OSCILLATORY AERODYNAMIC FORCES FOR A PITCHING AIRCRAFT

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ABSTRACT

Chordwise aerodynamic strip derivatives for the swept back wing having bending-torsion degrees of freedom as modified by the aircraft simple harmonic pitching oscillations have been calculated.

Introduction

Aerodynamic derivatives based on stream-wise and chordwise strip derivatives are often used for flutter calculations for a swept wing of a low aspect ratio. The former results are more suited to the wing having ribs oriented in the flight direction and the latter approach is preferred when the ribs are perpendicular to the span. As the aircraft speed is increasing, the necessity of including more degrees of freedom is becoming imperative.

In this paper, the effect on the wing bending-torsion chordwise aerodynamic strip derivatives due to the additional aircraft pitching degee of freedom is studied. Though similarity rules for such extension of results are sometimes quoted, yet one is liable to make mistakes, unless the entire calculations are carried out *ab initio.*

Kinetic Energy

For brevity let us stick to the symbols and notations used by Bisplinghoff[1] and others in their book 'Aerolasticity'. The $\overline{Y}$ – axis is along the wing span and swept at an angle Δ and the $\overline{X}$ — axis is perpendicular to it, origin being the point of intersection of the line of aircraft symmetry and the wing elastic axis (Fig I). The bending deformation of a section at $\overline{Y}$ is taken as $h\ (\bar{y},\ t)$ (positive downwards) and the twist about the elastic axis is $\alpha\ (\bar{y}, t)$ (positive nose-up). The aircraft executes simple harmonic pitching oscillations of frequency ω about a lateral axis at a distance p_o from the origin.

The upward displacement of points on the mean line may be written as

$$Z_a\ (\bar{x}, \bar{y}, t) = -h - \bar{a}\,\bar{x} + (p_o - \bar{x}\,Cos\ \Lambda - \bar{y}\,Sin\ \Lambda)\ ,$$

where ϕ is the pitching angle. Thus the kinetic energy of the whole wing is obtained as

$$\frac{1}{2}\left(M\dot{h}^2 + I_{\bar{\alpha}}\dot{\bar{\alpha}}^2 + I_{\phi}\dot{\phi}^2\right) + \left(\dot{h}\dot{\bar{\alpha}}S_{\bar{\alpha}} + \dot{h}\dot{\phi}S_{\phi} + \dot{\bar{\alpha}}\dot{\phi}P_{\bar{\alpha}\phi}\right)$$

where M, $I_{\bar{\alpha}}$ and $S_{\bar{\alpha}}$ have the usual meanings, and

I_ϕ = wing moment of inertia about the pitching axis

S_ϕ = wing static moment about the pitching axis

$P_{\bar{\alpha}\phi}$ = wing product of inertia about the elastic axis and pitching axis.

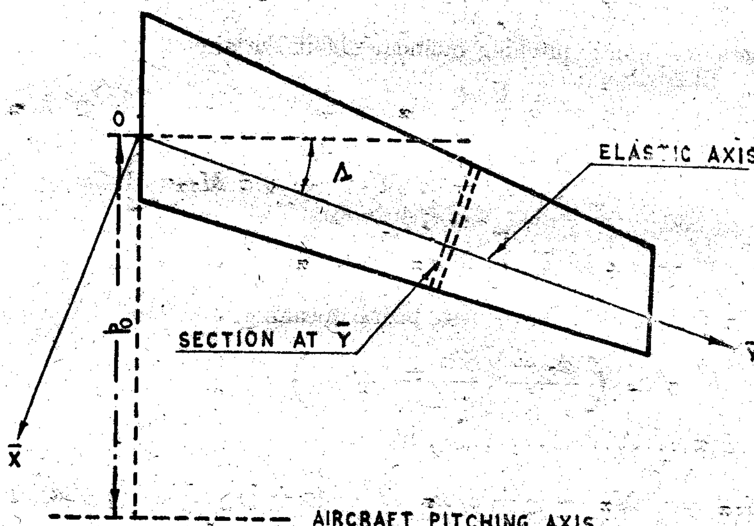

Fig. 1 Chordwise Section of the Swept Wing and Co-ordinate-axes

Aerodynamic Forces and Moments

The vertical velocity of fluid particles forced by the wing motion is

$$\omega_a(\bar{x}, \bar{y}, t) = \frac{\partial z_a}{\partial t} + U \cos\Delta \frac{\partial z_a}{\partial \bar{x}} + U \sin\Delta \frac{\partial z_a}{\partial \bar{y}}$$

$$= -\dot{h} - U\bar{\alpha}\cos\Delta - \dot{\bar{\alpha}}\bar{x} - U(\sigma + \tau\bar{x})\sin\Delta + (p_o - \bar{x}\cos\Delta - \bar{y}\sin\Delta)\dot{\phi} - U\phi.$$

where U is the free stream velocity and

$$\sigma = \frac{\partial h}{\partial \bar{y}}, \quad \tau = \frac{\partial \bar{\alpha}}{\partial \bar{y}}$$

