THE FORM FUNCTIONS FOR MODIFIED MULTITUBULAR CHARGES

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ABSTRACT

It is shown in the present paper that it is likely to be useful to eliminate the second stage of burning for the heptatubular charge by removing the portions corresponding to the slivers in advance, with the result that there is only one stage of burning and throughout this stage in almost all cases S/S_0 increases. The properties of the form-function for such a shape and the equivalent form factor θ have been worked out in complete generality and it is found that this shape gives a great improvement over the unmodified shape as is seen by comparing the values of the equivalent form factor θ here with the corresponding mean value of θ for the unmodified heptatubular charge obtained by Tavernier.

These properties have also been investigated for a shape which has only three circular holes in the unmodified form. Here too the surface for the modified charge is throughout progressive in almost all cases.

Introduction

The heptatubular charge is widely used in many countries on account of its progressive nature in burning. It first became popular in the U.S.A., but it was not so much used in the continent because of the possibility of irregular burning and difficulties of manufacture. Later it was found that the difficulties were not so real and the use of this shape became more popular.

One difficulty with the ordinary heptatubular charge is that it burns in two stages, in the first of which the burning surface increases while in the second, it rapidly decreases. Though S/S_0 remains continuous at the point of rupture, it can be shown that its derivative suffers a wide jump at this point. This sudden discontinuity together with the irregularity in the burning of the slivers takes away a great deal from the advantage of progressive burning surface area in the first stage. It is, therefore, suggested in the present paper that the second stage be eliminated altogether by removing the portions corresponding to the slivers in advance and inhibiting the surface from burning. The result would be that there will be only one stage of burning, and throughout this stage in almost all cases S/S_0 would be increasing. We have calculated the form-function for such a shape, *of the same generality* as that of Tavernier[1] for the usual heptatubular charge. We have obtained the equivalent form-factor (θ) for this shape by using the principle of least squares and found that it gives a great improvement over the corresponding mean values of θ for the heptatubular charge obtained by Tavernier. Our values are only slightly inferior to those obtained for the heptatubular for the first stage only, but the behaviour has to be compared for the whole duration of burning.

It may be admitted that there may be some difficulties on the manufacture of this modified shape, which will have nineteen holes instead of seven in the usual hepta-tubular charge, but the advantages of this shape are sufficiently attractive, if the manufacturing difficulties can be overcome. In fact the difficulties can be reduced by using a smaller number of holes by using the principle suggested here. We have worked out the form function for a shape which has only three circular holes in the un-modified form. Here too the surface for the modified charge is throughout progressive in almost all cases and graphs and tables have been given to show the behaviour of this stage.

It may also be argued that increasing the number of holes in the heptatubular charge would decrease the average density. To overcome this difficulty, we have calculated in the last section the size of the holes in the modified tritubular charge so that the average density for this charge with seven holes is the same as that for the usual hepta-tubular charge with seven holes.

Notations

The following notations have been used:—

d = The diameter of the holes of the grain.

D = The distance between any two holes and between any of the six outer holes and the curved surface of the grain.

L = The length of the grain.

m = Ratio of the exterior diameter of the grain to the diameter of the holes ($m > 3$ for the heptatubular charge and $> 1+2/\sqrt{3}$ for the tritubular charge)

ρ = Ratio of the length of the grain to the exterior diameter of the grain ($\rho > \dfrac{m-3}{4m}$ for hepta-tubular and

$$> \frac{(m-1)\sqrt{3}-2}{2(\sqrt{3}+1)m} \text{ for the tritubular charge}).$$

S_0 = Surface of the powder initially exposed to the combustion.

S = Surface of the combustion at the instant 't'.

z = Fraction of the charge burnt at the instant 't'.

f = Fraction of the initial thickness (*web*-size) remaining at the instant 't'.

$\phi(z) = \dfrac{S}{S_0}$ = Function of the progressivity of the powder.

C = Mass of the grain.

δ = Propellant density.

Modified Hepta-Tubular Charge and its Form-Function

The appearance of the end section of a grain is shown in the following figures (1), (2), (3) and (4)

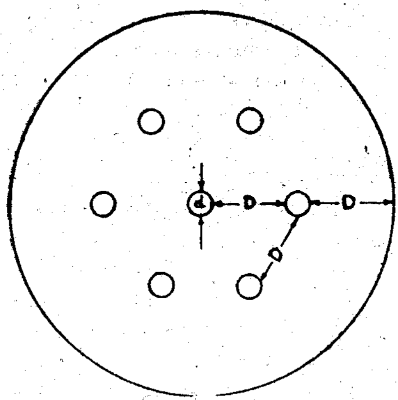


FIG. 1

Unmodified hepta-tubular charge.

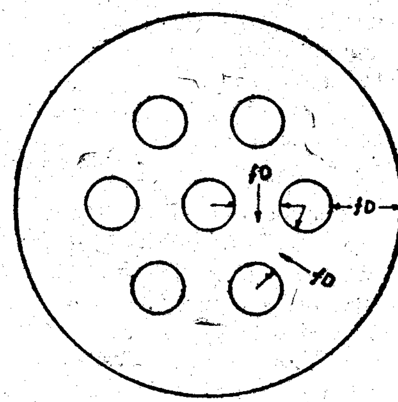


FIG. 2

Unmodified hepta-tubular charge when a fraction f of D remains.

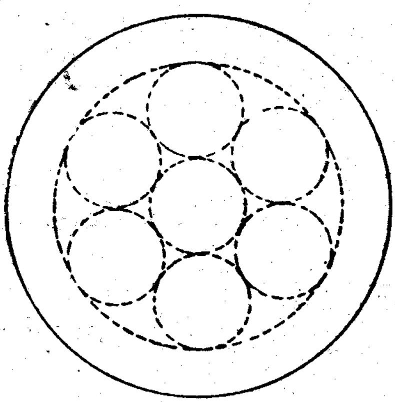


FIG. 3

Unmodified Hepta-tubular charge showing the position of slivers at the end of first stage of burning.

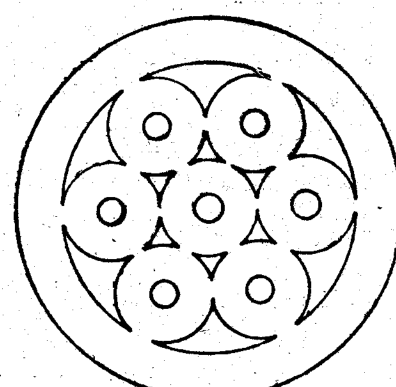


FIG. 4

Modified hepta-tubular charge

The diameter of the cylinder is $(4D+3d)$ and since this equals md, we have

$$D = \frac{m-3}{4}\, d \qquad \ldots \quad \ldots \quad \ldots \qquad (1)$$

The area of the twelve curvilinear triangular prisms i.e. of the slivers is

$$= \pi \left(\frac{3D+3d}{2}\right)^2 - 7\pi\left(\frac{D+d}{2}\right)^2$$

$$= \tfrac{1}{2}\, \pi\, (D+d)^2 \qquad \ldots \quad \ldots \quad \ldots \qquad (2)$$

