ONE OPERATIONAL AMPLIFIER SIMULATOR FOR THIRD ORDER SYSTEMS WITH A LEADING TIME CONSTANT

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ABSTRACT

The paper outlines a method for the simulation of third order linear systems with only one operational amplifier. A particular class of the general third order systems, that is, systems with a leading time constant is considered in this paper. A basic circuit consisting of one operational amplifier, four capacitors and five resistors is presented. The circuit is analyzed and the conditions of physical realisability discussed and obtained. The design formulae and procedure are also given.

INTRODUCTION

IN PREVIOUS communications[1—6] on this subject three particular classes of the general third order linear systems were considered for simulation with only one operational amplifier. The purpose of this paper is to consider another particular class of systems, that is, third order systems with a leading time-constant, which are characterised by a transfer function of the form

$$F(S) = -\frac{b_\circ (b_1 S + 1)}{a_3 S^3 + a_2 S^2 + a_1 S + 1} \quad .. \quad (1)$$

where a's and b's are positive and real constants ; and S is the Laplace operator.

In principle, it should be possible to simulate the system of (1) with the aid of three capacitors and six resistors but the resulting network design formulæ and the conditions of physical realisability become somewhat complicated. With the employment of four capacitors and five resistors, however, the design formulæ and the conditions of physical realisability become simple and conveniently computable. It is primarily with a view to ensuring simplicity and convenience that in the networks presented in this paper four capacitors and five resistors have been used.

Of the various possible circuits each employing four capacitors and five resistors only one will be presented here ; the design formulae and conditions of physical realisability will also be discussed and obtained.

THIRDORDER SYSTEM SIMULTATION

A network for the simulation of third order systems is shown in figure I and its transfer function has been shown[4] to be

$$\frac{E_\circ}{E_1} = -\frac{Y_1 Y_3 Y_5}{Y_6(Y_1+Y_2+Y_8)(Y_3+Y_4+Y_5+Y_7)+Y_3 Y_6(Y_4+Y_5+Y_7)+Y_5 Y_7(Y_1+Y_2+Y_3+Y_8)+Y_3 Y_5 Y_8} \quad (2)$$

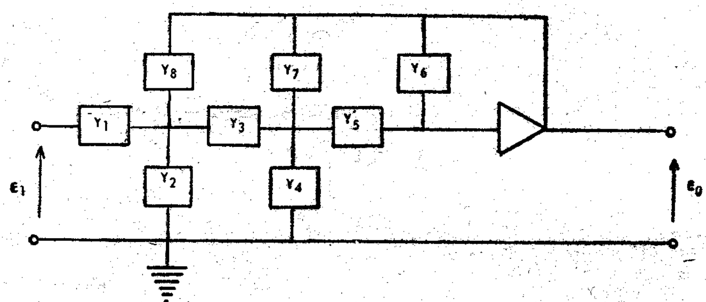

Fig. 1. Net work for the simulation of third order systems.

Simulation of the system of (1) with the network of figure 1 is possible if the admittances (Y's) are properly chosen ; and furthermore it should be obvious from (2) that at least three of the appropriate admittances will be required to be purely capacitative. As already mentioned the use of three capacitors gives inconveniently long design formulæ and conditions of physical realisability while the use of four capacitors makes these simple and easily computable.

A possible circuit for simulating the system of (1) is shown in figure 2, in which

$$\left.\begin{aligned} Y_3 &= \left(SC_3 + \frac{1}{R}\right) \\ Y_4 &= SC_4 \\ Y_6 &= SC_6 \\ Y_7 &= SC_7 \\ Y_1 &= Y_2 = Y_5 = \frac{1}{R} \\ Y_8 &= \frac{1}{aR} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

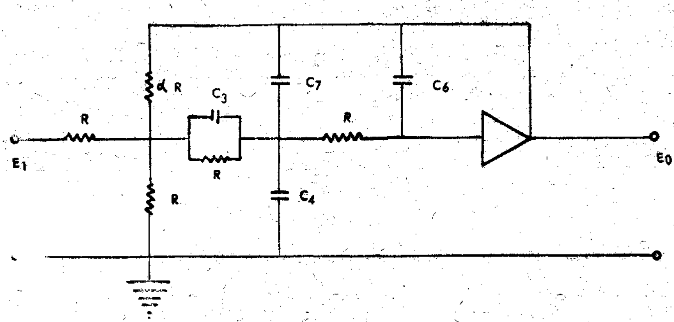

Fig 2. Net work for the simulation of $\frac{E_o}{E_1} = -\frac{b_o (b_1 S + 1)}{a_3 S^3 + a_2 S^2 + a_1 S + 1}$

