A CONDITION FOR ACHROMATISM OF THREE LENSES

by

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ABSTRACT

In this paper is given an account of an attempt at obtaining a general condition for the constancy of power for all wave lenghts, in other words, total achromatism of a system of three separated lenses, in terms of Hartmann constants. The solution of the general equation by making the Hartmann constants a_1, a_2 and a_3, equal to unity, is naturally the simplest and the condition of achromatism arrived at is shown to be that of three lenses in contact.

Introduction

Sodha and Nigam[1] in their earlier work described the condition for total achromatism for two separated lenses. In this paper general condition for total achromatism for a system of three lenses has been worked out.

Condition for Total Achromatism

The combined power of a system of three separated thin lenses is given by—

$$\begin{aligned}\Phi = {} & A_1(N_1-1) + A_2(N_2-1) + A_3(N_3-1) - d_1A_1A_2(N_1-1)(N_2-1) \\ & -(d_1+d_2)A_1A_3(N_1-1)(N_3-1) - d_2A_2A_3(N_2-1)(N_3-1) \\ & + d_1d_2A_1A_2A_3(N_1-1)(N_2-1)(N_3-1) \quad \ldots \quad (1)\end{aligned}$$

where, A_1, A_2, A_3, are the nett curvatures (given by the differences of reciprocals of radii of curvatures of surfaces) of the individual lenses; d_1, d_2 are the air separations between the lenses; and N_1, N_2, N_3 are the refractive indices of the first, second and third lenses respectively. The condition for total achromatism may be obtained by putting $d\Phi/d\lambda=0$ which gives,

$$\begin{aligned}d\Phi/d\lambda = {} & A_1\frac{dN_1}{d\lambda} + A_2\frac{dN_2}{d\lambda} + A_3\frac{dN_3}{d\lambda} - d_1A_1A_2(N_1-1)\frac{dN_2}{d\lambda} \\ & - d_1A_1A_2(N_2-1)\frac{dN_1}{d\lambda} - d_1A_1A_3(N_1-1)\frac{dN_3}{d\lambda} - d_1A_1A_3(N_3-1)\frac{dN_1}{d\lambda} \\ & - d_2A_1A_3(N_1-1)\frac{dN_3}{d\lambda} - d_2A_1A_3(N_3-1)\frac{dN_1}{d\lambda} - d_2A_2A_3(N_2-1)\frac{dN_3}{d\lambda} \\ & - d_2A_2A_3(N_3-1)\frac{dN_2}{d\lambda} + d_1d_2A_1A_2A_3(N_1-1)(N_2-1)\frac{dN_3}{d\lambda} \\ & + d_1d_2A_1A_2A_3(N_1-1)(N_3-1)\frac{dN_2}{d\lambda} + d_1d_2A_1A_2A_3(N_2-1)(N_3-1)\frac{dN_1}{d\lambda} \\ & = 0 \quad \ldots \quad (2)\end{aligned}$$

Using Hartmann dispersion formulae

$$\left.\begin{array}{l} N_1 = N_{01} + \dfrac{C_1}{(\lambda - \lambda_{01})^{a_1}} \quad ; \quad N_2 = N_{02} + \dfrac{C_2}{(\lambda - \lambda_{02})^{a_2}} \\ N_3 = N_{03} + \dfrac{C_3}{(\lambda - \lambda_{03})^{a_3}} \end{array}\right\} \quad (3)$$

and substituting for $dN_1/d\lambda$, $(N_1 - 1)$ etc. in equation (2) from (3) one gets the following general equation in λ,