The mass of the grain is given by

$$C = \left[\pi\left(\frac{4D+3d}{2}\right)^2 - 7\pi\left(\frac{d}{2}\right)^2 - \tfrac{1}{2}\pi(D+d)^2\right]\rho(4D+3d)\delta$$

$$= \tfrac{1}{2}\pi\rho D\delta(7D+10d)(4D+3d) \quad \ldots \quad (3a)$$

The mass when a fraction f of D remains (Cross-Section shown in Fig. 2) is given by

$$C(1-z) = \left[\pi\left\{\left(\frac{4D+3d}{2}\right)^2 - \frac{D(1-f)}{2}\right\}^2 - 7\pi\left\{\frac{d}{2} + \frac{D(1-f)}{2}\right\}^2\right.$$

$$\left. - \tfrac{1}{2}\pi(D+d)^2\right]\left[\rho(4D+3d) - D(1-f)\right]\delta$$

$$= \tfrac{1}{2}\pi Df\delta[(7D+10d)+3D(1-f)][\rho(4D+3d)-D(1-f)] \quad \ldots \quad (3b)$$

on dividing (3*b*) by (3*a*) and using (1),

$$1-z = f\left[1+\frac{3D(1-f)}{7D+10d}\right]\left[1-\frac{D(1-f)}{\rho(4D+3d)}\right]$$

$$= f\left[1-\frac{m-3}{4m\rho}(1-f)\right]\left[1+\frac{3m-9}{7m+19}(1-f)\right] \quad \ldots \quad (4a)$$

Hence

$$z = (1-f)\left[1-\frac{(m-3)(6m\rho-5m-5)}{2m\rho(7m+19)}f - \frac{3(m-3)^2}{4m\rho(7m+19)}f^2\right] \quad (4b)$$

which is the (z,f) relation of the form

$$z = (1-f)\ (1-af-bf^2) \quad \ldots \quad \ldots \quad \ldots \quad (4c)$$

where

$$a = \frac{(m-3)(6m\rho-5m-5)}{2m\rho(7m+19)} \quad (5a)$$

and

$$b = \frac{3(m-3)^2}{4m\rho(7m+19)} \quad \ldots \quad \ldots \quad (5b)$$

Since S is clearly proportional to $\frac{dz}{df}$ and since initially $S=S_\circ$ when $f=1$, the (z,f) relation is easily obtained *viz.*

$$\frac{S}{S_o} = \frac{\left[\frac{dz}{df}\right]}{\left[\frac{dz}{df}\right]_{f=1}} = \frac{(1+a)-2(a-b)f-3\,bf^2}{1-(a+b)} \quad .. \quad (6b)$$

Equations (4c) and (6b) determine the (S, z) relation in parametric form.

The Equivalent Form-Factor for the Modified Hepta-Tubular Charge

According to the principle of least squares, we should choose the form-factor θ so as to minimise the quantity

$$\int_o^1 [(1-f)(1+\theta f)-(1-f)(1-af-bf^2)]^2\,df. \quad .. \quad (7a)$$

Differentiating under the sign of integration, we get the following equations for determining θ :

$$\int_o^1 f(1-f)[(1-f)(1+\theta f)-(1-f)(1-af-bf^2)]\,df = o \quad .. \quad (7b)$$

or

$$\int_o^1 \left[(\theta+a)f^2-\left\{2(\theta+a)-b\right\}f^3+(\theta+a-2b)f^4+bf^5\right]df = 0 \quad (7c)$$

whence

$$\theta = -(a+\tfrac{1}{2}b) \quad .. \quad .. \quad .. \quad .. \quad (8a)$$

on substituting the values of a and b we obtain

$$\theta = -\frac{(m-3)(24m\rho-17m-29)}{8m\rho\,(7m+19)} \quad .. \quad (8b)$$

which will be negative if

$$\rho > \frac{17\,m+29}{24\,m} \quad .. \quad (9)$$

Now the sum of the squares of the residues for fitting by the method o least squares is

$$= \int_0^1 \Big[(1-f)\,(1-af-bf^2) - (1-f)\,(1+\theta f) \Big]^2 df$$

$$= \tfrac{1}{4}\, b^2 \int_0^1 \Big[f^2 - 6f^3 + 13f^4 - 12f^5 + 4f^6 \Big] df$$

$$= \frac{1}{840}\; b^2 \qquad \ldots \quad (10)$$

We have collected and calculated in table 1 the values of a and b, the form-factor θ, and the sum of the squares of the residues for some typical values of m and ρ. We have also given here for comparison the estimates of θ obtained by Tavernier[1] for the period before rupture and for the whole period of burning. These figures show that for the period before rupture our estimates are slightly inferior, but for the whole period, there is a great deal of improvement.

TABLE 1

m	ρ	a	b	Estimate of θ for the Modified Heptatubular charge	Sum of the squares of the residues	Estimate of θ for Unmodified Heptatubular Charge as obtained by Tavernier	
						Before rupture	After rupture
4	½	—0·069,15	0·007,98	+0·065,16	0·000,000,0758	+0·087	+1·0408
4	1	—0·002,66	0·003,99	+0·000,67	0·000,000,0189	+0·002	+0·8675
4	9/4	+0·034,28	0·001,77	—0·035,17	0·000,000,0037	—0·052	+0·7711
7	9/4	+0·101,77	0·011,20	—0·107,37	0·000,000,1495	—0·130	+0·2477
10	9/4	+0·139,83	0·018,35	—0·149,01	0·000,000,4010	—0·175	+0·1172
∞	9/4	+0·269,84	0·047,62	—0·293,62	0·000,002,6995	—0·331	—0·1720
∞	∞	+0·428,57	0	—0·428,57	0	—0·451	—0·3228

Some properties of the Form-Function for the Modified Hepta-tubular Charge

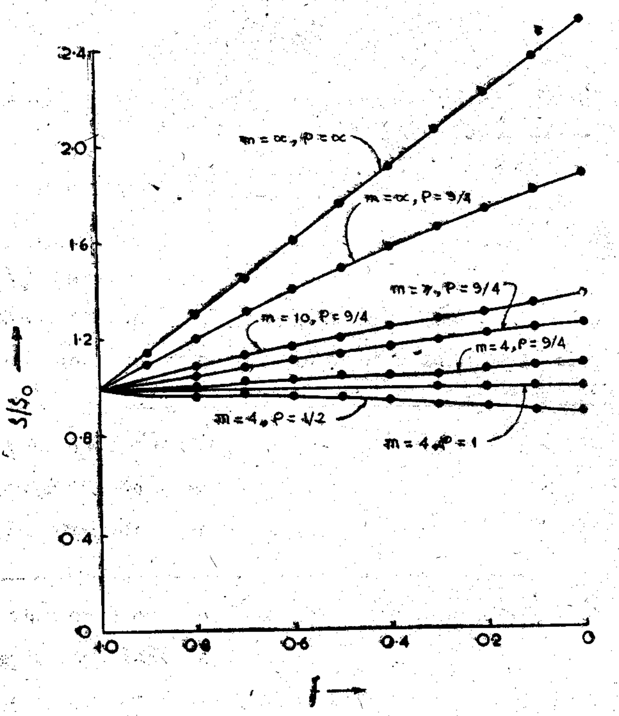
(*a*) *Relations between* z, f *and* S/S_o *for particular shapes.*

The values of z and S/S_o for various values of f for the above set of values of m and ρ are shown in following table:

TABLE 2

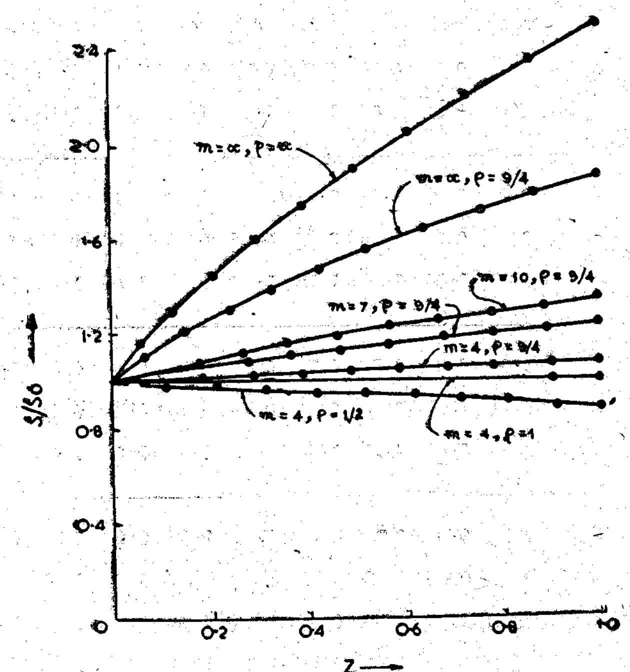
$m=4$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	1
$\rho=\frac{1}{2}$	z	0	0·11	0·21	0·31	0·42	0·52	0·62	0·71	0·81	0·91	1·00
	S/S_o	1·00	0·98	0·98	0·97	0·95	0·95	0·94	0·92	0·91	0·89	0·88
$m=4$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=1$	z	0	0·10	0·20	0·30	0·40	0·50	0·60	0·70	0·80	0·90	1·00
	S/So	1·00	0·99	0·99	0·99	0·99	1·00	1·00	1·00	1·00	1·00	1·00
$m=4$	f	1·0	0·9	0·8	0·7	0·6	0·6	0·4	0·3	0·2	0·1	0
$\rho=9/4$	z	0	0·10	0·19	0·29	0·39	0·49	0·59	0·69	0·79	0·90	1·00
	S/So	1·00	1·01	1·02	1·02	1·03	1·04	1·05	1·05	1·06	1·07	1·07
$m=7$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=9/4$	z	0	0·09	0·18	0·28	0·37	0·47	0·57	0·68	0·78	0·89	1·00
	S/So	1·00	1·03	1·05	1·08	1·11	1·13	1·15	1·18	1·20	1·22	1·24
$m=10$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=9/4$	z	0	0·09	0·18	0·27	0·36	0·46	0·57	0·67	0·78	0·89	1·00
	S/So	1·00	1·04	1·08	1·12	1·16	1·19	1·23	1·26	1·29	1·32	1·35
$m=\infty$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=9/4$	z	0	0·07	0·15	0·24	0·33	0·43	0·53	0·64	0·76	0·87	1·00
	S/S_o	1·00	1·11	1·21	1·30	1·39	1·48	1·57	1·65	1·72	1·79	1·86
$m=\infty$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=\infty$	z	0	0·06	0·13	0·21	0·30	0·39	0·50	0·61	0·73	0·86	1·00
	S/S_o	1·00	1·15	1·30	1·45	1·60	1·75	1·90	2·05	2·20	2·35	2·50

The above results are illustrated in the figures (5), (6) and (7).



m = ∞, ρ = ∞
m = ∞, ρ = 9/4
m = 7, ρ = 9/4
m = 10, ρ = 9/4
m = 4, ρ = 9/4
m = 4, ρ = 1/2
m = 4, ρ = 1
S/S₀
f
2·4
2·0
1·6
1·2
0·8
0·4
0
1·0
0·8
0·6
0·4
0·2
0

FIG. 5



m = ∞, ρ = ∞
m = ∞, ρ = 9/4
m = 10, ρ = 9/4
m = 7, ρ = 9/4
m = 4, ρ = 9/4
m = 4, ρ = 1/2
m = 4, ρ = 1
S/S0
Z
2·4
2·0
1·6
1·2
0·8
0·4
0
0·2
0·4
0·6
0·8
1·0

FIG. 6

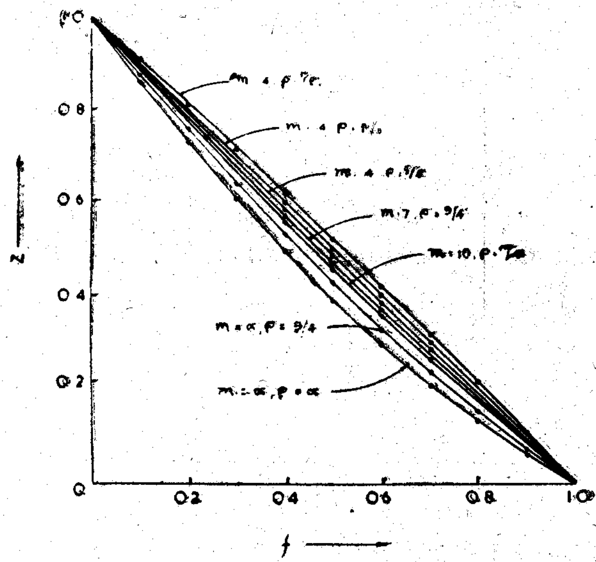


FIG. 7

The above tables and figures show that the least square method provides very good estimates and that for a given value of m, if ρ exceeds a certain value, the burning surface is progressive. We shall examine this function more closely in the next section.

(b) Modified Hepta-tubular Charge with a given value of equivalent Form-Factor θ

From Equation 8(b),

$$\frac{1}{\rho} = \frac{8\left[(3+7\theta)+\frac{1}{m}(19\theta-9)\right]}{\left(1-\frac{3}{m}\right)\left(17+\frac{29}{m}\right)} \quad \ldots \quad \ldots \quad (11)$$

In the following table are shown the values of $1/\rho$, the ratio of the exterior diameter to the length of the grain, for different values of $1/m$, the ratio of the diameter of the holes to the exterior diameter of the grain for the cases when the form factor θ assumes the values $+0\cdot1$, 0, $-0\cdot1$, $-0\cdot2$, $-0\cdot3$ and $-0\cdot4$.

TABLE 3

1/m	θ					
	+0·1	0	—0·1	—0·2	—0·3	—0·4
0	1·74	1·41	1·08	0·75	0·42	0·09
0·05	1·70	1·30	0·90	0·49	0·08	..
0·10	0·72	1·21	0·69	0·18	..	..
0·15	1·79	1·12	0·45	..	..	..
0·20	2·00	1·05	0·11	..	..	..
0·25	2·54	0·99	..	..	..	..
0·30	4·93	0·93	..	..	..	..

The above table or formula (11) would enable us to determine the value of ρ for which a charge with given value of m will behave as a charge with a given value of form-factor θ. Of course θ cannot be given any arbitrary values. In fact,

$$\theta \geqslant -\frac{3}{7} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

and the limits can be obtained when both m and ρ are infinite.

When ρ is infinite, (11) gives.

$$\frac{1}{m} = \frac{3+7\theta}{9-19\theta} \qquad \ldots \quad \ldots \quad \ldots \quad (13)$$

The values of $1/m$ for the above values of θ for which $1/\rho$ becomes zero are shown in the table 4 below:

TABLE 4

θ	+0·1	0	—0·1	—0·2	—0·3	—0·4
1/m	0·521	0·333	0·211	0·125	0·061	0·012

The results of the above tables are illustrated in figure 8.

