot0380\, S_3^2 + 0\cdot3633\, S_3 - 0\cdot0683$$

The likely values of ΣA, ΣB, ΣC, and ΣE which would achieve a satisfactory correction at the given finite aperture (f/6·3) and field (22°) were determined from a series of trials. Starting with an arbitrarily chosen set of values for ΣA, ΣB, ΣC, and ΣE, the thin lens shapes S_1, S_2 and S_3 were obtained by solution by the methods detailed in the earlier sections. Suitable thicknesses were introduced and finite rays were traced trigonometrically through the system to assess the performance of the system at the given aperture and field. The values of ΣA, ΣB, ΣC, and ΣE, were altered and the procedure of thickening and finite ray tracing repeated, till a satisfactory overall performance was achieved. It must, however, be noted that, since thin lens values of ΣA, ΣB, ΣC and ΣE are altered after thicknesses are introduced, each set of ΣA, ΣB, ΣC, and ΣE, chosen for solution of thin lens shapes should take into account the variation of these values on thickening, such variation being roughly constant for given values of thicknesses.

A preliminary solution of the above numerical equations gave $S_1=0\cdot54$, $S_2=0\cdot1162$ and $S_3=-1\cdot2571$ for $\Sigma A=2\cdot0$ $\Sigma B=-0\cdot28$ and $\Sigma C=0\cdot17$. With the above shapes, the value for ΣE was found to be $-0\cdot29$ resulting in a high value of distortion figure at 22°.

The elimination of S_1 from the two equations for ΣC and ΣE, with $\Sigma C=0\cdot155$ and $\Sigma E=-0\cdot18$, results in the following quartic equation in S_3.

$$S_3^4 + 16\cdot3817\, S_3^3 + 94\cdot1577\, S_3^2 + 217\cdot6951\, S_3 + 139\cdot3150 = 0$$

Examination of changes of signs of the Sturm's functions formed from the above equation indicated the location of a root between $S_3=-1\cdot0$ and $S_3=-2\cdot0$. Starting with an approximate value for the root and applying Newton's successive approximation method, the actual root was found to be $S_3=-1\cdot0053$. It may be of interest to note, in this connection, that the quartic equation in S_3 need not be solved for all the roots. Since all large values of S_3 give rise to deep curvatures for the surfaces of the lens and should be avoided, it is sufficient if the solution of the quartic is sought in the region $S_3=-1$ to $S_3=-2\cdot0$.

Substitution of S_3 in the equations for ΣC and ΣE, will give two pairs of solutions for S_1, the common value of the two being chosen as the required solution for S_1. With $S_3=-1\cdot0053$, S_1 was found to be $0\cdot6760$.

Substitution of $S_1 = 0\cdot6760$ and $S_3 = -1\cdot0053$ in the equation for ΣB gave $S_2 = -0\cdot0027$ for $\Sigma B = -0\cdot30$. With these values ΣA was found to be $0\cdot1398$, where as the required value for ΣA was about $2\cdot0$ for good spherical aberration correction.

The necessary correction in ΣA was obtained with the aid of the differentials given by (7), which are given numerically as follows :

$$\Sigma \frac{dA}{d\varphi_1} = 17\cdot0181\, S_1^2 - 30\cdot9081\, S_1 - 34\cdot6663\, S_2^2 - 24\cdot7749\, S_2 + 7\cdot3897\, S_3^2 + 2\cdot3575\, S_3 - 4\cdot8383$$

$$\Sigma \frac{dB}{d\varphi_1} = -2\cdot9271\, S_1^2 + 10\cdot4676\, S_1 + 11\cdot6643\, S_2 + 1\cdot8736\, S_3^2 + 3\cdot5670\, S_3 - 3\cdot5850$$

$$\Sigma \frac{dC}{d\varphi_1} = 0\cdot4805\, S_1^2 - 2\cdot8302\, S_1 + 0\cdot4506\, S_3^2 + 1\cdot8643\, S_3 + 2\cdot8363$$

$$\Sigma \frac{dE}{d\varphi_1} = -0\cdot0766\, S_1^2 - 0\cdot6506\, S_1 + 0\cdot1045\, S_3^2 + 0\cdot7032\, S_3 - 0\cdot1087$$

