STUDY OF MICROWAVE FILTERS PART I

by

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ABSTRACT

The basic unit employed in the design of microwave filters is usually a cavity resonator obtained by terminating the both ends of a suitable length of waveguide by reactances. Expressions have been given for Q, insertion loss and resonant frequency of this unit. A few such units have been investigated both experimentally and theoretically. A satisfactory agreement between theory and practice has been obtained for the units obtained from indigenous materials.

Introduction

Microwave filters have been extensively investigated in recent years and plenty of literature [1–4] is available regarding their design and performance characteristics.

Basically, the design procedure is almost the same as for low frequency circuits except that here the conventional circuit elements take the form of lengths of waveguides, cavity resonators, irises, posts etc.

The use of cavity resonators, however, has been found to be more suitable for design of these filters. A cavity resonator is the microwave analogue of a tuned circuit and is obtained by chosing a suitable length of waveguide and terminating it on both ends by means of suitable irises or posts. The total reactance of the system at the resonant frequency is designed to be zero.

For narrow band filters, the use of a single resonator may be sufficient to give the desired characteristics. However, if very high attenuation in the stop band is required or if a broader pass band with sharp attenuation at the edges of the pass band is required, recourse has to be taken to the use of a large number of cavity resonators either directly coupled or coupled by quarter wave length lines.

In general, microwave filters are required for use with TR systems television links, mixers and multichannel telephony systems. However, the present investigations on microwave filters, have been taken up to facilitate the work on parametric amplifiers. These filters are to be used for the selective transmission of the three frequencies, i.e. the pumping frequency, the idling frequency and the signal frequency, associated with this amplifier. A narrow band filter is needed in the pumping circuit as it has to transmit only a single frequency while band pass filters having sharp attenuation at the edges of the off band are required in the signal frequency circuit. The band width of the filter will be designed to suit the band width of the amplifier.

The three frequencies associated with the parametric amplifier may have any value depending on its design and frequency of operation, and hence, filters would be required for those frequencies.

However, inspite of the theory and design procedure available for these filters, in practice these items are to be developed individually for each frequency, the reason being the approximations in theory, uncertainties due to junction effects, mechanical imperfections etc. It was, therefore, realised that plenty of delay and trouble would be experienced if one had to depend on importing these filters from abroad.

Hence investigations on these filters were taken up so as to be able to design and obtain them locally from indigenous materials.

In the initial stages of the work, investigations have been done on filters employing a single cavity resonator as this forms the fundamental unit from which the more complex filters employing three or more such units are obtained.

The details of the theory employed in the design of this unit and the results obtained are being given in the following sections. Investigations have also been done on the filters employing three cavity resonators using both direct and quarter-wave length coupling as the requirements for some applications can only be met by using a chain of these resonators. The details of the investigations and the design procedure for these multi-cavity filters will be given in the second part of the paper.

Theory

General

It has already been mentioned that the cavity-resonator is obtained in practice by terminating a suitable length of waveguide by reactances on its two ends. This reactance can either be capacitive or inductive. The capacitive reactance is obtained by reducing the narrow dimensions of the waveguide by means of irises. The inductive reactance on the other hand may be obtained either by the use of irises along the broad dimension of the guide or by the use of posts extending all along the narrow dimensions of the guide.

The capacitive reactance, however, is found to be inferior from the point of view of mechanical tolerances and also the power handling capacity of the filter[5]. Hence only the filters employing inductive reactances will be considered.

A general treatment of the system which is independent of the type of inductive reactance used is being given for the evaluation of the resonant frequency, insertion loss and Q_L (the loaded Q) of the system.

Insertion loss and Q_L of the single cavity resonator:

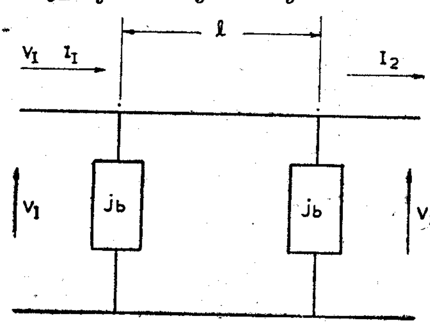


FIG. 1

Consider the Single Cavity resonator filter system of Fig. 1. Two inductive susceptances jb (normalised) are separated by means of length l. Let V_1, I_1 be the input voltage and current and let V_2, I_2 be the corresponding output voltage and current. The relations between V_1, I_1 and V_2, I_2, may be expressed as[6]

$$V_1 = pV_2 + jqI_2 \ldots\ldots\ldots\ldots (i)$$

$$I_2 = jrV_2 + sI_2 \ldots\ldots\ldots\ldots (ii)$$

Written in the matrix form,

$$\begin{vmatrix} V_1 \\ I_1 \end{vmatrix} = \begin{vmatrix} p & jq \\ jr & S \end{vmatrix} \begin{vmatrix} V_2 \\ I_2 \end{vmatrix} \ldots\ldots\ldots\ldots (3).$$

The elements p, q, r and s of the above matrix are real.

If the filter network were matched both at the input and output so that the generator and load impedances when normalized are both equal to unity the matrices of the generator series impedance and the load shunt impedances are written as

$$\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \text{ and } \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \text{ respectively.}$$

Let the generator voltage and current be V_o, I_o and the output voltage and current V_2 I_2 as shown in Fig. 2.