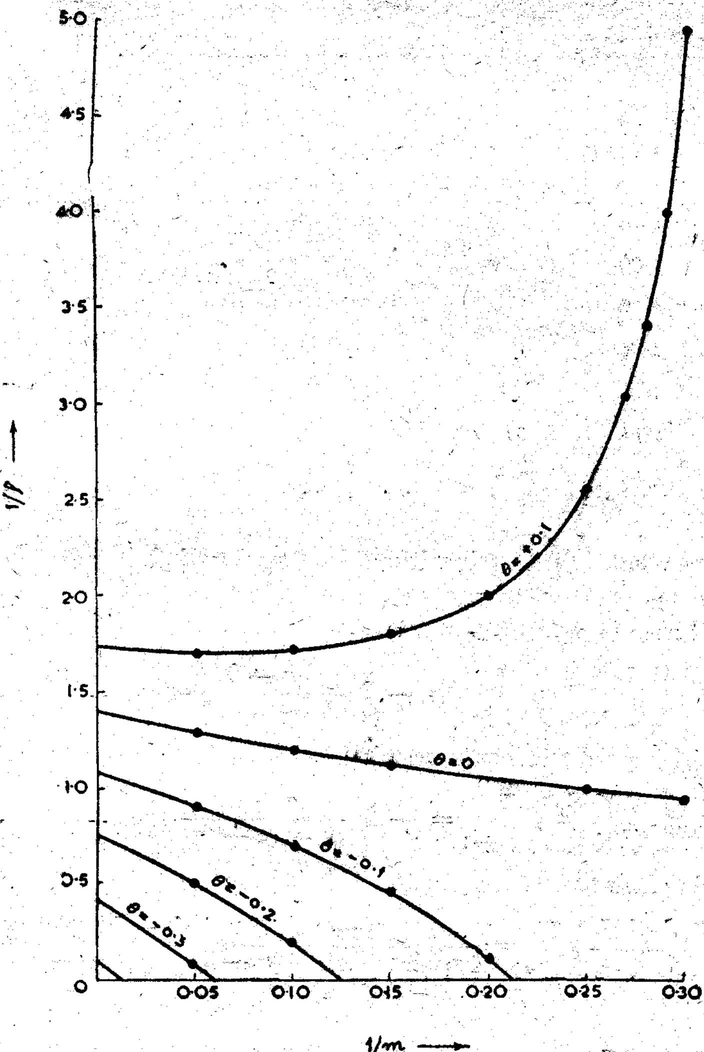


FIG. 8

(c) Progressive and Degressive Nature of the Burning Surface

From (6)

$$\frac{S}{S_o} = \frac{(1 + a) - 2\,(a - b)\,f - 3bf^2}{1 - (a + b)} \qquad \ldots \qquad \ldots \quad \textbf{(14a)}$$

$$\therefore \; \frac{d}{df}\left(\frac{S}{S_o}\right) = -2\;\frac{(a - b) + 3bf}{1 - (a + b)} \qquad \ldots \qquad \ldots \quad (14b)$$

and

$$\frac{d^2}{df^2}\left(\frac{S}{S_o}\right) = -6\;\frac{b}{1 - (a + b)} \qquad \ldots \qquad \ldots \quad \textbf{(14c)}$$

Now using (5a) and (5b)

$$1-(a+b)=\frac{4m\rho(7m+19)+(m-3)(7m+19-12m\rho)}{4m\rho(7m+19)} \qquad (15a)$$

But

$$\begin{aligned} &4m\rho(7m+19)+(m-3)(7m+19-12m\rho) \\ &=16m\rho(m+7)+(m-3)(7m+19) \quad .. \quad .. \quad (15b) \\ &>0 \end{aligned}$$

so that

$$1-(a+b)>0 \quad .. \quad .. \quad .. \quad .. \quad (15c)$$

also

$$b>0$$

$$\therefore \frac{d^2}{df^2}\left(\frac{S}{S_o}\right)<0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (16)$$

which shows that $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is a decreasing function of f.

Now for $f=0$ (end of combustion) :

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=0}=-2\ \frac{a-b}{1-(a+b)} \quad .. \quad .. \quad (17)$$

also for $f=1$ (beginning of combustion):

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=1}=-2\ \frac{a+2b}{1-(a+b)} \quad .. \quad .. \quad (18)$$

substituting for a and b from (5a) and (5b) we have

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=0}=\frac{2(m-3)\ (13m+1-12m\rho)}{4m\rho(7m+19)+(m-3)(7m+19-12m\rho)} \qquad (19)$$

and

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=1}=\frac{8(m-3)\ (m+7-3m\rho)}{4m\rho(7m+19)+(m-3)(7m+19-12m\rho)} \qquad (20)$$

The values of $\frac{d}{df}\left(\frac{S}{S_o}\right)$ have opposite signs at $f=0$ and at $f=1$, and $\frac{S}{S_o}$ takes a maximum value in the interval of variation of f, if:

$$\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=0}\left[\frac{d}{df}\left(\frac{S}{S_o}\right)\right]_{f=1}<0 \quad .. \quad .. \quad (21a)$$

i.e. if

$$(13m+1-12m\rho)(m+7-3m\rho)\leq 0 \quad .. \quad .. \quad .. \quad (21b)$$

The first member of this inequality can be considered as an expression of second degree in ρ which breaks for:

$$\rho_1 = \frac{m+7}{3m} \quad \ldots \quad (22a)$$

and

$$\rho_2 = \frac{13m+1}{12m} \quad \ldots \quad (22b)$$

ρ_1 and ρ_2 are decreasing functions of m which vary respectively from $\left(\frac{10}{9} \text{ to } \frac{1}{3}\right)$ and from $\left(\frac{10}{9} \text{ to } \frac{13}{12}\right)$ when m varies from $+3$ to $+\infty$ and are always greater than $\frac{m-3}{4m}$ the minimum value admissible for ρ. When we give to ρ a value lying between ρ_1 and ρ_2 the above inequality will be satisfied and the maximum of $\frac{S}{S_o}$ will appear for

$$f = -\frac{a-b}{3b} \quad \ldots \quad (23a)$$

using (5a) and (5b)

$$f = \frac{13m+1-12m\rho}{9(m-3)} \quad \ldots \quad (23b)$$

which can be written as

$$f = 1 - \frac{\rho - \rho_1}{3\rho_{min}} \quad \ldots \quad (23c)$$

We thus see that

for $\frac{m-3}{4m} \leqslant \rho < \rho_1$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is always + ve

for $\rho_1 < \rho \leqslant \rho_2$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is at first $-ve$ for $1 \geqslant f > \frac{\rho_2 - \rho}{\rho_2 - \rho_1}$

and then +ve for $\frac{\rho_2 - \rho}{\rho_2 - \rho_1} > f \geqslant 0$

for $\rho_2 < \rho_2$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is always —ve.

Hence $\frac{S}{S_o}$ is decreasing, increasing or at first increasing and then decreasing, according as ρ is less than ρ_1, greater than ρ_2, or lies between ρ_1 and ρ_2. We can, therefore, form the following table which illustrates the results obtainep so far:

TABLE 5

ρ	$\rho_{min} < \rho < \rho_1$	$\rho_1 < \rho < \rho_2$	$\rho_2 < \rho$
S/S_0	Decreasing function of z Degressive Powder.	Function initially increasing and then decreasing of z. Powder at first progressive and then degressive.	Increasing function of z. Progressive powder.

We note that the behaviour is similar to that of an unmodified heptatubular charge except that in the present case there is no second stage of burning so that if $\rho > \rho_2$, the powder is throughout progressive. We may also point out that an alternative way of removing the second stage of burning for a heptatubular charge is to choose $\rho = \rho_{min}$, but the disadvantage of this method is only seen from the above table viz. that the powder will be throughout degressively burning, while by our method $\rho > \rho_2$, it would throughout be progressively burning.

The Modified Tritubular Charge and its Form Function

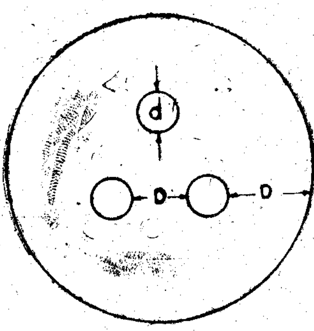


FIG. 9
Unmodified tritubular charge.

