APPLICATION OF THE DISTRIBUTION OF THE SUM OF r INDEPENDENT TRUNCATED POISSON VARIATES IN QUALITY CONTROL

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ABSTRACT

This paper deals with the application of the sum of 'r' independent truncated poisson variates, in fixing the no. of defects on each of the components in a set with certain probability, when the total no. of allowable defects in a set is known, so as to avoid wastage of finished product. This also warrants no further inspection of the finished product. To meet this end a few tables are prepared and appended.

INTRODUCTION

IN THE days of automation and complex mechanism, it is sometimes more convenient and less costly to study the characteristics of the variates, say no. of defects in a finished product, jointly rather than to examine them individually. For illustration let us take a simple case where two or three parts of a machine tool are mounted together and brought before the inspector in a finished form. In such situations it may not be feasible to count the number of defects on each part separately and therefore we may be interested in the total number of defects in the mounted part. There may be another situation also where in it is not possible to count the number of defects in the end. In such situations we have to put a restriction on the number of defects on each of the parts so that the number of defects on all the parts together will, be less than a fixed number with reject percentage less than a certain amount.

Let a population of members exists of which the variate is the number of defects, as is well known, following a Poisson law with the known parameter. The members of small samples from independent sources (*e.g.* no. of wrong threads in bolts and in nuts) are to be used together in such a way that the total number of defects in a set (*i.e.* a bolt and a nut) is less than a preassigned number. The question is at what value of the variate should the original population be cut in order that when members are taken from the truncated population, the desired result is achieved with an allowable probability of failure without any further inspection. With the view to solve this question, this paper is devoted to the distribution of the sum of two and three truncated poisson variates and the tabes are prepared, though not exhaustive, giving various values of truncation points for different values of the parameter and the probability associated with each of the different sets. Such a problem has been considered by Francis [1] for a normal distribution.

MATHEMATICAL MODEL

The probability density function of the Poisson Distribution is

$$f(x) = \frac{e^{-m}\, m^x}{x_!} \qquad o \leqslant x \leqslant \infty \qquad \ldots \qquad \ldots \quad (1)$$

where m is the parameter.

If x is less than or equal to the finite number c, then the distribution (1) assumes the form

$$f_1(x) = \begin{cases} \lambda\, f(x) & o \leqslant x \leqslant c \\ o & x > c \end{cases} \qquad \ldots \qquad \ldots \quad (2)$$

where λ is given by

$$\frac{1}{\lambda} = \sum_{x=o}^{c} f(x)$$

The density function of the sum of r variates say,

$z_r = x_1 + x_2 + x_3 + \dots + x_r$ is given by

$$f(z_r) = \lambda^r e^{-rm} m^{zr} \sum_{z_{r-1}=o}^{(r-1)c} \dots \sum_{z_2=o}^{2c} \sum_{z_1=o}^{c} \prod_{J=o}^{r-1} \frac{1}{(z_{J+1} - z_J)!} \qquad \dots (3)$$

$$o \leqslant z_r \leqslant rc$$

where $z_i = x_i + x_{i-1} + \dots + x_1$; $i = 1, 2, 3, \dots \gamma$

and $z_o = o$

and z_i takes values $o, 1, 2, 3, \dots$ ic or z_{i+1} whichever is small

for $i = 1, 2, \dots (r-1)$.

In case of two variates only, ; the distribution (3) takes the form

$$f(z_2) = \frac{\lambda^2 e^{-2m} m^{z_2}}{z_2!} \left[2^{z_2} - \sum_{J=1}^{z_2-c} \binom{z_2}{J-1} \right] \qquad \dots \quad \dots (4)$$

$$o \leqslant z_2 \leqslant 2c$$

and for the sum of three variates the distribution is

$$f(z_3) = \lambda^3 e^{-3m} m^{z_3} \sum_{z_2=o}^{2c} \sum_{z_1=o}^{c} \frac{1}{z_3 - z_2!\, z_2 - z_1!\, z_1!} \qquad \dots (5)$$

$$o \leqslant z_3 \leqslant 3c$$

Defining commulative probability as usual by $F(z)$, the probability of rejection is given by

$$1 - F_{z_s}(z_r) = 1 - \sum_{z_r=o}^{z_s} f(z_r) \qquad \dots \quad \dots \quad \dots \quad \dots (6)$$

where $z = 0, 1, 2, \dots, k; k < rc$

for $r=2$ and 3, the values of $1-F_{zs}(z_r)^2$ are tabulated for different values of c, z_s and the parameter m by using Kitagawa tables and appended at the end.

These tables can be used in controlling the production of bad quality by putting the restriction at each stage of the production mechanism. Thus it would be of great benefit in view of the economy and time saving. In the succeeding section a hypothetical example is discussed and the light is thrown on how the method developed in this paper may be of use in the age of automatic production on large scale with desired level of probability of failure.

