CONCEPT OF TEMPERATURE IN PLASMA AND ITS MEASUREMENT

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ABSTRACT

This paper reviews the concept of temperature, and its measurement *in general* and *in particular* the concept of temperature in a 'plasma' and the measurement of electron temperature and ion temperature.

Introduction

The concept of temperature and a scale to measure it arose from the basic idea to give a precise 'quantitative form to the qualitative result described from the physiological perception of heat or cold'. The experiments which established the nature of the temperature concept were first carried out on gases because of their simplicity and convenience. Any state of a gas in which the pressure and volume remain unaltered as long as external conditions are the same is called an equilibrium state. The law which establishes a relation between thermal equilibria of different systems is known as the "Zeroth law of thermodynamics" which can be stated as follows : "If two systems are in thermal equilibrium with a third separately, the both of them are in thermal equilibrium with each other". From this it can be inferred that for each system there exists a function of parameters describing an equilibrium state which can be named as "temperature" such that the equality of this factor for the two systems forms the condition for thermal equilibrium. This can be written mathematically as

$$f\,(P_A\,,\,V_A) = f\,(P_B\,,\,V_B) = T \qquad \ldots \qquad \ldots \qquad (1)$$

where P_A , V_A and P_B , V_B represent the pressure and volume respectively of the two systems A and B, which are in Thermal equilibrium and T is known as the *Temperature.* On the basis of this, Kelvin[1] developed the Thermodynamic scale or otherwise known as Kelvin's scale of temperature by applying the second law of thermodynamics to a reversible cycle for an ideal Carnot engine. Accordingly we get

$$\frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = f\,(T_1 - T_2) \qquad \ldots \qquad (2)$$

where W is the work performed, Q_1 is the source of heat at high temperature T_1 and Q_2 is the heat given out at low temperature T_2. From this it is easy to obtain the relation.

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (3)$$

which defines the thermodynamic scale of temperature.

The temperature of a gas can also be viewed from the Kinetic theory of gases where the gas molecules are supposed to move at random colliding elastically. Considering the momentum and transfer of energy due to collisions, we may write down the pressure exerted by a mole of an ideal gas as

$$p = \frac{N m \overline{c^2}}{3V} = \frac{2E_k}{V} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where N is the Avogodro's number, m the mass of the molecule, $\overline{c^2}$ the mean square of the molecular velocity, E_k the average *K.E.* of the molecules per mole. Since E_k increases with increase in temperature (T) we may write down

$$E_k = f(T) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

For an ideal gas, $PV = nRT$ and hence $E_k = 3/2\ RT$. Thus the temperature defined in this way for a perfect gas is proportional to the translational *K.E.* of the gas molecule.

The temperature of a gas can also be understood from the point of view of statistical mechanics. An equilibrium distribution of particles over a series of non-degenerate energy levels is given by the Boltzman distribution function

$$n_i = n_j \ e^{-E_{ij} / KT} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

where n_i is the average no. of molecules in the i^{th} state and n_j the average no. of molecules in the j^{th} state, E_{ij} the energy difference between the two states, K the Boltzman constant and T the Kelvin Temperature.

But in any system where the Boltzman distribution does not prevail spontaneous changes towards Boltzman type are observed. Typical cases of the same have been discussed by Herzfeld[2]. He points out that all the temperatures actually measured will be only "effective temperatures".

Concept of temperature in a Plasma

Now in an electrical discharge in a monoatomic gas we can conceive of four different kinds of temperature[3] viz.,

(*i*) the electron temperature which gives the Kinetic energy of the electrons;

(*ii*) the gas temperature which gives the Kinetic energy of the atoms;

(*iii*) the excitation temperature which determines the population of the different excited states;

and (*iv*) the ionisation temperature which gives the number of electrons per cm^3.

At low pressures and currents the electron temperature is high and the gas temperature is low while with the increase in current density and pressure the electron temperature drops and the other temperature rises, until all the temperatures approach the temperature of thermal equilibrium.

With the interesting picture of the property of an electric discharge given above, we can now make an attempt to understand what is meant by "Temperature in a Plasma". For this it is necessary to have a picture of "Plasma".

A plasma can be considered in two different ways: macroscopically and microscopically.

(*i*) In the former case it can be supposed to be a mixture of two compressible fluids of opposite signs. When a current flows through, these fluids stream in opposite directions. Their motion can then be represented by the usual hydrodynamic equations of force and continuity for a charged fluid streaming in electric and magnetic fields with the neutral gas providing the necessary continuous frictional force.

(*ii*) In the second case the plasma can be treated by the usual laws of gas Kinetics, as an assembly of small particles of three kinds—positive, negative and neutral—moving at random and colliding with each other. Except in very high current discharges, in all other types the number of neutral particles is larger than the charged one and the equilibrium of the latter is maintained by collisions with the neutral particles rather than with each other. In an electric field, there is superimposed upon this random motion, a relatively small drift in velocity. Then a new equilibrium is set up at a higher average energy so that it becomes possible for the three groups to exist in equilibrium at three different temperatures. Thus there can be three different types of temperatures for the plasma: viz., (1) the electron temperature, (2) the ion temperature & (3) the gas temperature.