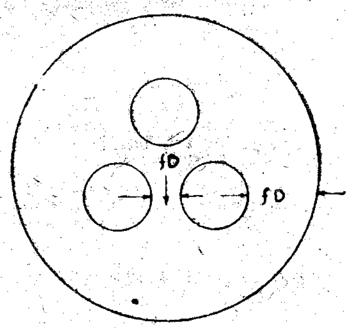


FIG. 10
Unmodified tritubular charge when a fraction f of D remains.

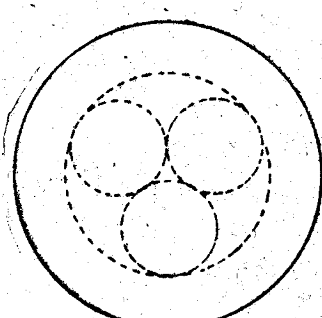


FIG. 11
Unmodified tritubular charge showing the position of slivers at the end of first stage of burning.

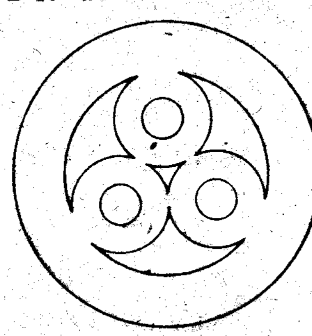


FIG. 12
Modified tritubular charge.

Using the same notations as for the hepta-tubular charge, the three holes in this case form an equilateral triangle of sides of length $(D + d)$. The distance of the centre of the section from each hole is two-thirds the length of the median of this triangle and is, therefore, $\frac{1}{\sqrt{3}}(D+d)$. The diameter of the cylinder is $2\left[\frac{1}{\sqrt{3}}(D + d) + D + \frac{d}{2}\right]$ and since this equals md, i.e.

$$\frac{2}{\sqrt{3}}(D + d) + 2D + d = md \quad .. \quad .. \quad .. \quad (24a)$$

We have

$$D = \frac{(m-1)\sqrt{3} - 2}{2(\sqrt{3}+1)} d \quad .. \quad .. \quad .. \quad .. \quad (24b)$$

The area of the four slivers that remains is

$$= \pi\left[\frac{1}{\sqrt{3}}(D + d) + \frac{1}{2}(D + d)\right]^2 - 3\pi\left[\frac{1}{2}(D + d)^2\right]$$

$$= \pi\left(\frac{1}{\sqrt{3}} - \frac{1}{6}\right)(D + d)^2 \quad .. \quad .. \quad .. \quad (25)$$

The mass of the grain is given by

$$C = \left[\pi\left\{\frac{1}{\sqrt{3}}(D + d) + D + \frac{d}{2}\right\}^2 - 3\pi\left(\frac{d}{2}\right)^2 - \pi\left(\frac{1}{\sqrt{3}} - \frac{1}{6}\right)(D + d)^2\right]$$

$$\times \rho\left[\frac{2}{\sqrt{3}}(D + d) + 2D + d\right]\delta \quad .. \quad .. \quad (26a)$$

Simplifying and using (24*a*)

$$C = \frac{1}{2}\pi D\rho\delta\left[\left(3 + \frac{2}{\sqrt{3}}\right)D + 2\left(2 + \frac{1}{\sqrt{3}}\right)d\right] \quad (26b)$$

When a fraction f of D remains (the cross-section shown in Fig. 10), the mass of the grain is given by

$$C(1 - z) = \left[\pi\left\{\frac{1}{\sqrt{3}}(D + d) + D + \frac{d}{2} - \frac{D(1-f)}{2}\right\}^2 - 3\pi\left\{\frac{d}{2} + \frac{D(1-f)}{2}\right\}^2\right.$$

$$\left. - \pi\left(\frac{1}{\sqrt{3}} - \frac{1}{6}\right)(D - d)^2\right]\left[\rho\left\{\frac{2}{\sqrt{3}}(D + d) + 2D + d\right\}\right.$$

$$\left. - D(1-f)\right]\delta \quad .. \quad .. \quad (27a)$$

Simplifying and using 24(a)

$$C(1-z) = \frac{1}{2}\pi D f \delta \left[\left\{\left(3+\frac{2}{\sqrt{3}}\right)D + 2\left(2+\frac{1}{\sqrt{3}}\right)d\right\} + D(1+f)\right] \times \left[\rho m d - D(1-f)\right] \quad \ldots \quad (27b)$$

On dividing (27b) by 27(a) and using (24b)

$$(1-z) = f\left[1 + \frac{3(m-1)-2\sqrt{3}}{(3\sqrt{3}+2)\{(m-1)\sqrt{3}-2\}+4(2\sqrt{3}+1)(\sqrt{3}+1)}(1-f)\right] \times \left[1 - \frac{(m-1)\sqrt{3}-2}{2(\sqrt{3}+1)\ m\rho}(1-f)\right] \quad \ldots \quad (28a)$$

Hence, on simplification we shall obtain

$$z = (1-f)\left[1 - \frac{\{(m-1)\sqrt{3}-2\}\{(\sqrt{3}+1)m\rho-(2\sqrt{3}+1)(m+1)\}}{(\sqrt{3}+1)\{(3m+5)\sqrt{3}+2(m+2)\}m\rho} f - \frac{\{(m-1)\sqrt{3}-2\}^2}{2(\sqrt{3}+1)\{(3m+5)\sqrt{3}+2(m+2)\}m\rho} f^2\right] \quad (28b)$$

Which is the (z, f) relation of the form

$$z = (1-f)(1-Af-Bf^2) \quad \ldots \quad (28c)$$

where

$$A = \frac{\{(m-1)\sqrt{3}-2\}\{(\sqrt{3}+1)m\rho-(2\sqrt{3}+1)(m+1)\}}{(\sqrt{3}+1)\{(3m+5)\sqrt{3}+2(m+2)\}m\rho} \quad (29a)$$

and

$$B = \frac{\{(m-1)\sqrt{3}-2\}^2}{2(\sqrt{3}+1)\{(3m+5)\sqrt{3}+2(m+2)\}m\rho} \quad (29b)$$

The (z, S) relation is ea-ily obtained as for the helptatubular charge from (28c) and the equation

$$\frac{S}{S_0} = \frac{(1+A)-2(A-B)f-3Bf^2}{1-(A+B)} \quad \ldots \quad (30)$$

The equivalent form-factor θ, using the principles of least squares is given by

$$\theta = -\left(A + \frac{1}{2}B\right) \quad \ldots \quad (31a)$$

On Substituting for A and B and simplifying

$$\theta = -\frac{\{(m-1)\sqrt{3}-2\}\{4(\sqrt{3}+1)m\rho-(7\sqrt{3}+4)m-3(3\sqrt{3}+2)\}}{4(\sqrt{3}+1)\{(3m+5)\sqrt{3}+2(m+2)\}m\rho} \quad (31b)$$

Which will be negative if

$$\rho > \frac{(7\sqrt{3} + 4)\, m + 3\,(3\sqrt{3} + 2)}{4(\sqrt{3} + 1)m} \qquad \ldots \qquad \ldots \qquad (32)$$

Also the sum of the squares of the residues, for fitting by using the above principle is

$$\frac{1}{840} \; B^2 \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (33)$$

The values of A, B, the form-factor θ and the sum of the squares of the residues, for the same set of values of m and ρ as for the hepta-tubular charge are exihibited in table 6 below :