With $S_1 = 0\cdot6760$, $S_2 = -0\cdot0027$ and $S_3 = -1\cdot0053$ the above equations yield

$$\Sigma \frac{dA}{d\varphi_1} = -12\cdot79$$

$$\Sigma \frac{dB}{d\varphi_1} = 0\cdot4295$$

$$\Sigma \frac{dC}{d\varphi_1} = -0\cdot2761$$

$$\Sigma \frac{dE}{d\varphi_1} = -0\cdot3052$$

Since the change required in ΣA is $\triangle A = 1\cdot8602$ the necessary change $\triangle\varphi_1$, in φ_1, which would approximately achieve the required $\triangle A$, was evaluated from the above to be $\triangle\varphi_1 = -0\cdot1454$. Since this predicted change $\triangle\varphi_1$, appeared to be large, a smaller value $\triangle\varphi_1 = -0\cdot10$ was considered. With the new value of $\varphi_1 = 2\cdot0 - 0\cdot10 = 1\cdot9$ and $J = 0\cdot8$ the new values of ΣA, ΣB, ΣC, and ΣE, were calculated with the help of eqs (6). The values were found to be

New values with $\triangle\varphi_1 = -0\cdot10$	Required values
$\Sigma A = 1\cdot3120$	$\Sigma A = 2\cdot0$
$\Sigma B = -0\cdot3570$	$\Sigma B = -0\cdot3$
$\Sigma C = 0\cdot1893$	$\Sigma C = 0\cdot155$
$\Sigma E = -0\cdot1482$	$\Sigma E = -0\cdot18$

The changes in ΣB, ΣC, and ΣE, as can be seen from the above, resulting out of a change $\triangle\varphi_1$, were adjusted by slight changes in the shapes, with the help of eqs (10), (11) and (12).

With $\triangle\varphi_1 = -0\cdot10$ eqs (10) may be expressed numerically as

$$-1\cdot2810\, \triangle S_1 + 0\cdot7182\, \triangle S_3 = -0\cdot0343$$

$$0\cdot2443\, \triangle S_1 + 0\cdot2376\, \triangle S_3 = -0\cdot0318$$

The above yield

$$\triangle S_1 = -0{\cdot}0307, \qquad \triangle S_3 = -0{\cdot}1024$$

Substituting these values in eqs (11), written as

$$7{\cdot}2567 \ \triangle S_2 = 0{\cdot}3070$$

which gives $\triangle S_2 = 0{\cdot}0424$.

the new shapes are $S_1 = 0{\cdot}6453$, $S_2 = 0{\cdot}0397$ and $S_3 = -1{\cdot}1077$. ΣA, ΣB, ΣC, and ΣE, were evaluated with the above shapes in eqs (6).

Values obtained by the differential methods.	Required values
$\Sigma A = 1{\cdot}9448$	$\Sigma A = 2{\cdot}0$
$\Sigma B = -0{\cdot}2930$	$\Sigma B = -0{\cdot}3$
$\Sigma C = 0{\cdot}1567$	$\Sigma C = 0{\cdot}155$
$\Sigma E = -0{\cdot}1796$	$\Sigma E = -0{\cdot}18$

It may be seen from the above that the agreement between the required values and those obtained by the differential adjustment is quite close.

The value of ΣP which was required to be $0{\cdot}354$ was changed to $0{\cdot}409$ due to the change $\triangle \varphi_1 = -0.10$. The original value of ΣP may, however, be restored by differential changes in J given by (8). Adopting similar procedure as with φ_j differentials, followed by slight shape changes, adequate correction in ΣA, ΣB, ΣC, ΣE and ΣP may be achieved.

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