# " A MATHEMATICAL NOTE ON RADIOIODINE UPTAKE AND EXCRETION FOR THE SYSTEM OF A DYNAMIC ORGANISM"

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ABSTRACT

In this note the uptake of radioiodine in the thyroid and excretion through urine have been investigated. The technique employed here is of plasma concentration in the blood. Two equations have been obtained one for the quantity of iodine concentrated in the thyroid after a time '$t$' and the other for the concentration of iodine in the kidneys after a time '$t$', which give the index of thyroid activity.

## INTRODUCTION

CONSIDERABLE use has been made of radioactive iodine to study the metabolism of small amounts of this element in the body. The iodine metabolism is usually given in the form of uptake (or excretion) curves, where the percentage fraction of the initial dose taken up by the thyroid (or excreted through kidneys) is plotted against time after the test dose has been administered. The general shape of these curves depends largely on the condition of he thyroid. A study of the radioiodine uptake by the thyroid gland (estimated generally from measurement over the gland with a suitable counter) is a valuable aid in the diagnosis of the state of the thyroid. Larger doses of Iodine—131 are given therapeutically for certain types of disorders like hyperthyroidism, some cases of intractable angina etc. Here an attempt has been made to calculate the concentration of I—131 in thyroid and excretion through urine by plasma concentration method.

## MATHEMATICAL CALCULATIONS

The uptake of radioiodine by the thyroid and its elimination have been mathematically derived in the present note under the following assumptions:—

(1) The inorganic iodine is uniformly distributed throughout the plasma volume in the initial stages.

(2) The iodine is removed from the blood in the inorganic form by the thyroid gland.

(3) The reverse process of exchange of iodine from the thyroid to the blood takes place at a rather slower rate and can be neglected.

(4) Both the thyroid and kidneys reduce the plasma concentration by a constant fraction per unit time, that is the rate of removal of iodine from the plasma by each organ is proportional to the plasma concentration.

(5) The elimination of radioiodine via other means (*e.g.* through faeces and sweat) is negligible compared to that through the kidneys.

The rate of decrease of plasma concentration, may be expressed by the equation

$$-\frac{dp}{dt} = ( K_{th} + K_r + \lambda_r ) \, p \quad \ldots \qquad (1)$$

where

$p$ =plasma concentration

$K_{th}$ =thyroid accumulation rate constant

$K_r$ =renal accumulation rate constant

$\lambda_r$ =radio=active decay constant

Integration of equation (1) gives

$$p = p_\circ e^{-(K_{th} + K_r + \lambda_r)t} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where $p_\circ$ =a constant.

The quantity of iodine concentrated in the thyroid and the kidneys after a time '$t$' denoted by $(Q_t)_{th}$ and $(Q_t)_k$ respectively can be obtained from the equation—

$$\left(\frac{dQ_t}{dt}\right)_{th} = (vp)K_{th}$$

where $v$=plasma volume

By putting the value of $p$ from equation (2) we get

$$\left(\frac{dQ_t}{dt}\right)_{th} = v\, p_\circ K_{th}\, e^{-(K_{th} + K_r + \lambda_r)t}$$

$$= Q_\circ K_{th}\, e^{-(K_{th} + K_r + \lambda_r)t} \quad \ldots \quad \ldots \quad (3)$$

where $Q_\circ$ represents the test dose.

Similarly,

$$\left(\frac{dQ_t}{dt}\right)_k = v\, p_\circ K_r\, e^{-(K_{th} + K_r + \lambda_r)t}$$

$$= Q_\circ K_r\, e^{-(K_{th} + K_r + \lambda_r)} \quad \ldots \quad \ldots \quad (4)$$

By integrating equation (3) we have

$$(Q_t)_{th} = \frac{Q_\circ K_{th}}{K_{th} + K_r + \lambda_r}\left(1 - e^{-(K_{th} + K_r + \lambda_r)}\right)$$

$$\text{or } \left(\frac{Q_t}{Q_\circ}\right)_{th} = \frac{K_{th}}{K_{th} + K_r + \lambda_r}\left(1 - e^{-(K_{th} + K_r + \lambda_r)}\right) \quad \ldots \quad \ldots \quad (5)$$

Similarly on integration equation (4) gives

$$\left(\frac{Q_t}{Q_\circ \delta}\right)_k = \frac{K_r}{K_{th} + K_r + \lambda_r}\left(1 - e^{-(K_{th} + K_r + \lambda_r)}\right) \quad .. \quad (6)$$

If $\left(Q_t \; \delta / Q\delta_\circ\right)_{th}$ and $\left(Q_t \; \delta / Q_\circ\right)_k$ are determined at any particular time '$t$', by actual measurement, then $K_{th}$ and $K_r$ may be calculated from equation (5) and (6). This will enable $(Q_t \;)_{th}$ and $(Q_t \;)_k$ to be estimated for any arbitrary time $t$.

## CONCLUSION

In these calculations, the conversion of inorganic iodine to protein bound iodine and the release of the latter into the blood stream has been neglected. It is therefore a better approximation in the case of normal than of thyrotoxic cases. However, the calculation of thyroid accumulation rate constant on the basis of this calculation gives a useful index of the thyroid activity. It may be noted at a definite time the radioiodine in either the thyroid or the urine depends upon both $K_{th}$ and $K_r$ i.e. it depends upon the function of the thyroid and the kidneys.

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