TABLE 6

m \ ρ	A	B	θ	Sum of the squares of the residues.
4, $\frac{1}{2}$	—0·237,89	0·022,56	+0·226,61	0·000,000,6056
4, 1	—0·080,39	0·011,28	+0·074,75	0·000,000,1514
4, $\frac{9}{4}$	+0·007,11	0 005,01	—0·009,62	0·000,000,0299
7, $\frac{9}{4}$	+0·022,64	0·012,98	—0·029,13	0,000,000,2007
10, $\frac{9}{4}$	+0·025,66	0·014,10	—0·032,71	0·000,000,2366
∞, $\frac{9}{4}$	+0·065,90	0·033,91	—0·082,85	+0·000,001,3689
∞, ∞	+0·240,69	0	—0·240,69	0

Some properties of the Form—Function for the Modified Tri-tubular Charge

(*a*) *Relations between z, f and $S/S_\circ$ for particular shapes:*

The values of z and $\dfrac{S}{S_\circ}$ for the same range of variation of f as for the hepta-tubular charge for the above set of values of m and ρ are shown in the following table :

TABLE 7

$m=4$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=\frac{1}{2}$	z	0	0·12	9·24	0·35	0·45	0·56	0·65	0·75	0·84	0·92	1·0
	S/S_o	1·0	0·96	0·93	0·90	0·86	0·83	0·79	0·75	0·71	0·67	0·63
$m=4$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=1$	z	0	0·11	0·21	0·32	0·42	0·52	0·62	0·72	0·81	0·91	1·0
	S/S_o	1·0	0·99	0·98	0·97	0·95	0·94	0·92	0·91	0·89	0·88	0·86
$m=4$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=9/4$	x	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
	S/S_o	1·0	1·0	1·01	0·10	1·10	1·02	1·02	1·02	1·02	1·02	1·02
$m=z$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=9/4$	z	0	0·1	0·19	0·29	0·39	0·49	0·59	0·69	0·8	0·9	1·0
	S/S_o	1	1·01	1·02	1·03	1·04	1·05	1·05	1·05	1·06	1·06	1·06
$m=10$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=9/4$	z	0	0·1	0·19	0·29	0·39	0·49	0·59	0·69	0·8	0·9	1·0
	S/S_o	1·0	1·01	1·02	1·03	1·04	1·05	1·05	1·06	1·06	1·07	1·07
	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$m=\infty$	z	0	0·09	0·19	0·28	0·38	0·48	0·58	0·68	0·79	0·89	1·0
$\rho=9/4$	S/S_o	1·0	1·03	1·06	1·08	1·10	1·12	1·14	1·15	1·17	1·18	1·18
$m=\infty$	f	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0
$\rho=\infty$	z	0	0·08	0·16	0·25	0·34	0·44	0·54	0·65	0·76	0·88	1·0
	S/S_o	1·0	1·06	1·13	1·19	1·25	1·32	1·38	1·44	1·51	1·57	1·63

The results are illustrated in the figures (13), (14) and (15).

(b) Modified tri-tubular charge with a given value of equivalent form-factor θ

From (31*b*).

$$\frac{1}{\rho}=\frac{4(\sqrt{3}+1)\left[\left\{(3\sqrt{3}+2)\,\theta+\sqrt{3}\right\}+\frac{1}{m}\left\{(5\sqrt{3}+4)\,\theta-(\sqrt{3}+2)\right\}\right]}{\left\{\sqrt{3}-(\sqrt{3}+2)\frac{1}{m}\right\}\left\{(7\sqrt{3}+4)+\frac{3}{m}(3\sqrt{3}+2)\right\}} \quad \ldots \quad \ldots \quad (34)$$

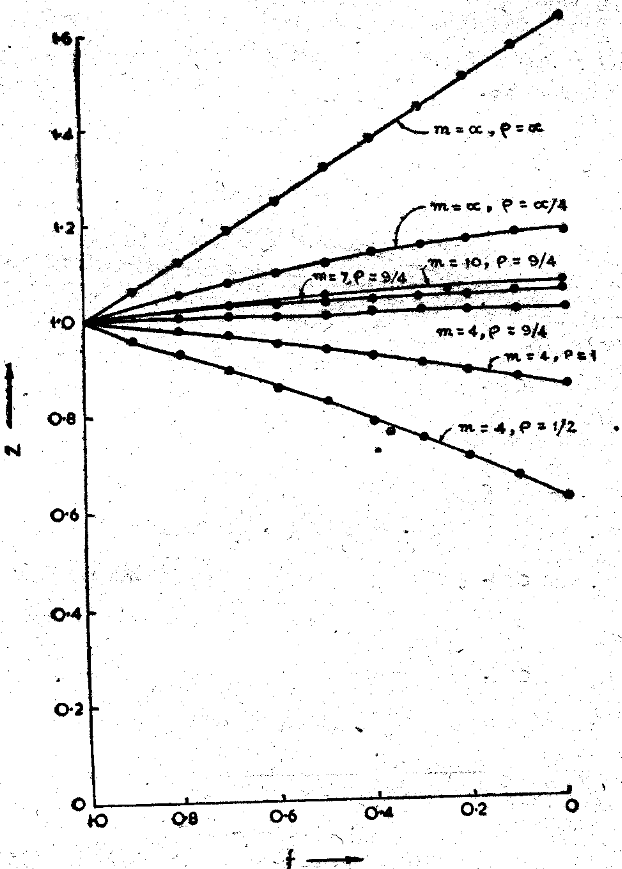


FIG. 13

We have calculated and collected the values of $\frac{1}{\rho}$ for the cases when the form-factor θ takes the values +0·1, 0, —0·1, and —0·2, for the same values of $\frac{1}{m}$ as for the hepta-tubular charge.

TABLE 8

$\frac{1}{m}$	θ			
	0·1	0	—0·1	—0·2
0·04	0·959,38	0·677,77	0·396,16	0·144,58
0·05	0·95,01	0·635,22	0·313,41	..
0·10	0·970,02	0·597,74	0·225,50	..
0·15	1·0023	0·564,40	0·126,49	..
0·20	1·0623	0·534,61	0·006,95	..
0·25	1·1663	0·507,80	..	..
0·30	1.3516	0·483,54	..	..

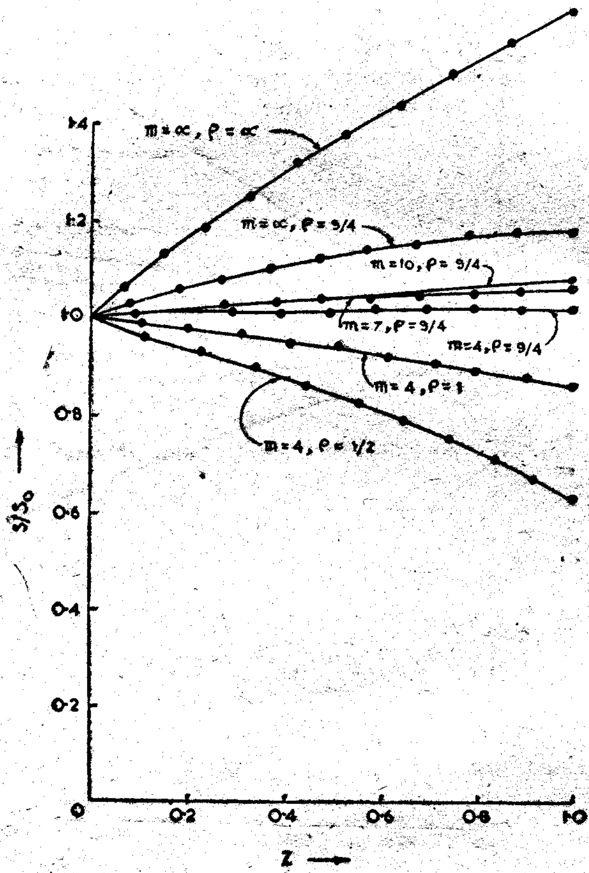

m = ∞, ρ = ∞
m = ∞, ρ = 9/4
m = 10, ρ = 9/4
m = 7, ρ = 9/4
m = 4, ρ = 9/4
m = 4, ρ = 1
m = 4, ρ = 1/2
s/s₀
z
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4