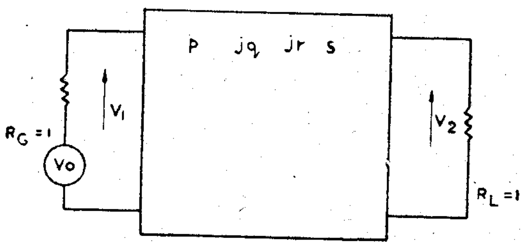


FIG. 2

The voltage V_2 can be determined in terms of the general network parameters in the following manner.

Let p′, q′, r′ and s′ be the elements of the matrix representing the cascade connection of the generator resistance, the filter and the load resistance as shown in Fig. 3.

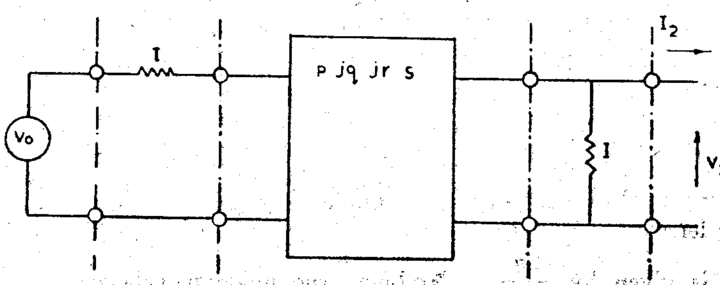


FIG. 3

Then,

$$\begin{vmatrix} p' & q' \\ r' & s' \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \times \begin{vmatrix} p & jq \\ jr & s \end{vmatrix} \times \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \quad \text{.......(4)}$$

$$= \begin{vmatrix} p + jq + jr + s & jq + s \\ jr + s & s \end{vmatrix} \quad \text{.......(5)}$$

From the very definition of the above matrix, for open circuit conditions *i.e.* when $I_2=0$.

$$\frac{Vo}{V_2} = p' = p + jq + jr + s \quad \text{.......(6)}$$

It is obvious from Fig. 3 that the output voltage V_2' in the absence of the filter network is given by $\frac{1}{2}$Vo,

$$\text{Hence, } \frac{V_2'}{V_2} = \frac{1}{2} \left[(p+s) + j(q+r) \right] \quad \text{.......(7)}$$

Therefore the insertion loss L, which is defined as the ratio of power delivered to the load in the absence of the filter network to the power delivered to the load with the filter in the circuit is given by,

$$L = \left| \frac{V_2'}{V_2} \right|^2 = \left[\frac{p+s}{2} + j \frac{(q+r)}{2} \right] \times \left[\frac{p+s}{2} - j \frac{(q+r)}{2} \right] \quad \text{.......(8)}$$

$$= \frac{1}{4} (p+s)^2 + \frac{1}{4} (q+r)^2 \quad \text{.......(9)}$$

Since the circuit is symmetrical, $p=s$..........(10)

and by reciprocity theorem, $ps+qr=1$..........(11)

from eqn. 9,

$$L = ps+qr + \frac{1}{4} (p-s)^2 + \frac{1}{4} (q-r)^2 \quad \text{.......(12)}$$

Hence,

$$L = 1 + \frac{1}{4} (q-r)^2 \quad \text{.......(13)}$$

The next step in the evaluation of the insertion loss of the filter is to express the matrix parameters p, q, r, s, in terms of the susceptance b and the length 'l'.

The matrix (say B) of a pure susceptance is given by $\begin{vmatrix} 1 & 0 \\ jb & 1 \end{vmatrix}$ and that of the length 'l' of a loss-less wave guide say (L) by $\begin{vmatrix} Cos\beta l & jsin\beta l \\ jsin\beta l & Cos\beta l \end{vmatrix}$

where β is given by $\frac{2\pi}{\lambda g}$, λg being the guide wavelength.

Hence the matrix [B] [L] [B] of the filter system which is a cascade connection of three matrices is written as

$$[B]\,[L]\,[B] = \begin{vmatrix} 1 & 0 \\ jb & 1 \end{vmatrix} \begin{vmatrix} \cos\beta l & j\sin\beta l \\ j\sin\beta l & \cos\beta l \end{vmatrix} \begin{vmatrix} 1 & 0 \\ jb & 1 \end{vmatrix} \quad \ldots \quad (14)$$

$$= \begin{vmatrix} \cos\beta l - b\sin\beta l & j\sin\beta l \\ j[2b\cos\beta l + (1-b)^2 \sin\beta l] & \cos\beta l - b\sin\beta l \end{vmatrix} \quad \ldots \quad \ldots \quad (15)$$

Hence, from eqn 13, the insertion loss of the filter under matched conditions is given by

$$L = 1 + \tfrac{1}{4}\,[2\,b\cos\beta l - b^2 \sin\beta l\,]^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

From the above eqn by suitable trignometric manipulation it may be shown that[4],

$$L = 1 + \frac{b^2}{4}\,(b^2 + 4)\,[\delta\,(\beta l)]^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

assuming 'b' to be independent of 'f'.

assuming l to be constant, we have

$$\delta\,(\beta l) = l\left(\frac{d\beta}{df}\right)\delta f \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

$$= (\beta l)\left(\frac{\lambda g}{\lambda o}\right)^2 \frac{\delta f}{fo}, \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

by simple differentiation and using the equation, $\beta^2 = \beta_o^{\,2} - \beta^2_c$ .. (20)

where λ_o and λ_c are the free space and the cut-off wavelengths respectively.

$$\left(\beta_o = \frac{2\pi}{\lambda_o}\;;\quad \beta_c = \frac{2\pi}{\lambda_c}\right)$$

For obtaining the expression for Q_L of the system the value of δf corresponding to 3 db points is required. If δf at 3 db points be δf then

$$Q_L = \frac{fo}{2\,\delta fo} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

It is also quite clear that at 3 db points L must be equal to 2.