$$K_1(\lambda - \lambda_{02})^{a_2+1} (\lambda - \lambda_{03})^{a_3+1} + K_2 (\lambda - \lambda_{01})^{a_1+1} (\lambda - \lambda_{03})^{a_3+1}$$
$$+ K_3 (\lambda - \lambda_{01})^{a_1+1} (\lambda - \lambda_{02})^{a_2+1} + L_1 (\lambda - \lambda_{03})^{a_3+1} \left\{ 2\lambda - (\lambda_{01} + \lambda_{02}) \right\}$$
$$+ L_2 (\lambda - \lambda_{02})^{a_2+1} \left\{ 2\lambda - (\lambda_{01} + \lambda_{03}) \right\} + L_3(\lambda - \lambda_{01})^{a_1+1} \left\{ 2\lambda - (\lambda_{02} + \lambda_{03}) \right\}$$
$$+ M_1 \left\{ a_1(\lambda - \lambda_{02})(\lambda - \lambda_{03}) + a_2(\lambda - \lambda_{03})(\lambda - \lambda_{01}) + a_3(\lambda - \lambda_{01})(\lambda - \lambda_{02}) \right\} = O \quad (4)$$

Where,

$$\left.\begin{array}{l} K_1 = A_1 a_1 c_1 \left\{ 1 - d_1 A_2 (N_{02} - 1) - d_1 A_3 (N_{03} - 1) - d_2 A_3 (N_{03} - 1) \right. \\ \qquad \left. + d_1 d_2 A_2 A_3 (N_{02} - 1)(N_{03} - 1) \right\} \\ K_2 = A_2 a_2 c_2 \left\{ 1 - d_1 A_1 (N_{01} - 1) - d_2 A_3 (N_{03} - 1) \right. \\ \qquad \left. + d_1 d_2 A_1 A_3 (N_{01} - 1)(N_{03} - 1) \right\} \\ K_3 = A_3 a_3 c_3 \left\{ 1 - d_1 A_1 (N_{01} - 1) - d_2 A_1 (N_{01} - 1) - d_2 A_2 (N_{02} - 1) \right. \\ \qquad \left. + d_1 d_2 A_1 A_2 (N_{01} - 1)(N_{02} - 1) \right\} \\ L_1 = A_1 A_2 c_1 c_2 d_1 \left\{ d_2 A_3 (N_{03} - 1) - 1 \right\} \\ L_2 = A_1 A_3 c_1 c_3 \left\{ d_1 d_2 A_2 (N_{02} - 1) - (d_1 + d_2) \right\} \\ L_3 = A_2 A_3 c_2 c_3 d_2 \left\{ d_1 A_1 (N_{01} - 1) - 1 \right\} \\ M_1 = A_1 A_2 A_3 c_1 c_2 c_3 d_1 d_2 \end{array}\right\} (5)$$

Putting $a_1 = a_2 = a_3 = 1$ in equation (4) and expanding, one gets a fourth power equation in λ, which is the condition for achromatism and which can be satisfied for all values of λ only when the coefficients of powers of λ are each equal to zero.

Thus, equating the coefficients of the powers of λ and the constant term to zero one gets the following five conditions—