NUMERICAL EXAMPLE

Consider a problem of two components a bolt and a nut whose threads are cut by two different dies, which are finally to be assembled to form a set. Suppose the mean value of the number of defects (*i.e.* the no. of wrong threads) in either of the two components is 1·5. The producer's condition in view of its consumption is that the total number of defects in the set should not exceed 3 in at least 95 cases out of 100. What will be the maximum number of defects allowable in each component of the set? From table I we find that this number is 2. This means that the production mechanism should be set in such a way that in each part or component, the number of defects does not exceed 2 and once it is set in this way no further inspection is warranted.

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SUM OF TWO VARIATES

TABLE I

Values of $1-Fz_s\ (z_2)$ for different m, z and c.

m	c	z_s	$1-Fz_s\ (z_2)$
1	2	3	4
0·5	1	1	0·11
	2	2	0·05
1·0	1	1	0·50
	2	2	0·20
	—	3	0·01
1·5	2	2	0·35
	—	3	0·05
	3	3	0·26
	—	4	0·08
2·0	2	2	0·48
	—	3	0.11
	3	4	0·16
	4	4	0.30
	—	5	0·13
	—	6	0·03
2·5	3	3	0·54
	—	4	0·25
	—	5	0·01
	4	4	0·43
	—	5	0·23
	—	6	0·07
	5	5	0·33
	—	6	0·17
	—	7	0·07
3	3	3	0·64
	—	4	0·34
	—	5	0·18
	4	4	0·57
	—	5	0·34
	—	6	0·12
	5	5	0·47
	—	6	0·28
	—	7	0·13
	—	8	0·10
	6	6	0·35
	—	7	0·20
	—	8	0·10
	—	9	0·03
3·5	4	4	0·67
	—	5	0·44
	—	6	0·18
	5	5	0·59
	—	6	0·39
	—	7	0·20
	—	8	0·05
	6	6	0·49
	—	7	0·32
	—	8	0·17
	—	9	0·01
	7	7	0·37
	—	8	0·23
	—	9	0·13
	—	10	0·06

m	c	z_s	$1-Fz_s\ (z_2)$
1	2	3	4
4	4	4	0·75
	—	5	0·52
	—	6	0·25
	5	5	0·69
	—	6	0·49
	—	7	0·28
	—	8	0·09
	6	6	0·60
	—	7	0·43
	—	8	0·26
	—	9	0·12
	—	10	0·01
	7	7	0·50
	—	8	0·34
	—	9	0·21
	—	10	0·10
	—	11	0·03
	8	8	0·38
	—	9	0·25
	—	10	0·15
	—	11	0·08
	—	12	0·03

TABLE II

SUM OF THREE VARIATES

Values of 1-F_{z_s} (z_3) for different m, z_s and c

m	c	z_s	1-F_{z_s} (z_3)	m	c	z_s	1-F_{z_s} (z_3)
1	2	3	4	1	2	3	4
0·5	1	1	0·26	3·5	4	4	0·94
	—	2	0·07		—	6	0·74
	2	2	0·16		—	8	0·30
	—	3	0·03		5	5	0·92
1·0	1	1	0·50		—	7	0·72
	—	2	0·19		—	9	0·38
	2	2	0·46		—	11	0·02
	—	3	0·18		6	6	0·88
1·5	2	2	0·67		—	8	0·64
	—	4	0·05		—	10	0·46
	3	3	0·58		—	12	0·10
	—	5	0·15		7	7	0·81
	—	6	0·01		—	9	0·66
2·0	2	2	0·80		—	11	0·39
	—	4	0·14		—	13	0·19
	3	3	0·76	4	4	4	0·97
	—	5	0·31		—	6	0·82
	—	6	0·08		—	8	0·41
	4	4	0·66		5	5	0·96
	—	6	0·30		—	7	0·82
	—	8	0·03		—	9	0·51
2·5	3	3	0·86		—	11	0·13
	—	5	0·46		6	6	0·93
	—	6	0·13		—	8	0·78
	4	4	0·81		—	10	0·52
	—	6	0·47		—	13	0·22
	—	8	0·10		7	7	0·90
	4	5	0·73		—	9	0·72
	—	7	0·51		—	11	0·47
	—	9	0·24		—	13	0·23
3	3	3	0·92		—	15	0·04
	—	5	0·59		8	8	0·83
	—	6	0·29		—	9	0·74
	4	4	0·90		—	10	0·64
	—	6	0·22		—	12	0·40
	—	8	0·19		—	14	0·20
	5	5	0·85				
	—	7	0·56				
	—	9	0·25				
	6	6	0·77				
	—	8	0·50				
	—	10	0·22				
	—	12	0·05				