Hence, from eqns (17) & 19,

$$\frac{b^2}{4}\,(b^2 + 4)\,(\beta l)^2 \left(\frac{\lambda g}{\lambda o}\right)^4 \left(\frac{\delta fo}{fo}\right)^2 = 1 \quad \ldots \quad \ldots \quad (22)$$

$$\text{Hence}\quad Q_L = -\tfrac{1}{4}\,b\left(\frac{\lambda g}{\lambda o}\right)^2 (\beta l)\,\sqrt{b^2 + 4} \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

The value of b, depends on the type of susceptance being used. If an inductive iris is being used, b is given by,

$$b \doteq -\frac{\lambda g}{a}\,\cot^2 \frac{\pi d}{2a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

and if the post is being used, an approximate formula for the susceptance is given by b

$$b = -2\,\frac{\lambda g}{a}\;\frac{1}{\log_n\left(\frac{4a}{\pi d}\right) - 2} \quad \ldots \quad (25)$$

d being the diameter of the post, and a is the broader dimension of the wave-guide.

Evaluation of the resonant frequency of the filter

Consider the filter system when isolated as shown in Fig. 4.

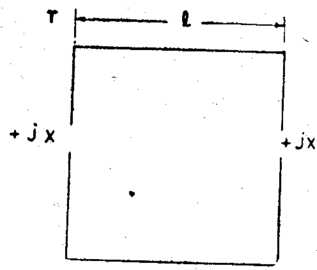


FIG. 4

This consists of the length of line 'l' in series with the two reactances jx (jx being the reactance of the iris). At resonance the total reactance of the system must be zero.

Looking from the plane T, let the input impedance be Z_{in}, hence for resonance $Z_{in} + jx = 0$, ... (26)

$$Z_{in} = Z_o\,\frac{j\chi + j Z_o \tan(\beta l)}{Z_o - \chi \tan(\beta l)}, \quad \ldots \quad (27)$$

where Zo is the characteristic impedance of the line (waveguide). From eqns. 26 & 27,

$$\frac{\chi}{Z_o} + \frac{\frac{\chi}{Z_o} + \tan(\beta l)}{1 - \frac{\chi}{Z_o}\tan(\beta l)} = 0 \quad \ldots \quad (28)$$

from the above, it may easily be shown,

$$\tan(\beta l) = \left[\frac{2\chi}{Z_o}\right]\left[\left(\frac{\chi}{Z_o}\right)^2 - 1\right]^{-1} \quad \ldots \quad (29)$$

$$= \frac{2b}{b^2 - 1} \quad \ldots \quad (30)$$

From eqn. 30,

$$(\beta l) = tan^{-1} \frac{2\, b}{b^2 - 1} \quad \ldots \quad (31)$$

This determines the resonant length 'l' of the filter corresponding to given susceptance of frequency.

from the eqns. (24) & (30) it can be shown that

$$\tan \frac{\frac{2\,\pi\, l}{\lambda g}\left[\frac{K^2}{\lambda g^2} - 1\right]}{K/\lambda g} = 2 \quad \ldots \quad (32)$$

Where $K = a \tan^2 \left(\frac{\pi d}{2a}\right)$

The above eqn. has to be solved numerically either for l or λg, if the other be known. Hence for a given iris opening and the resonant frequency, the resonant length may be calculated.

A similar treatment may be done for the posts.

Experimental Details and Conclusions

Filters using single cavity resonators were fabricated in the workshop. The inductive irises were used as terminations on both the ends of the length of waveguide. Irises of different openings were used with the same length of the waveguide and the values of Q and the resonant frequency of the system with the different irises were experimentally determined. The theoretical values of those quantities were also computed and compared with the experimental results.

The following experimental arrangement, shown in Fig. 5, was used for testing the characteristics of the filter.

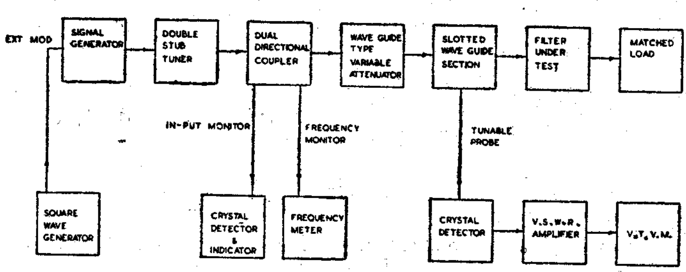


FIG. 5

The frequency response of the filter was obtained by detuning the klystron and taking the output readings from the filter, keeping the input power always at a constant level.

A typical response curve of the filter is shown in Fig 6, $Q_{L'}$, the loaded Q of the filter can readily be obtained from this curve by measuring the bandwidth between the 3 db points. However, it is found more reliable to plot $\frac{1}{D}$ against $(2 \triangle f)^2$; D being the detected output from a square law detector and $2 \triangle f$ is the width of the response curve corresponding to the output level given by D. This plot gives a straight line and it has been shown in Appendix I that the slope of the line is $\left(\frac{Q_L}{fo}\right)^2$. One such plot corresponding to Fig. 6 is shown in Fig. 7.

