ON THE APPLICATION OF PROBIT ANALYSIS IN VISUAL PERCEPTION EXPERIMENTS

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ABSTRACT

This paper reports how the valid Statistical analysis of the data on visual perception can be made with advantage by the methods of probit analysis.

INTRODUCTION

IN THIS paper we have discussed how the method of probits could be used in the statistical analysis of data obtained from an experiment on visual perception. The scope of the method is not limited to the type of experiment discussed in this paper. Its application extends to a class of visual experiments where repeated presentations of a few stimulus magnitudes are made to an observer who indicates discrimination by correctly identifying a variable attribute of stimulus, say its spatial location or its time of occurrence. By stimulus magnitude we mean some measure of the signal presented to the observer which may be a physical variable such as the time of occurrence, the linear or angular size of the object, the magnitude of the contrast between the object and the background etc. The percentage of correct detections of a signal by the observer is generally a function of the magnitude of the signal, known as a psychophysical function and for a number of visual experiments the cumulative normal curve approximately represents this function. The estimation of the parametres of such a curve is best made by the methods of probit analysis. The percentage of correct detections for varying magnitudes of the signal provides us the necessary data to estimate the threshold values of the signal for visual perception. Recently Bhatia and Verghese[1] of the Physiology Division carried out an experiment for estimating and comparing the fifty percent threshold linear (angular) size of a moving object for visual detection keeping linear (angular) speed of the object and the linear (angular) size of the slit constant. The data obtained by them have been used in this paper to illustrate the method. It may also be mentioned that the method discussed here is equally applicable in psychoacoustic data where we are concerned with the detectability of auditory stimuli.

EXPERIMENTAL DATA

In this section we shall give a brief outline of the experiment and the data used in the analysis.

On a drum with a white background and capable of rotating at different speeds there were four spaces some of which were left blank and each of the rest contained an object of given size. Since the identical objects were placed in one two or three of the spaces the total number of possible combinations was 14 ($= 4C_1 + 4_1C_2 + 4C_3$). The experiment was conducted on two subjects. The fourteen arrangements were presented at random to each of the subjects to view through a slit in front of the drum. The subjects were asked to identify the objects as distinguished from the background by a suitably chosen contrast, rotating the drum at different speeds. Such trials were conducted for six sizes and different speeds of the object, and different distances between the observer and the object. The data are given below.

Experiment 1

Linear velocity of the object = 130 cms/sec. kept constant throughout the experiment.
Slit length = 5 cms.

Subject A—Distance between the observer and object = 2 metres

Size of object in mm.	12	11	10	9	8	7
No. of correct answers out of 14 presentations	14	12	11	9	7	5

Distance between the observer and object = 4 metres

Size of object in mm.	12	11	10	9	8	7
No. of correct ans. out of 14 presentations	13	12	12	7	7	5

Subject B—Distance between the observer and object = 2 metres

Size of object in mm.	10	9	8	7	6	5
No. of correct ans. out of 14	12	10	9	6	6	2

Distance between the observer and object = 4 metres

Size of object in mm	10	9	8	7	6
No. of correct ans. out of 14	11	8	8	8	5

Experiment 2

Linear velocity of the object = 350 cm/sec kept constant throughout the experiment.
Slit length = 12·5 cms.

Subject A—Distance between the observer and object = 1 metre

Size of object in mm.	17	16	15	14	13
No. of correct ans. out of 14	12	9	9	8	5

Distance between the observer and object = 5 metres

Size of object in mm.	17	16	15	14	13
No. of correct ans. out of 14	12	8	8	8	5

Subject B—Distance between the observer and object = 1 metre

Size of object in mm.	15	14	13	12	11	10
No. of correct ans. out of 14	12	12	11	7	7	2

Distance between the observer and object = 5 metres

Size of object in mm.	15	14	13	12	11
No. of correct ans. out of 14	11	11	10	8	5

Experiment 3

For the distance of 1 meter between the observer and the object, the slit length was 2·5 cms. and the linear speed of the object was 70 cms/sec., and for the 5 meter distance slit length was 12·5 cms. and the linear speed of the object was 350 cm/sec., so that the angular speed of the object remained constant throughout the experiment.

Subject A—Distance between the observer and object = 1 metre

Size of object in mm.	12	11	10	9	8
No. of correct ans. out of 14	12	11	9	7	5

Distance between the observer and object = 5 metres

Size of object in mm.	17	16	15	14	13
No. of correct ans. out of 14	12	8	8	8	5

Subject B—Distance between the observer and object = 1 metre

Size of object in mm.	10	9	8	7	6
No. of correct ans. out of 14	10	11	9	8	3

Distance between the observer and object = 5 metres

Size of object in mm.	15	14	13	12	11
No. of correct answers out of 14	11	11	10	8	5

THE STATISTICAL ANALYSIS

Examining the tables above it will be noted that the information available from the experiment is essentially the percentage of correct answers given by a subject for varying sizes of the object. Denoting by x the size of the object and assuming the proportion P of correct answers is given by the cumulative distribution of a normal curve, we have for an object of size a,

$$P = \int_{-\infty}^{a} f(x)dx, \text{ where}$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}.$$

We are now interested in estimating the mean of this normal distribution which evidently gives the fifty percent threshold size of the object, that is, the size corresponding to which fifty percent of the answers is expected to be correct.

