GAMMA-RAY DOSE RATE AT AN EXTERNAL POINT FROM A SPHERICAL RADIOACTIVE SOURCE

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ABSTRACT

In this paper a formula for "Gamma-Ray Dose Rate from a Spherical Radioactive Source" has been obtained at an external point. To simplify the problem, factors such as self-absorption and multiple scattering inside the source have not been taken into consideration. For facilitating measurements in particular cases the distance of the point of interest is taken from the surface of the source. Some uses of the formula have also been discussed in medical and other fields.

Introduction

The gamma-ray dose rate at a point inside a spherical radioactive source has already been calculated by several workers ([1], [2]). In general the flux of gamma radiation at any point from a radiation source depends on the geometry of the source, self-absorption and multiple scattering of gamma-rays within the source. In this paper the gamma-ray dose rate at an external point due to a spherical source has been evaluated by neglecting self-absorption and multiple scattering. In order to avoid practical difficulty encountered in measuring the distance from the centre of the source, here we have considered the distance of the point of interest from the surface of the source.

No organs or parts of the body have exactly the shape of a sphere. However for gamma-ray dose rate calculations, the masses containing radioactive materials can often be approximated in terms of spheres. Therefore, this particular case, may have practical uses in the medical and other fields for calculation of doses.

Derivation of Dose Rate

It is known that for a gamma point source of 1 mc. which emits one gamma photon of energy E Mev. per disintegration, the rate of energy absorption, Γ, in a small volume of air at a distance of 1 cm. is equal to the production probability of secondary electrons (which is given by the true linear gamma-ray absorption coefficient in air, μ_{air}) times the energy flux F (Mev/cm^2-mc-hr at 1 cm.). Taking the value of the average energy required to produce one ion pair in air to be 34* ev. 1 r is equivalent to $7 \cdot 08 \times 10^4$ mev per cm^3 of air at N.T.P, we get

$$\Gamma = F \times \mu_{air} \quad \ldots \quad (1)$$

$$= 1 \cdot 50 \times 10^5 E.\ \mu_{air}\ r/mc - hr$$

*The average energy absorbed per ion pair is almost constant and probably lies between 33 & 35 ev per ion pair for air. Here we have taken the value recommended by ICRU([3]).

The absorbed energy rate Γ is generally called 'Point Source Gamma-Ray Dose Rate Constant' and its units are often expressed as r/mc-hr at a distance of 1 cm. in air.

Under the conditions of validity of Γ, for which self-absorption and multiple scattering of gamma radiations is neglected, the gamma dose rate, D, at a distance d_1 cm. from a point source of C mc. is given by the following expression

$$D = \frac{\Gamma C}{d_1^2} \ \text{r/hr} \qquad (2)$$

Let us now consider the gamma-ray dose rate from a sphere of radius 'a' containing C mc. activity uniformly distributed throughout its volume. The gamma-ray dose rate at a point P due to an infinitesimal volume element $r^2 \sin\theta \, d\theta d\phi dr$ of the sphere at a distance d cm. from its surface is given by

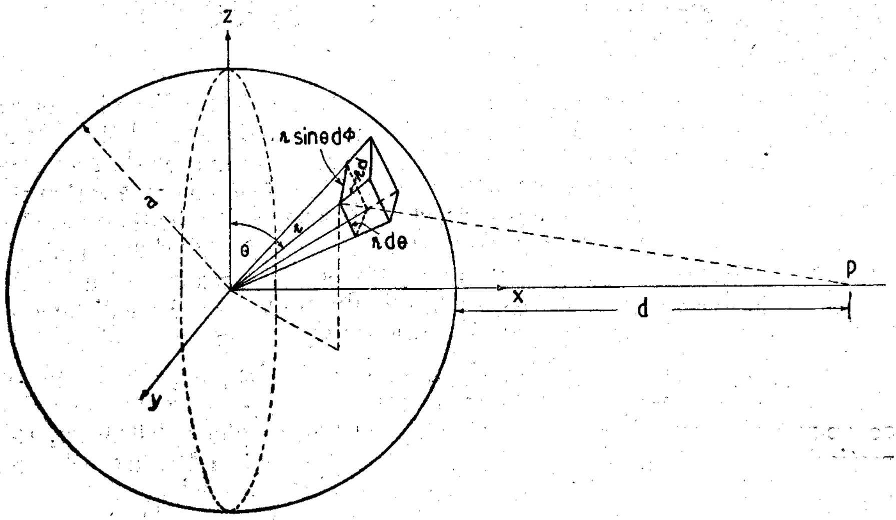

FIG. 1

$$dD_p = \Gamma \frac{C}{V} \frac{r^2 \sin\theta \, d\theta \, d\phi \, dr}{\{ r^2 + (a+d)^2 - 2r(a+d) \cos\theta \}} \qquad (3)$$

where V is the volume of the spherical source.