Q_L of the cavity under test could also be measured by finding the reflection co-efficient from the filter under the conditions of matched termination for various frequencies. A typical plot of reflection co-efficient against frequency for one of the filters is given in Fig. 8. From these reflection co-efficient measurements Q_L is derived in the following manner. It has been shown in Appendix II, that the plot of $\left(\frac{\rho^2 - \rho_o{}^2}{1 - \rho^2}\right)^{\frac{1}{2}}$ against $2 \triangle f$ is a straight line having the slope given by $\frac{Q_L}{fo}$ where ρ_o and ρ are the reflection coefficients for the resonant frequency and for the two frequencies on both the ends of the bandwidth $2 \triangle f$ respectively. A typical plot of these quantities is shown in Fig. 9. This method has the advantage that here the input power levels need not be constant at the various frequencies.

The results on the measurements of resonant frequencies and Q_L for the different filters obtained by using different iris pairs are shown in Figs. 10 & 11. Along with these experimental curves, the theoretical curves of Q_L and resonant frequency, obtained by using the eqns. (23) and (32), are also displayed.

The resonant frequency is obtained by numerically solving eqn (32) for λ_g. As may be seen from there, different values of λ_g are chosen and the value of λ_g which satisfies eqn (32) gives the resonant guide wavelength. A curve of $f(\lambda_g)$ against λ_g is shown in Fig. 12, where $f(\lambda_g)$ stands for the left hand side of eqn. (32).

The results obtained with the different pair of irises are given in Table I:—

TABLE I

No.	Opening of the inductive Iris CM.	Resonant Frequency Mc/s		QL	
		Theory	Experiment	Theory	Experiment
1	2·38	2749	2730	80·93	78·61
2	1·68	2807	2790	354·1	363·3
3	0·80	2843	2854	high	1476