Proceeding in the usual manner, repeating all the calculations *ab initio* the following expressions for the aerodynamic forces and moments are obtained.

The oscillatory aerodynamic wing lift is

$$\bar{L}\,(\bar{y}, t) = -\pi \rho\ \omega^2\, \bar{b}^3 \left\{ \frac{h}{\bar{b}}\ L_{hh} + \sigma\, L_{hh}' + \bar{\alpha}\ L_{h\alpha} + \bar{b}\, \tau\ L_{h\alpha}' \right\}$$

$$- \pi\, \rho\ \omega^2\, \bar{b}^3 \left\{ L_{h\alpha} + \tan \triangle\ .\ L_{hh'} \right\} \dot{\varphi}\ \mathrm{Cos}\ \triangle$$

The aerodynamic pitching moment about the elastic axis is

$$M_{\bar{y}}\ (\bar{y}, t) = \pi\, \rho\ \omega^2\ \bar{b}^4 \left\{ \frac{h}{\bar{b}} M_{\alpha h} + \sigma\, M_{\alpha h'} + \bar{\alpha}\ M_{\alpha\alpha} + \bar{b}\, \tau\ M_{\alpha\alpha'} \right\}$$

$$+ \pi\, \rho\ \omega^2\ \bar{b}^4 \left\{ M_{\alpha\alpha_0} - (\tfrac{1}{2} + a')\, M_{\alpha h} + \tan \triangle\, L_{\alpha h} \right\} \varphi\ \mathrm{Cos}\ \triangle$$

The aerodynamic pitching moment about the aircraft pitching axis is

$$\bar{M}_{\phi}\,(\bar{y}, t) = \pi\, \rho\ \omega^2\ \bar{b}^4 \Big[\frac{h}{\bar{b}}\ M_{\alpha h} + \bar{\alpha} \left\{ M'_{\alpha\alpha_0} - (\tfrac{1}{2} + a)\ M'_{\alpha h} \right\}$$

$$+ \ \sigma\ M'_{\alpha h'} + \bar{b}\ \tau\ M_{\alpha\alpha'1} \Big]\ \mathrm{Cos}\ \triangle$$

$$+ \pi\, \rho\, \omega^2\ \bar{b}^4 \left[M'_{\alpha\alpha} + M'_{\alpha h} \right] \phi\ \mathrm{Cos}^2\ \triangle$$

where $a, \bar{b}, \rho, \omega$ etc. have the usual meaning and

$$a' = \left(\frac{p_0 - \bar{y}\ \mathrm{Sin}\ \triangle}{\bar{b}\ \mathrm{Cos}\ \triangle} + a \right)$$

L_{hh}, $L_{h\alpha}$ ....etc. have their usual significance and

$$M_{\alpha\alpha} = \left\{ M_{\alpha} - (\tfrac{1}{2} + a)\, L_{\alpha} \right\} - (a + \tfrac{1}{2}) \left\{ M - (\tfrac{1}{2} + a)\ L_h \right\}$$

$$= M_{\alpha\alpha_0} - (\tfrac{1}{2} + a)\ M_{\alpha h}$$

$$M_{\alpha\bar{\alpha}1'} = - \frac{i \tan \triangle}{k} \left[\frac{3}{8} - \frac{i}{2k} - \frac{a}{2}(1 - L_h) + \frac{a'}{2}(1 - L_h) - L_h\ (\tfrac{1}{4} - a a') \right]$$

One dash on L_{hh}............ etc. signify that in the corresponding terms a is replaced by a'.

Conclusion

The paper fulfils the needs of extending swept wing bending torsion strip derivatives to the more generalised case when aircraft pitching degree of freedom is also included.

Acknowledgement

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Reference

1. Bisplinghoff, R. L., Ashley, H. and Halfman, R. L., *Aeroelasticity*, Addison-Wesley Publishing Co. 1955.