Integrating equation (3) we get,

$$D_p = \Gamma \frac{C}{V} \int_{r=0}^{r=a} \int_{\theta=0}^{\theta=\pi} \int_{\phi=0}^{\phi=2\pi} \frac{r^2 \sin\theta \, dr \, d\theta \, d\phi}{\{ r^2 + (a+d)^2 - 2r(a+d) \cos\theta \}} \qquad (4)$$

where D_p is the gamma-ray dose rate at P due to the whole sphere. Therefore,

$$D_p = \frac{\pi \Gamma C}{V} \left\{ 2a - \frac{d(2a+d)}{a+d} \log_e \left(\frac{2a+d}{d} \right) \right\} \text{ r/hr} \quad .. \quad (5)$$

Simplifying the equation (5) we have

$$D' = \tfrac{3}{4}\, \Gamma C \left\{ \frac{2}{a^2} - \frac{d(2a+d)}{a^3(a+d)} \log_e \left(\frac{2a+d}{d} \right) \right\} \text{ r/hr} \quad .. \quad (6)$$

By equation (6) the gamma-ray dose rate at a point outside the radioactive source can be easily calculated.

Discussion

In order to study the variation of gamma-ray dose rate with distance the value of D'_p $(=D_p / \tfrac{3}{4}\Gamma C)$ are calculated for different values of d; taking the radius of spherical source as 3 cm. and, ΓC, as a constant, as shown in Table 1.

TABLE 1

d in cm.	D'_p
0	0·222
0·5	0·134
1·0	0·096
1·5	0·073
2·0	0·058
2·5	0·047
3·0	0·039
4·0	0·028
5·0	0·022
8·0	0·011
10·0	0·008

As it is evident from the curve of fig. 2, the dose rate falls very rapidly in the beginning and becomes asymptotic later with respect to the abscissa.

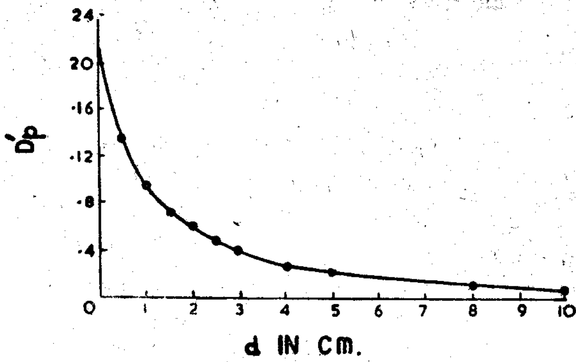

FIG. 2

It is of interest to compare the gamma-ray dose rates due to the spherical source and the point source having the same values of Γ C. In table 2 comparison of dose rates and the percentage errors have been tabulated for different values of *a*.

TABLE 2

d_1 (=d+a) cm.	Dp/D a=3cm.	Dp/D a=5cm.	% error a=3cm.	% error a=5 cm.
3	1·500	..	50·0	..
4	1·153	..	15·3	..
5	1·086	1·500	8·6	50·0
6	1·056	1·210	5·6	21·0
7	1·040	1·134	4·0	13·4
8	1·029	1·095	2·9	9·5
9	1·026	1·072	2·6	7·2
10	1·021	1·056	2·1	5·6
12	1·014	1·038	1·4	3·8
15	1·008	1·023	0·8	2·3
20	1·000	1·013	<0·1	1·3
25	..	1·008	..	0·8
30	..	1·005	..	0·5
40	..	1·000	..	<0·1

The above values have been plotted in fig. 3 and from the nature of the curves one can conclude that for distances greater than seven to eight times

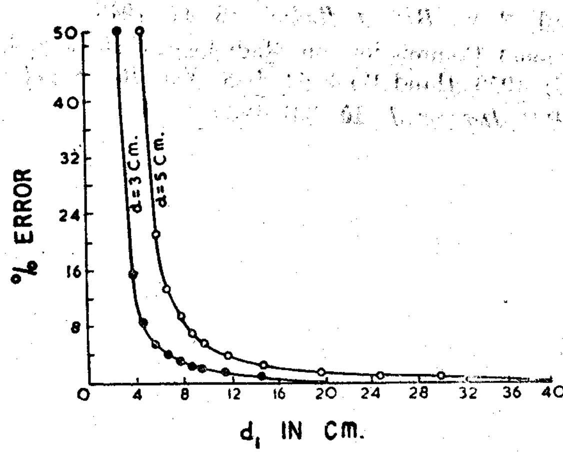

FIG. 3

the radius of the spherical sources one is justified to consider the spherical source as a point source.

The above formula can be used for determining the gamma-ray dose rate from internally distributed radioisotopes in tissues (approximated spheres) in many biological and medical problems.

Complete dosage calculations should always be carried out prior to using an internal emitter in patients in order to determine whether the incidental exposure to them is hazardous. This formula limits the quantity of dose to be given to a patient for treatment to protect other organs near the 'critical organ' in which the radioactive medicine is going to be accumulated.

Help of this formula may also be taken in shielding problems, for example, the distances for 'permissible dose'[4] can be calculated for the personnel attending the patient to whom the activity of the order of milli-curies is given.

The required doses to various plants in 'Gamma Garden', having a spherical source, may also be theoretically evaluated by this expression.

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