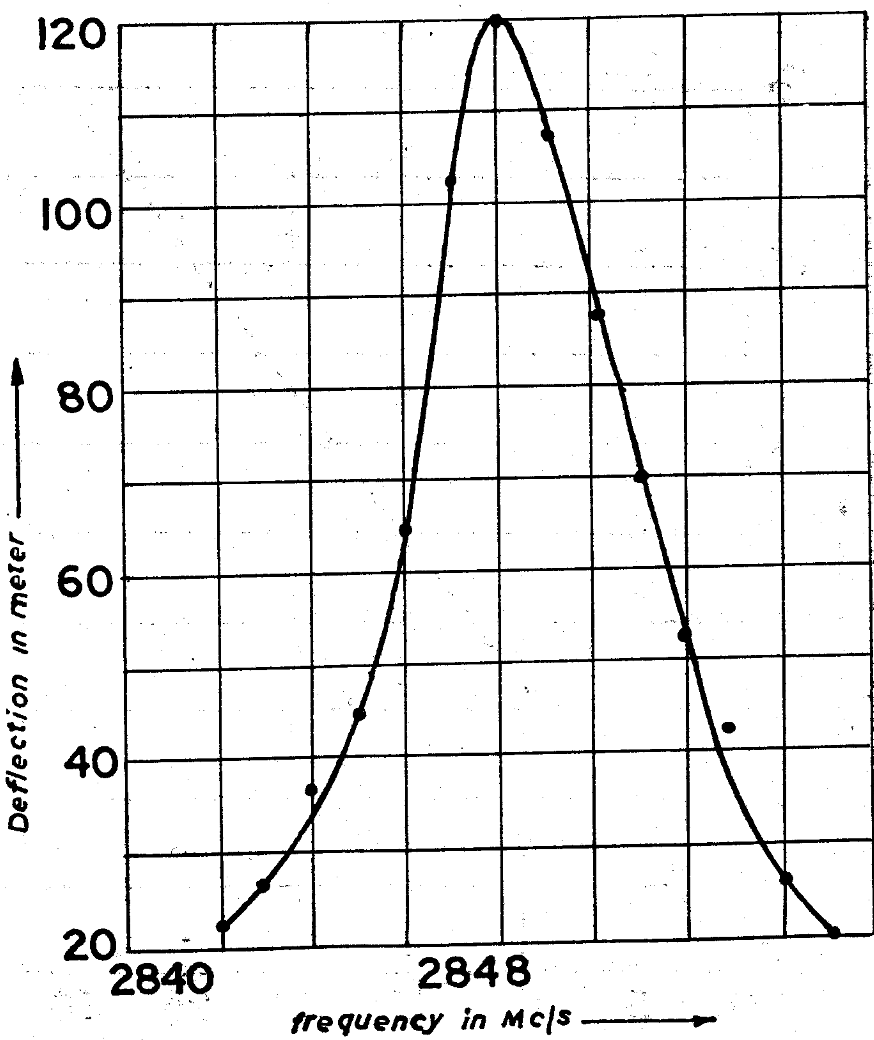


Fig. 6—*A typical transmission response of the filter*

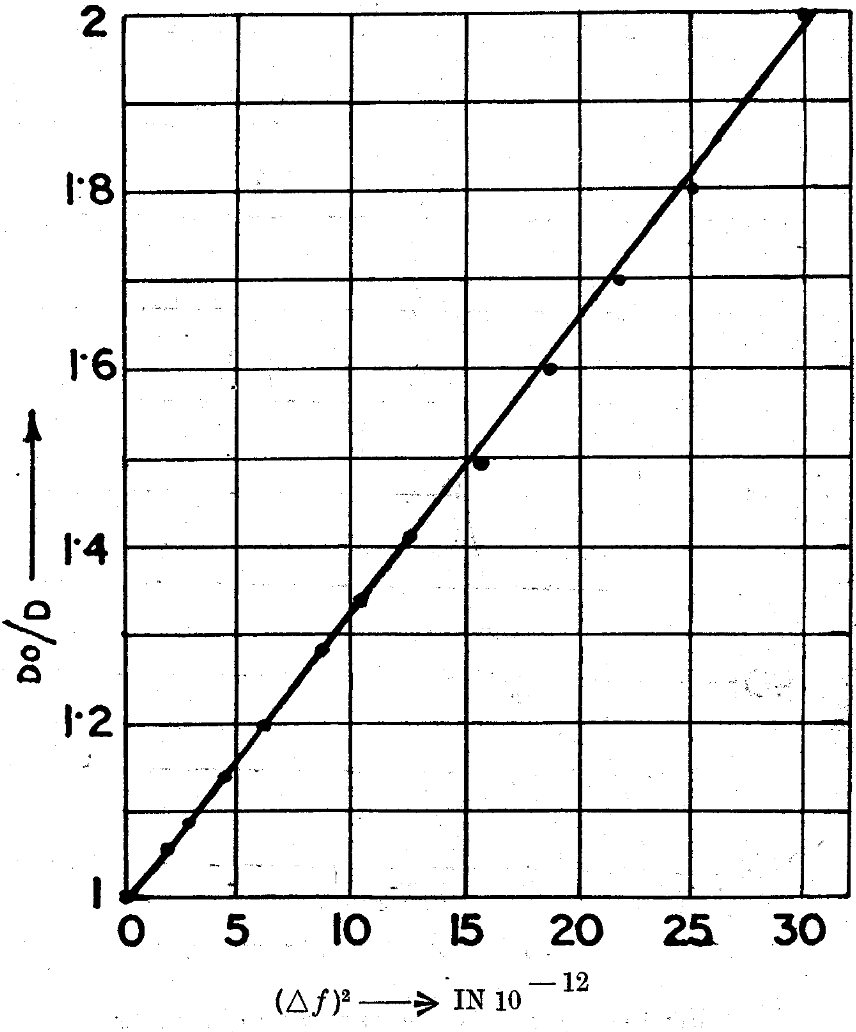


FIG. 7—*Plot of Do/D against* $(\Delta f)^2$
The slope of this St line is equal to $(Q_L/fo)^2$