This estimation can obviously be done by using the technique of 'Probit Analysis'. The details of the calculation, for experiment III on subject A with one metre as the distance between the observer and the object are discussed below.

The percentages of correct answers were converted into probits and were plotted as ordinates with object-sizes as abscissae and a straight line passing through the various points as closely as possible was drawn in Fig. 1. The good linear fit observed in Fig. 1 shows that the assumption of normal distribution is justified. The first estimates of m and σ were obtained from the intercept and the slope of the line, the values being $m = 9$ and $b \left(= \frac{1}{\sigma}\right) = \cdot 38$ roughly. These provisional estimates were then improved by iterative solutions of the maximum likelihood equations as explained by Finney[2].

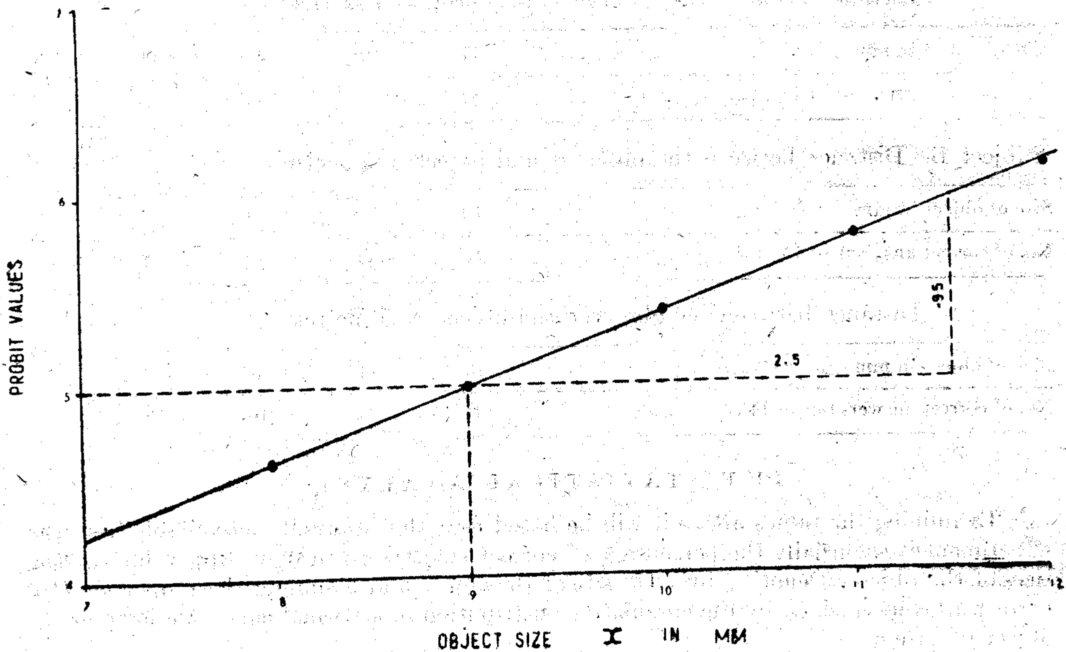

FIG 1. The first estimate of Probit Regression Line.

The expected probits $Y = 5 + \dfrac{x - m}{\sigma}$ are based on the provisional estimates. The expected proportion $P_i = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{Y-5} e^{-t^2/2}\, dt$ and the ordinate $Z = \dfrac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}(Y-5)^2}$ were noted from the normal tables of areas and ordinates. The details of the calculations are given in Table I (appended).

TABLE I
SHOWING THE CALCULATION OF 50% THRESHOLD SIZE OF OBJECTS

Linear size of the object in mm. x_i	Total no. of presentation n_i	Number of correct answers r_i	Proportion of correct answers p_i	Empirical probits	Expected probits Y_i	Z_i	expected proportion P_i	Working probits $y_i = Y_i + \dfrac{p_i - P_i}{Z_i}$	$W_i = \dfrac{Z_i^2}{P_i(1-P_i)}$	$W_i X_i$	$W_i Y_i$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
8	14	5	·357	4·634	4·62	·371	·352	4·6335	0·604	4·832	2·799
9	14	7	·500	5·000	5·00	·399	·500	5·000	0·637	5·733	3·185
10	14	9	·643	5·367	5·38	·371	·648	5·3665	0·604	6·040	3·241
11	14	11	·786	5·793	5·76	·299	·776	5·7934	0·515	5·665	2·984
12	14	12	·857	6·067	6·14	·208	·873	6·0631	0·391	4·692	2·371
								Total	2·751	26·962	14·580