Substituting (3) into (2) and s mplifying

$$\frac{E_o}{E_1} = \frac{\alpha(RC_3S+1)}{\alpha R^3(C_4+C_7)C_3C_6S^3+[(3\alpha+1)R^2C_3C_6+\alpha R^2C_3C_7+(3\alpha+1)R^2(C_4+C_7)C_6]S^2+[RC_3+(5\alpha+2)RC_6 +(3\alpha+1)RC_7]s+1)} \quad \ldots \quad (4)$$

Equations (1) and (4) will be identical i

$$b_o = \alpha \quad \ldots \quad (5)$$

$$b_1 = T_3 \quad \ldots \quad (6)$$

$$a_1 = T_3 + (5\alpha + 2)\,T_6 + (3\alpha + 1)\,T_7 \quad \ldots \quad (7)$$

$$a_2 = (3\alpha + 1)\,T_3\,T_6 + \alpha T_3\,T_7 + (3\alpha + 1)\,(T_4 + T_7)\,T_6 \quad \ldots \quad (8)$$

$$a_3 = \alpha\,(T_4 + T_7)\,T_3\,T_6 \quad \ldots \quad (9)$$

where

$$T_n = RC_n \quad (10)$$

Now, simulation of the system of (1) with the network of figure 2 is possible only if the values of α, T_3, T_4, T_6, T_7 obtained as the solution of (5) through (9) are real and positive. It is required to determine therefore, in terms of the given real and positive a's and b's va ues of α, T_3, T_4, T_6, T_7; and find the conditions, if any, under which these can be real and positive.

Elimination of α, T_3 and T_4 from (5), (6), (7) and (5), (6), (8), (9) give the following two linear simultaneous equations in T_6 and T_7

$$(5b_o + 2)\,T_6 + (3b_o + 1)\,T_7 = a_1 - b_1 \quad \ldots \quad (11)$$

and

$$b_o b_1^2\,(3b_o + 1)\,T_6 + b_o^2\,b_1^2\,T_7 = a_2\,b_o\,b_1 - a_3(3b_o + 1) \quad \ldots \quad (12)$$

The solution of (11) and (12) gives

$$T_6 = \frac{(3b_o + 1)\{a_2 b_o b_1 - a_3 (3b_o + 1)\} - b_o^2 b_1^2 (a_1 - b_1)}{b_o b_1^2 (2b_o + 1)^2} \quad \ldots \quad \ldots \quad (13)$$

$$T_7 = \frac{b_o b_1^2 (3b_o + 1)(a_1 - b_1) - (5b_o + 2)\{ b_o \quad - a_3 (3b_o + 1)\}}{b_o b_1^2 (2b_o + 1)^2} \quad (14)$$

It is evident f om (13) and (14) that T_6 and T_7 are real; and these will be also positive, if

$$(3b_o + 1)\left\{\frac{a_3(5b_o + 2) + b_o b_1^2 (a_1 - b_1)}{b_o b_1 (5b_o + 2)}\right\} > a_2 > \left\{\frac{a_3(3b_o+1)^2 + b_o^2 b_1^2 (a_1 - b_1)}{b_o b_1 (3b_o + 1)}\right\} \quad (15)$$

Therefore, if (15) is satisfied then T_6 and T_7 will be real and positive; a and T_3 are directly known to be real and positive from (5) and (6), but it is not quite certain, from what has been stated already, if the corresponding T_4 will be also real and positive. Elimination of a, T_3 and T_6 from (5), (6), (7) and (9) gives

$$T_4 = \frac{a_3}{b_o b_1 T_6} + \frac{(5b_o + 2)}{(3b_o + 1)} T_6 - \frac{(a_1 - b_1)}{(3b_o + 1)} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

from which it is evident that for all T_6 lying between zero and infinity the corresponding T_4 is positive and real i

$$a_3 > \frac{b_o b_1 (a_1 - b_1)^2}{4(3b_o + 1)(5b_o + 2)} \quad \ldots \quad \ldots \quad (17)$$

Therefore, a set of real and positive a, T_3, T_4, T_6 and T_7 exists if the inequalities of (15) and (17) are satisfied.

For the design of the network circuit component values are required to be determined. The proper procedure for design would be to first check and see if the inequalities of (15) and (17) are satifised. The satisfaction of these conditions signifies that the circuit of figure 2 for simulating the system of (1) is physically realisable. The circuit component values may be then obtained with the aid of (13), (14), (5), (6) and (9). Having thus determined a, T_3, T_4, T_6, T_7; and choosing arbitrarily a convenient value for any one of the capacitors the values of the resistors and the remaining capacitors may be then determined with the aid of (10).

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