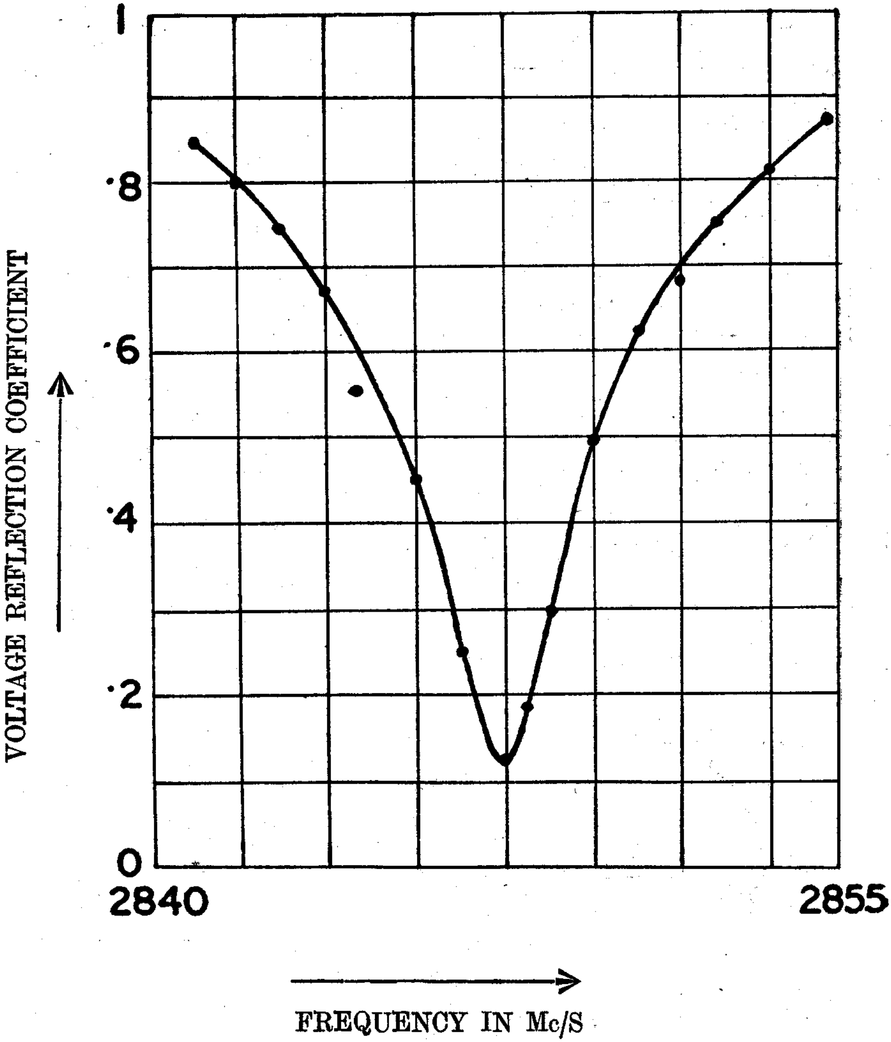
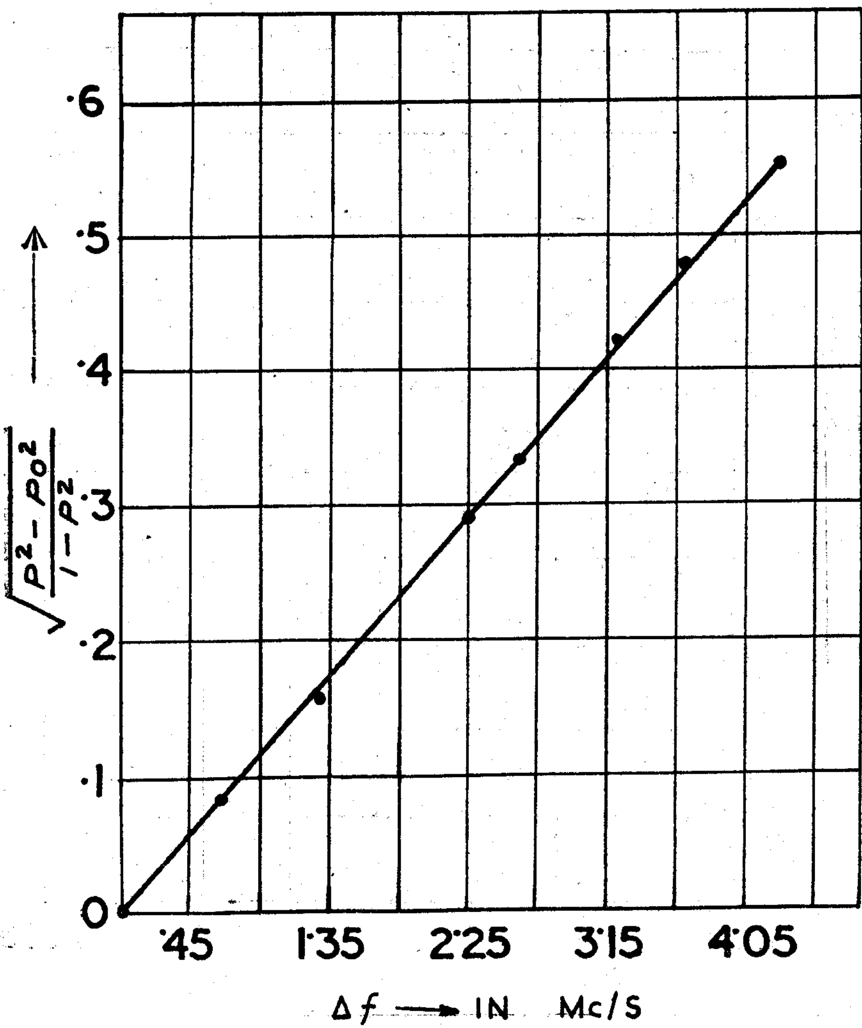