$$\left.\begin{array}{l}
1.\quad K_1 + K_2 + K_3 = 0 \\[6pt]
2.\quad K_1(\lambda_{03}+\lambda_{02}) + K_2(\lambda_{03}+\lambda_{01}) + K_3(\lambda_{02}+\lambda_{01}) - (L_1+L_2+L_3) = 0 \\[6pt]
3.\quad \left\{ K_1(\lambda_{02}^2+\lambda_{03}^2+4\lambda_{02}\lambda_{03}) + K_2(\lambda_{01}^2+\lambda_{03}^2+4\lambda_{01}\lambda_{03}) + K_3(\lambda_{01}^2+\lambda_{02}^2+4\lambda_{01}\lambda_{02}) \right. \\
\qquad \left. - L_1(\lambda_{01}+\lambda_{02}+4\lambda_{03}) - L_2(\lambda_{01}+\lambda_{03}+4\lambda_{02}) - L_3(\lambda_{02}+\lambda_{03}+4\lambda_{01}) + 3M_1 \right\} = 0 \\[6pt]
4.\quad \left\{ K_1(\lambda_{02}^2\lambda_{03}+\lambda_{03}^2\lambda_{02}) + K_2(\lambda_{01}^2\lambda_{03}+\lambda_{03}^2\lambda_{01}) + K_3(\lambda_{01}^2\lambda_{02}+\lambda_{02}^2\lambda_{01}) \right. \\
\qquad - L_1(\lambda_{03}^2+\lambda_{01}\lambda_{03}+\lambda_{02}\lambda_{03}) - L_2(\lambda_{02}^2+\lambda_{01}\lambda_{02}+\lambda_{03}\lambda_{02}) - L_3(\lambda_{01}^2+\lambda_{01}\lambda_{02}+\lambda_{01}\lambda_{03}) \\
\qquad \left. - M_1(\lambda_{01}+\lambda_{02}+\lambda_{03}) \right\} = 0 \\[6pt]
5.\quad \left\{ K_1(\lambda_{02}^2\lambda_{03}^2) + K_2(\lambda_{01}^2\lambda_{03}^2) + K_3(\lambda_{01}^2\lambda_{02}^2) - L_1\lambda_{03}^2(\lambda_{01}+\lambda_{02}) - L_2\lambda_{02}^2(\lambda_{01}+\lambda_{03}) \right. \\
\qquad \left. - L_3\lambda_{01}^2(\lambda_{02}+\lambda_{03}) + M_1(\lambda_{01}\lambda_{02}+\lambda_{01}\lambda_{03}+\lambda_{02}\lambda_{03}) \right\} = 0
\end{array}\right\} 6$$

Results

The above five conditions after a considerable reduction, yield the following results,

$$\left.\begin{array}{l}
\dfrac{1}{d_1} = A_1\left\{ N_{01} - 1 + \dfrac{C_1}{\lambda_{02}-\lambda_{01}} \right\} = A_1\left\{ N_{01} - 1 + \dfrac{C_1}{\lambda_{03}-\lambda_{01}} \right\} \\[12pt]
\dfrac{1}{d_2} = A_3\left\{ N_{03} - 1 + \dfrac{C_3}{\lambda_{02}-\lambda_{03}} \right\} = A_3\left\{ N_{03} - 1 + \dfrac{C_3}{\lambda_{01}-\lambda_{03}} \right\} \\[12pt]
\dfrac{1}{d_1} + \dfrac{1}{d_2} = A_2\left\{ N_{02} - 1 + \dfrac{C_2}{\lambda_{03}-\lambda_{02}} \right\} \\[12pt]
\qquad\qquad = A_2\left\{ N_{02} - 1 + \dfrac{C_2}{\lambda_{01}-\lambda_{02}} \right\}
\end{array}\right\} 7$$

From the above it can be seen that

$$\lambda_{01} = \lambda_{02} = \lambda_{03}$$

which make $d_1 = d_2 = O$. Thus a system of three lenses in contact with each other is the only possible solution. This result, may also be verified by initially starting with three lenses in contact and subsequently proving that $\lambda_{01} = \lambda_{02} = \lambda_{03}$. The result which has been derived on the basis that $a_1 = a_2 = a_3 = 1$, is in general agreement with the one obtained by Sodha and Nigam for two lenses in contact, in which case, however, corresponding a_1, a_2 were assumed to be non-integral.

The condition presented in this note is only part of a general condition for total achromatism of a system of three separated lenses. The analysis of the general solution of eq. (4) treating a_1, a_2 and a_3 as non-integral, is under progress and the condition of achromatism of three separated lenses expressed in terms of Hartmann constants, along with practical application, if any, to photographic triplet lenses and triplet eyepiece systems will be published later.

Acknowledgements

The author desires to record his thankfulness to Dr. C. S. Rao, Director I.R.D.E., Dehra Dun and to Shri K. Swaminathan, Senior Scientific Officer. I.R.D. E., for the encouragement given to him in the preparation of this paper,

Reference

1. Sodha, M. S., and Nigam, A.N., *Curr. Sci.*, **22**, 232, 1953.