Measurement of electron temperature

Assuming that the plasma is homogeneous, the atoms of the gas will be under equilibrium obeying Maxwell-Boltzman distribution law. The number per unit volume having energy E_n are given by

$$N_n = c \cdot g_n \cdot e^{-E_n / KT} \quad \cdots \quad (7)$$

where g_n is the degeneracy of the state and c is a normalising constant.

The total number of atoms per unit volume N, can then be written as

$$N = c \, \Sigma_n \, g_n \, e^{-E_n / KT} \quad \cdots \quad (8)$$

Now light is emitted or absorbed by an atom where there is a transition between levels $E_{n'}$ to $E_{n''}$ and the intensity of the spectral line associated with is given by

$$I_{n'n''} = c \cdot A_{n'n''} \cdot e^{-E_{n'}/KT} \quad \cdots \quad (9)$$

where $A_{n'n''}$ is called the 'transition probability'. If two lines are emitted by different levels $E_{m'}$ and $E_{n'}$, then the ratio of their intensities is given by

$$\frac{I_m}{I_n} = \frac{I_{m'm''}}{I_{n'n''}} = \frac{A_{m'm''}}{A_{n'n''}} \cdot e^{-(E_{m'} - E_{n'})/KT} \quad \cdots \quad (10)$$

$$\text{or } (\log I_n - \log I_n) = (\log A_{m'm''} - \log A_{n'n''}) - \left(\frac{E_{m'} - E_{n'}}{KT} \right) \cdots \quad (11)$$

Using equations (9) and (11) the electron temperature can be determined, by measuring the intensity of the spectral lines.

The electron temperature T_e can also be determined by the measurements from the "Bremstrahlung" i.e. the radiation emitted by charged particles, mainly the electrons. The energy radiated can be written as

$$E_\nu \, d\nu = C N_1 N_2 Z^2 T^{e^{-\frac{1}{2}}} \exp - \left(\frac{h\nu}{KT_e} \right) d\nu \quad .. \quad (12)$$

Where C is a constant, ν the frequency, N_1 and N_2 are the densities of ions and electrons, Z the ionic charge. The electron temperature can then be found by measuring the absolute amount of energy radiated in a known energy interval or by measuring the variation of energy radiated as a function of energy. Since the second method involves only relative intensity measurements, it is likely to be more reliable. It is to be noted here that in order to measure temperatures of the order of 10^6 °K, it is necessary to make the measurements in a spectral region where the exponential term varies significantly. This falls in the X-ray region of a Kev energy and far vacuum ultra-violet (about 100—1000 A°).

Measurement of ion temperature

The ion temperature which represents the mean K.E. of the ions at a given time and position can be measured by studying the broadening of spectral lines due to Doppler effect. The line half width due to this broadening is proportional to $\sqrt{T/m}$ where m is the atomic weight of the element. The spectral lines are also broadened due to other causes like collisions and internal stark effect etc. Therefore, before making use of this technique, a careful analysis is to be made to determine whether the broadening is due to causes other than Doppler broadening. The most favourable case for this method is when the gas is at a low pressure and the temperature is high and above all a fairly light element is used.

Another method which does not involve the optical measurement is to determine the degree of ionization in a gas. This is a function of temperature and the ionization equilibrium is described by the Saha equation

$$\frac{N_{r+1}}{N_r} \cdot P_e = \frac{2g_{r+1}}{2g_r} \cdot \frac{(2\pi m)^{3/2} \cdot (KT)^{5/2}}{h^3} \cdot e^{-\frac{E_i}{KT}} \quad (13)$$

where N_r and N_{r+1}, are the number of atoms in the ground level of the rth and $(r+1)th$ states of ionization; P_e is the electron pressure which is given by $P_e = n_e KT$, where n_e is the number of electrons per unit volume. In a pure gas, P_e depends on the gas itself. Therefore the ionization measurement can be used for temperature measurement if the electron pressure is known.

Conclusion

If, however, we assume that there is an equilibrium between the three types of particles present in the plasma, we can conceive of an equilibrium temperature for the plasma. Then by using the Holtsmark[4] theory of stark effect broadening by an electric field, the ion concentration in a discharging plasma could be obtained. The Saha equation can then be used to obtain the ion and atom concentrations as a function of temperature. This way the plasma temperature is directly related to the spectral line width. Good agreement has been obtained between temperatures obtained for H_β and H_γ lines see Dickerman [5].

All the above mentioned methods have been developed assuming that there exists a thermal equilibrium between the various particles in the plasma. However, it is very doubtful whether Maxwell-Boltzman distribution is strictly obeyed by these different particles in the plasma. Then to speak of the "*Temperature of a plasma*" would be meaningless. The value obtained in such cases largely depends upon the method adopted.

If we can therefore make investigation in a non-homogeneous medium where it is impossible to define temperature, we may expect to know much more about the different processes in the plasma. Since the equilibrium distribution is established by collisions, we may expect this in-homogeneity of the fluid at low pressures where the collisions are far too few and the observation have to be made within a very short time after the reaction has taken place.

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References

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