FIG. 8—*Typical curve of V.S.W.R.Vs frequency*



Plot of $\sqrt{P^2 - P_o^2}/\sqrt{1-P^2}$ *Vs* Δf

FIG. 9—*The Slope of this Straight line is* $(2Q_L/fo)$

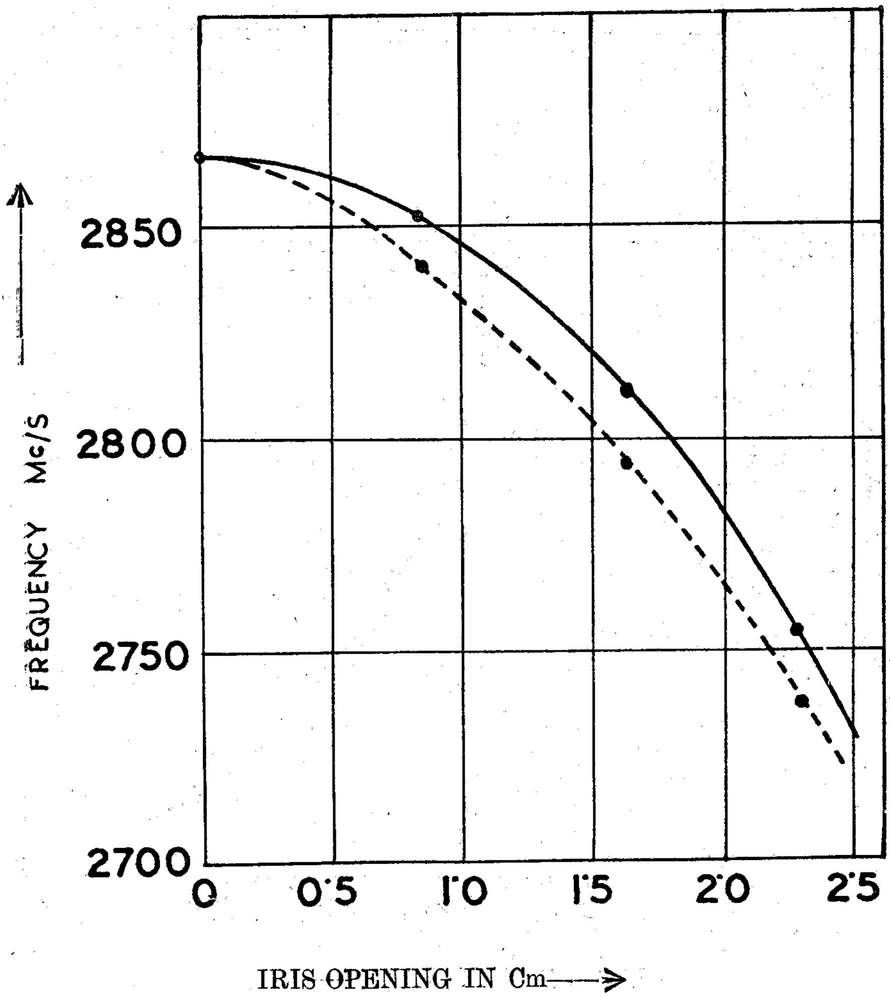


FIG. 10—*Curve showing the variation of resonant frequency with iris opening of the microwave filter having a fixed length of 7·6 cms*

....	Experimental Curve
————	Theoretical Curve

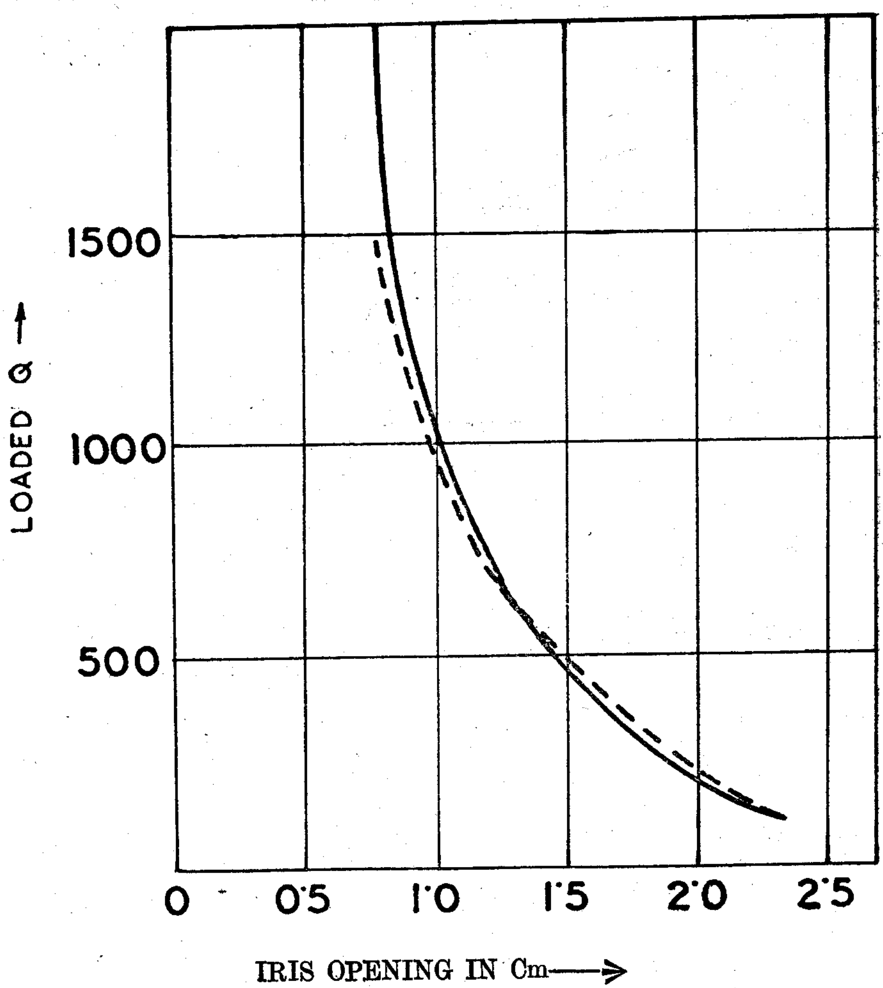


FIG. 11—*Curve showing the variation of Q_L with iris opening for a microwave filter of length 7·6 cms*