FIG. 14

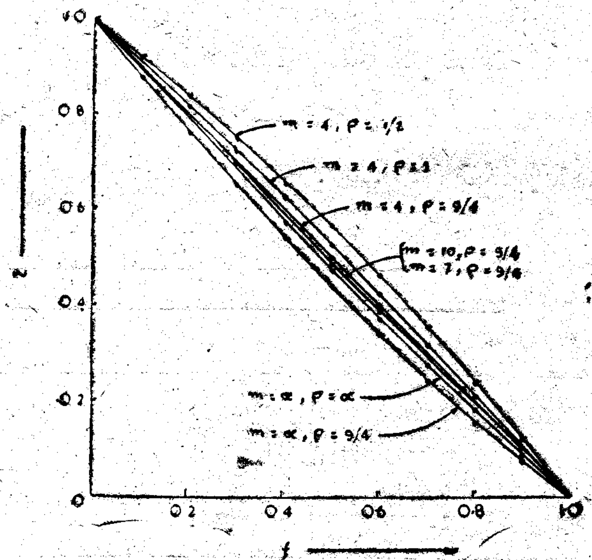

m = 4, ρ = 1/2
m = 4, ρ = 1
m = 4, ρ = 9/4
m = 10, ρ = 9/4
m = 7, ρ = 9/4
m = ∞, ρ = ∞
m = ∞, ρ = 9/4
z
f
0
0·2
0·4
0·6
0·8
1·0

FIG. 15

The table 8 or formula (34) would enable us to determine the value of ρ for which a charge with given value of m will behave as a charge with a given value of form-factor θ. Of course θ cannot be given any arbitrary value. In fact

$$\theta \geqslant - \frac{\sqrt{3}}{3\sqrt{3}+2} \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

and the limits can be obtained when both m and ρ are infinite

When ρ is infinite, (34) gives

$$\frac{1}{m} = \frac{\sqrt{3}+(3\sqrt{3}+2)\,\theta}{(2+\sqrt{3})-(5\sqrt{3}+4)\,\theta} \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

The values of $1/m$ for the above values of θ for which $1/\rho$ becomes zero are shown in table 9 below :—

TABLE 9

θ	0·1	0	—0·1	—0·2
$\frac{1}{m}$	0·994	0·464	0·203	0·047

The results of the above tables are illustrated in figure 16.

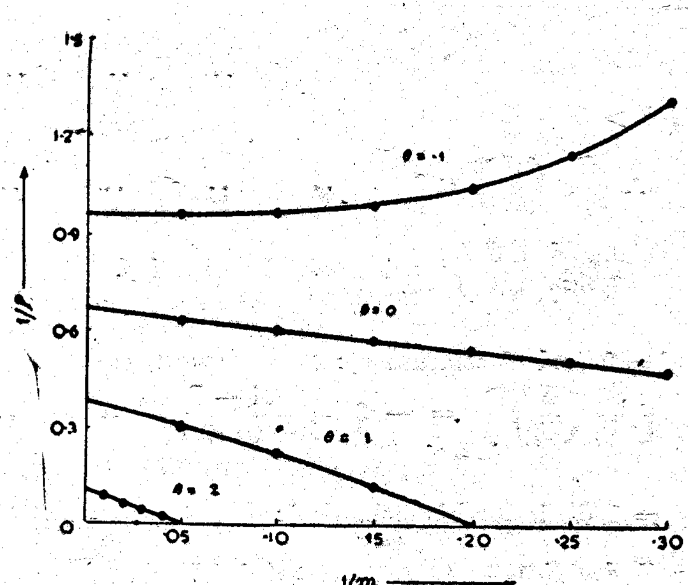


FIG. 16

(c) *Progressive and Degressive nature of the Burning Surface*

From (30)

$$\frac{S}{S_\circ} = \frac{(1 + A) - 2(A - B)f - 3Bf^2}{1 - (A + B)} \quad \cdots \quad (37a)$$

$$\therefore \frac{d}{df}\left(\frac{S}{S_\circ}\right) = -2 \;\frac{(A - B) + 3Bf}{1 - (A + B)} \quad \cdots \quad \cdots \quad (37b)$$

and

$$\frac{d^2}{df^2}\left(\frac{S}{S_\circ}\right) = -6 \;\frac{B}{1 - (A + B)} \quad \cdots \quad \cdots \quad (37c)$$

Using (29*a*) and 29(*b*)

$$1 - (A + B) = \frac{1}{2(\sqrt{3} + 1)\{3\sqrt{3} + 2)m + (5\sqrt{3} + 4)\}m\rho} \times$$

$$[\, 2(\sqrt{3} + 1)\{3\sqrt{3} + 2)m + (5\sqrt{3} + 4)\}m\rho - 2\{(m - 1)\sqrt{3} - 2\}$$

$$\{(\sqrt{3} + 1)m\rho - (2\sqrt{3} + 1)(m + 1) - \{(m - 1)\sqrt{3} - 2\}^2\,] \quad \cdots \quad (38a)$$

On simplifying

$$1-(A+B) = \frac{8(2 + \sqrt{3})(m + 3)m\rho + \{(m - 1)\sqrt{3} - 2\}\;\{(3\sqrt{3} + 2)m + (5\sqrt{3} + 4)\}}{2(\sqrt{3} + 1)\{(3\sqrt{3} + 2)m + (5\sqrt{3} + 4)\}m\rho}$$

$$> 0 \quad \cdots \quad \cdots \quad \cdots \quad (38b)$$

also B is > 0

$$\therefore \frac{d^2}{df^2}\left(\frac{S}{S_\circ}\right) < 0 \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (39)$$

which shows that $\frac{d}{df}\left(\frac{S}{S_\circ}\right)$ is a decreasing function of f.

Now for $f = 0$ (end of combustion):

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=0} = -2 \;\frac{A - B}{1 - (A + B)} \quad \cdots \quad \cdots \quad (40)$$

also for $f = 1$ (beginning of combustion):

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=1} = -2 \;\frac{A + 2B}{1 - (A + B)} \quad \cdots \quad (41)$$

Substituting for A and B from (29a) and 29 b) and simplifying

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=0} =$$

$$\frac{2\{(m-1)\sqrt{3}-2\}\{(5\sqrt{3}+2)m+3\sqrt{3}-2(\sqrt{3}+1)m\rho\}}{8(2+\sqrt{3})(m+3)m\rho+\{(m-1)\sqrt{3}-2\}\{(3m+5)\sqrt{3}+2(m+2)\}} \quad (42)$$

and

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=1} =$$

$$\frac{4(\sqrt{3}+1)\{(m-1)\sqrt{3}-2\}(m+3-m\rho)}{8(2+\sqrt{3})(m+3)m\rho+\{(m-1)\sqrt{3}-2\}\{(3m+5)\sqrt{3}+2(m+2)\}} \quad (43)$$