Also $\sum_i W_i x_i^2 = 269.272$, $\sum_i W_i x_i y_i = 144.736$, $\sum_i W_i y_i^2 = 77.946$,

$$S_{xx} = \sum_i W_i (x_i - \bar{x})^2 = 5.023, \quad S_{xy} = \sum_i W_i (x_i - \bar{x})(y_i - \bar{y}) = 1.847,$$

$$S_{yy} = \sum_i W_i (y_i - \bar{y})^2 = 0.681,$$

$$\bar{x} = \frac{\sum_i W_i x_i}{\sum_i W_i} = 9.801, \quad \bar{y} = \frac{\sum_i W_i y_i}{\sum_i W_i} = 5.296$$

$$b = \frac{S_{xy}}{S_{xx}} = 0.3677, \quad \sigma = 2.7196.$$

Thus an estimate of the fifty percent threshold size of the object is given by

$$\hat{m} = \frac{5 - \bar{y}}{b} + \bar{x} = 8 \cdot 996 \text{ mm.}$$

To test the goodness of fit of the probit regression line, we calculate.

$$\chi_3^2 = 14 \left\{ S_{yy} - \frac{S^2_{xy}}{S_{xx}} \right\} = 0 \cdot 031 ,$$ which gives no evidence of heterogeneity of departure from the fitted probit line. Hence we calculate the variances of the estimates as

$$V(b) = \frac{1}{14 S_{xx}} = 0 \cdot 01442, \quad \text{and}$$

$$V(\hat{m}) = \frac{1}{14 b^2} \left\{ \frac{1}{\Sigma_i W_i} + \frac{(\hat{m} - \bar{x})^2}{S_{xx}} \right\}$$

$= 0 \cdot 2602$, so that the standard error of the estimate of the fifty percent threshold size is

$$s.e. \ (\hat{m}) = 0 \cdot 5101.$$

Similar calculations were performed for other experiments as well, and in no case χ^2—test gave evidence of heterogeneity of departure from the fitted probit line. The final results have been tabulated below.

TABLE II.

SHOWING THE 50% THRESHOLD LINEAR SIZES AND THEIR STANDARD ERRORS.

Expt. No.	Subject	Linear speed of the obj. in cms/sec.	Distance between the observer and the obj.	b	V(b)	50 % threshold size of the obj. in mm.	s.e.($\hat{m}$) in mm.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	A	130	2 metres	0·3677	0·0139	7·986	0·5108
1	A	130	4 ,,	0·3301	0·0090	8·346	0·5123
1	B	130	2 ,,	0·3722	0·0086	7·206	0·4051
1	B	130	4 ,,	0·2245	0·0127	7·156	0·7945
2	A	350	1 ,,	0·2948	0·0137	13·936	0·6471
2	A	350	5 ,,	0·2409	0·0147	13·965	0·8101
2	B	350	1 ,,	0·3932	0·0118	11·673	0·4335
2	B	350	5 ,,	0·2896	0·0148	11·641	0·5654
3	A	70	1 ,,	0·3677	0·0142	8·996	0·5101
3	A	350	5 ,,	0·2698	0·0130	14·143	0·6608
3	B	70	1 ,,	0·2885	0·0160	7·324	0·6077
3	B	350	5 ,,	0·3086	0·0150	11·822	0·6441

CONCLUSIONS

Comparing the fifty per cent threshold values with the standard errors (Column 7 & 8 of Table II), it is apparent from the results of experiments 1 and 2, that, for a given linear speed of a moving object, the fifty per cent threshold linear size for visual detection does not vary with the changes in distance between the observer and the object over a range of 1 to 5 metres, that is, there is a trend towards the constancy of threshold size.

For experiment 3, we have to convert the linear values of fifty percent threshold sizes and the standard errors into angular values, before we can draw our conclusions. This conversion into angular sizes (in degrees) is done for the distances of 1 and 5 metres with the help of the multipliers ·0573* and ·01146* respectively. The results obtained are shown in Table III.

TABLE III.

SHOWING THE ANGULAR VALUES OF 50% THRESHOLD SIZES AND THE STANDARD ERRORS FOR EXPERIMENT 3

Subject	Distance between the observer & object in metres	50% threshold size of the object in degrees	Standard error of the estimate in degrees
(1)	(2)	(3)	(4)
A	1	0·5155	0·0292
A	5	0·1621	0·0076
B	1	0·4196	0·0348
B	5	0·1355	0·0074

Thus, for a given angular speed of the moving object, we see that the angular values for the fifty percent threshold sizes are markedly different for the distances of 1 and 5 metres.

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REFERENCES

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2. FINNEY D. J.—Probit Analysis—(1952), University Press, Cambridge

*If x is the size of the object in milimetres and the distance of the observer in metres then the angular size of the object is approximately $x/1000d$ radians or ·0573 x/d degrees, taking 1 radian=57·30. Thus for d=1 and 5, we get the multipliers ·0573 and ·01146 respectively.