........Experimental Curve
———Theoretical Curve

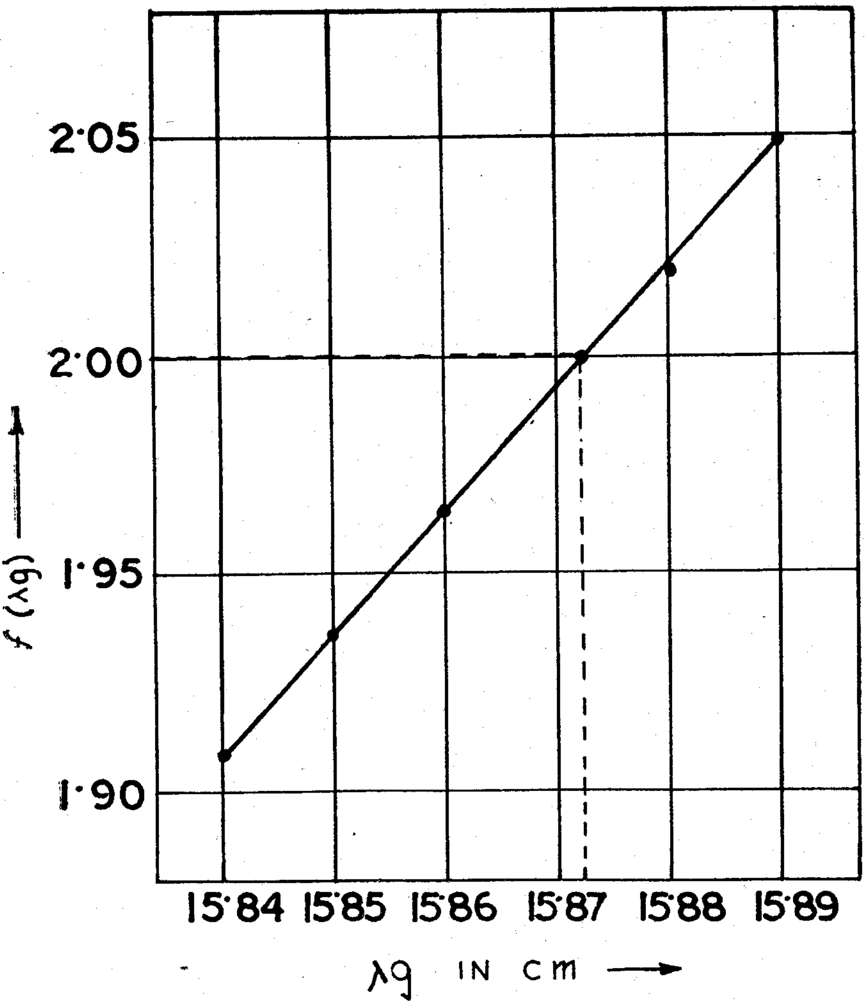


FIG. 12—*Plot of* $f(\lambda_g)$ *Vs* λ_g

where $f(\lambda g)=2\pi l/\lambda g\ (k^2/\lambda_g{}^2\ ^{-1})/k/\lambda_g$.

Conclusions

The theoretical and the experimental results obtained for the resonance frequency of the single cavity filter show a very satisfactory agreement. Hence there are sufficient grounds for the belief that these filters can be designed and produced from indigenous materials. As these single cavity-resonator filters form the fundamental units for the design of the more complex filters, the latter can also be designed and manufactured locally. The results obtained on these coupled filters, however, will be discussed in another paper.

It may be mentioned here that by having a provison for a tuning screw in the middle of the filter on the broad face of the guide, the actual frequency can be made exactly equal to the design frequency.

The theoretical and experimental values of Q_L show a good agreement for larger openings of the iris. However, for narrower openings the two values do not agree at all. This is to be expected. The theoretical expression for Q_L has been deduced on the assumption that the only source of power loss is through the openings in the irises and there are no conductor losses. With larger openings it may be all right to ignore the conductor losses, but when the iris opening is reduced the conductor losses become of the same order as the power-losses through the iris. Hence, the experimental and theoretical values of Q_L for these openings are not in agreement.

Acknowledgements

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APPENDIX I

Evaluation of Q_L from transmission measurement

For a transmission type resonator it has been shown[7] that,

$$\frac{T(fo)}{T(f)} = 1 + Q_L{}^2 \left(\frac{f}{fo} - \frac{fo}{f}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

In the above equation T_{f_o} is the transmission loss function, defined by the ratio of power available to the load with the filter in circuit and the maximum power available from the generator, for the resonant frequency fo and $T_{(f)}$ is the transmission-loss function at any other frequency f.

T (fo) and T (f) are proportional to the detected output D_O and D respectively if the detector be square law.

Hence $$\frac{Do}{D} = 1 + Q_L{}^2 \left(\frac{f}{fo} - \frac{fo}{f}\right)^2 \quad \ldots \quad (34)$$

It may easily be shown that $$\left(\frac{f}{fo} - \frac{fo}{f}\right) = \frac{2\,\triangle f}{fo} \quad \ldots \quad (35)$$

where $\triangle f = f - fo$

eqn. (33) may now be written as

$$\frac{D_o}{D} = 1 + \left(\frac{Q_L}{f_o}\right)^2 (2\triangle f)^2 \quad \ldots \quad (36)$$

It is obvious that the plot of $\frac{1}{D}$ rs $(2\triangle f)^2$ will give a line of slope $\left(\frac{Q_L}{f_o}\right)^2$

APPENDIX II

Evaluation of Q_L from reflection co-efficient measurements

Correlating reflection co-efficient with the input impedance of the system and the load impedance, it can easily be shown[7] that

$$\frac{\rho}{\rho_o} = \frac{1 + jQ_L / \rho_o \left(\frac{f}{f_o} - \frac{f_o}{f}\right)}{1 + jQ_L \left(\frac{f}{f_o} - \frac{f_o}{f}\right)} \quad \ldots \quad (37)$$

where ρ_o is the reflection coefficient at the resonant frequency and ρ is the reflection coefficient at any frequency f.

from eqn (35) and (37).

$$\frac{\rho}{\rho_o} = \frac{1 + jQ_L / \rho_o \frac{2\triangle f}{f_o}}{1 + jQ_L \frac{2\triangle f}{f_o}} \quad \ldots \quad (38)$$

Hence, $$\left|\frac{\rho}{\rho_o}\right|^2 = \frac{1 + \left(\frac{2\triangle f}{f_o}\right)^2 \frac{Q_L{}^2}{\rho_o{}^2}}{1 + \left(\frac{2\triangle f}{f_o}\right)^2 Q_L{}^2} \quad \ldots \quad (39)$$

This gives, $$\left(\frac{2\triangle f}{f_o}\right)^2 Q_L{}^2 = \frac{\rho^2 - \rho_o{}^2}{1 - \rho^2} \quad \ldots \quad (40)$$