The values of $\frac{d}{df}\left(\frac{S}{S_\circ}\right)$ have opposite signs at $f = 0$ and at $f = 1$, and $\frac{S}{S_\circ}$ takes a maximum value in the interval of variation of f, if :

$$\left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=0} \times \left[\frac{d}{df}\left(\frac{S}{S_\circ}\right)\right]_{f=1} < 0 \quad \ldots \quad \ldots \quad (44a)$$

i.e. if

$$(m+3-m\rho)\left[(5\sqrt{3}+2)m+3\sqrt{3}-2(\sqrt{3}+1)m\rho\right] < 0 \ \ldots \quad (44b)$$

The first number of this inequality can be considered as an expression of second degree in ρ which breaks for :

$$\rho_1 = \frac{m+3}{m} \quad (45a)$$

and

$$\rho_2 = \frac{(5\sqrt{3}+2)m+3\sqrt{3}}{2(\sqrt{3}+1)m} \quad (45b)$$

ρ_1 and ρ_2 are decreasing functions of m which vary respectively from (2 to 1) and from $[(4-\sqrt{3})$ to $\frac{1}{4}(13-3\sqrt{3})]$ when m varies from $+3$ to $+\infty$ and are always greater than $\frac{(m-1)\sqrt{3}-2}{2(\sqrt{3}+1)m}$ the minimum value admissible for ρ. When we give to ρ a value lying between ρ_1 and ρ_2 the above inequality will be satisfied and the maximum of $\frac{S}{S_\circ}$ will appear for

$$f = -\frac{A-B}{3B} \quad (46a)$$

Using (29*a*) and (29*b*)

$$f = \frac{(5\sqrt{3}+2)m + 3\sqrt{3} - 2(\sqrt{3}+1)m\rho}{3\,[\,(m-1)\sqrt{3}-2\,]} \quad \cdots \qquad (46b)$$

Which can be written as

$$f = 1 - \frac{\rho - \rho_1}{3\rho_{min}} \quad \cdots \qquad (46c)$$

We thus see that

for $\frac{(m-1)\sqrt{3}-2}{2(\sqrt{3}+1)m} \leqslant \rho \leqslant \rho_1$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is always + ve.

for $\rho_1 < \rho \leqslant \rho_2$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is at first —ve for $1 \geqslant f > \frac{\rho_2 - \rho}{\rho_2 - \rho_1}$ and then

+ve for $\frac{\rho_2 - \rho}{\rho_2 - \rho_1} > f \geqslant 0$

for $\rho_2 < \rho$, $\frac{d}{df}\left(\frac{S}{S_o}\right)$ is always — ve.

Hence S/So is decreasing, increasing or at first increasing and then decreasing according as ρ is less than ρ_1, greater than ρ_2, or lies between ρ_1 and ρ_2. The results of table 5 apply in this case also though the values of ρ_1 and ρ_2 are different.

Shape of the Modified Tri-tubular Charge for the same Average Density as a given Unmodified Hepta-Tubular Charge.

In this section *d*, *D*, θ and *m* refer to the unmodified hepta-tubular charge, d', D', θ', and m' are the corresponding notations for the modified tri-tubular charge. We proceed to find a relation between *m* and *m′* such that the area of the four arcual and three circular holes for the modified tri-tubular charge equals the area of the seven circular holes of the unmodified hepta-tubular charge. This condition gives,

$$3\pi\left(\frac{d'}{2}\right)^2 + \pi\left(\frac{1}{\sqrt{3}} - \frac{1}{6}\right)(D'+d')^2 = 7\pi\left(\frac{d}{2}\right)^2. \qquad (47)$$

The diameter of the cylindrical tri-tubular charge is

$\left[\frac{2}{\sqrt{3}}(D'+d') + 2D' + d'\right]$ and since this equals $m'd'$, i.e.

$$\frac{2}{\sqrt{3}}(D'+d') + 2D' + d' = m'd' \qquad (48a)$$

We have

$$D' = \frac{(m' - 1)\sqrt{3} - 2}{2(\sqrt{3} + 1)} d' \qquad (48b)$$

Using (48b) and implifying, (47) gives:

$$6 + (\sqrt{3} - \tfrac{1}{2})(2 - \sqrt{3})(m' + 1)^2 = 14\left(\frac{d}{d'}\right)^2 \qquad (49)$$

But

$$\frac{d}{d'} = \frac{m'}{m} \qquad (50)$$

Simplifying (49) and using (50), we obtain

$$m^2 = \frac{7\, m'^2}{3 + \tfrac{1}{4}(5\sqrt{3} - 8)(m' + 1)^2} \qquad (51)$$

Writing (51) as a quadratic in m', we obtain

$$[28 - (5\sqrt{3} - 8)\, m^2]\, m'^2 - [2(5\sqrt{3} - 8)\, m^2]\, m' - (5\sqrt{3} + 4)m^2 = o \qquad (52)$$

The positive root of (52) gives us the value of m' for any given $m (m < 6{\cdot}5122)$. It has been thus found out that the minimum value of m' must be $2{\cdot}5581$, (which corresponds to $m = 3$) which is permissible since $m' > 1 + \frac{2}{\sqrt{3}} = 2{\cdot}1547$. We have calculated and collected the values of m' for some typical values of m in the table 10 below.

TABLE 10

m'	2·5581	3	4	5	6	7	8	9	10	∞
m	3	3·3419	3·9643	4·4238	4·7673	5·0287	5·2314	5·3914	5·5201	6·5122

These results are illustrated in Fig. 17.

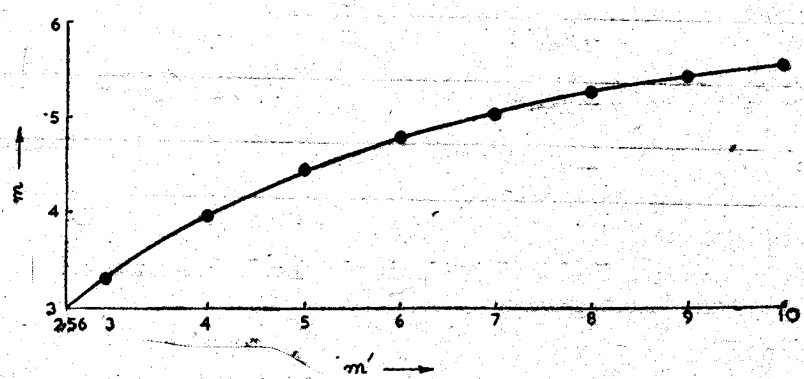


FIG. 17

TABLE 11

$m = 4, m' = 4 \cdot 0677$

ρ	½	1	9/4	5	20	∞
θ	1·0408	0·8675	0·7711	0·7178	0·7031	0·6943
θ'	+0·2309	+0·0759	—0·0101	—0·0480	—0·0713	—0·0790

Table 11 shows the great improvement in the value of θ by using the modified tritubular charge instead of the unmodified hepta-tubular charge with the same average density.

It may be noted that if $m>6$, the holes in the modified tritubular charge would become very small and therefore it would be possible to insist on the same average density only if $m < 6$.

For the same size of the circular holes, same external diameter and same length, the improvements in using a modified tritubular charge over the unmodified heptatubular charge are given in the following table:

TABLE 12

m	ρ	For Modified Tritubular Charge	For unmodified heptatubular charge
4	½	+226,61	+1·0408
4	1	+·074,75	+0·9675
4	9/4	—·009,62	+0·7711
7	9/4	—·029,13	+0·2477
10	9/4	—·032,71	+0·1172
0	9/4	—·082,85	—0·1720
∞	∞	—·240,69	—0·3228

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Reference

1. Tavernier, P., *Memorial de L' Artillerie* Francais, Tom 30, Fasi. pp. 117-165, 1956.